Photons (II)

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Ingredients for QIPC

- Quantum degrees of freedom
- Local operations
- Measurements
- One of
  - **Entangled state sources**
  - Universa 2qb unitaries
- Error correcting schemes
- Large number of qubits

Q. Communication

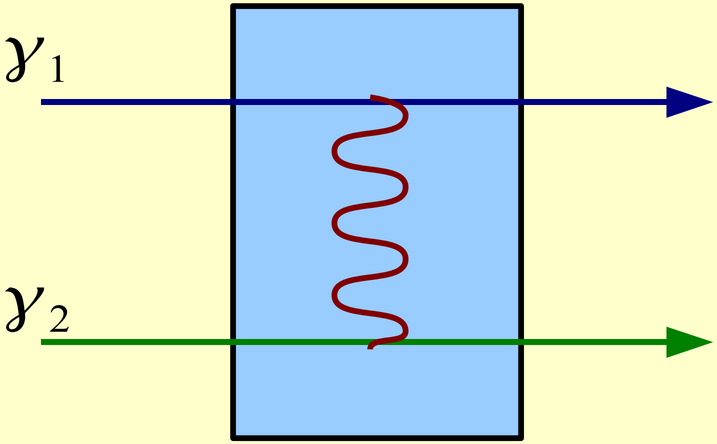
Q. Cryptography

Q. Simulation

Q. Computation

Entanglement

Kerr media



$$a_1 \rightarrow a_1 \exp(-i\kappa t n_2)$$

$$a_2 \rightarrow a_2 \exp(-i\kappa t n_1)$$

$$1\ photon \quad \rightarrow \quad \kappa t = 10^{-18}$$

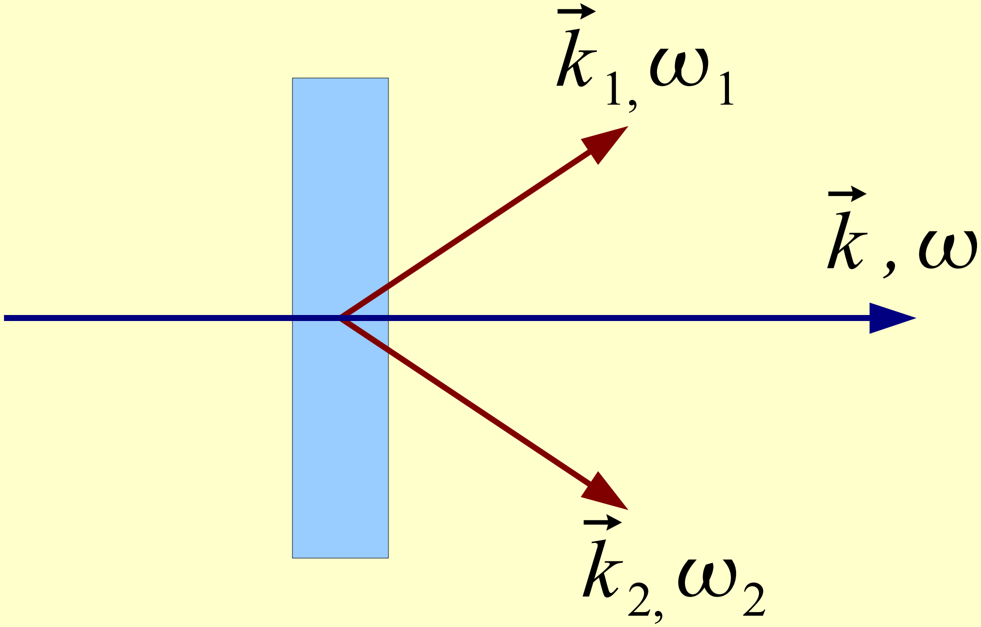
- To entangle, we need some interaction between photons.
- Light does not “interact”
- Interaction has to be mediated by some material.
- Nonlinear change of the polarization

$$P = \epsilon E + \xi E^2$$

may induce a nonlinear change of refraction index

$$H = \chi n_1 n_2$$

Parametric down-conversion

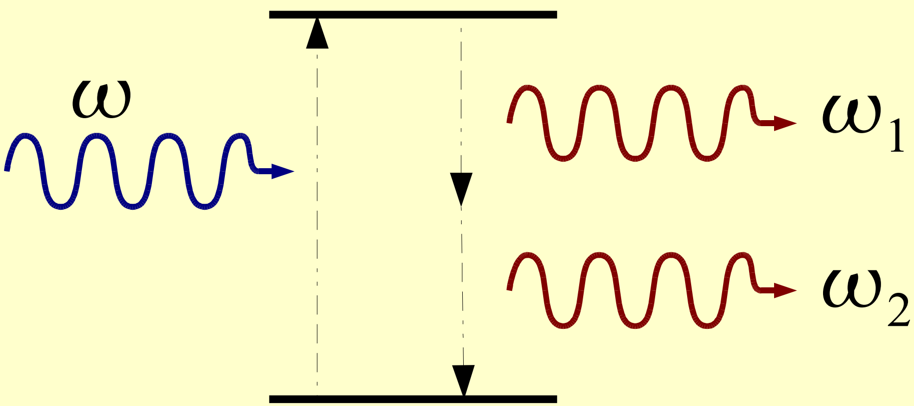


- Second order process in which the atom performs a forbidden transition.

- Conservation of energy and momentum

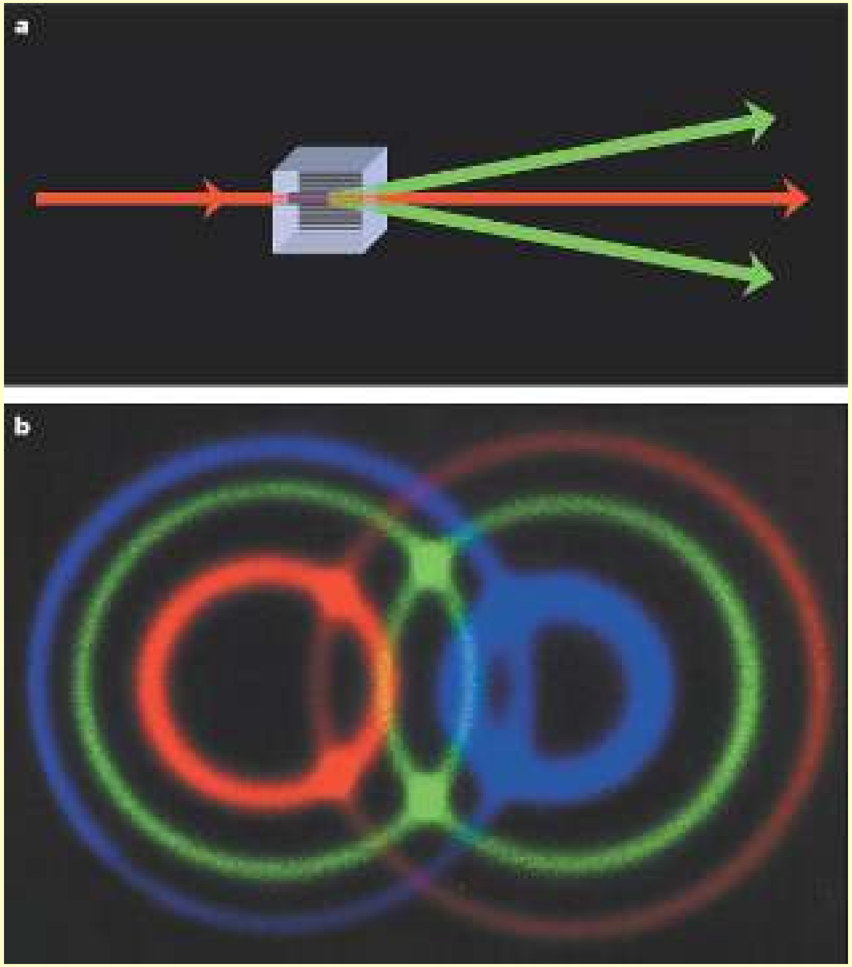
$$\vec{k} = \vec{k}_1 + \vec{k}_2$$

$$\omega = \omega_1 + \omega_2$$



- Need strong lasers to induce it: infrequent, low photon rate.

Parametric down-conversion



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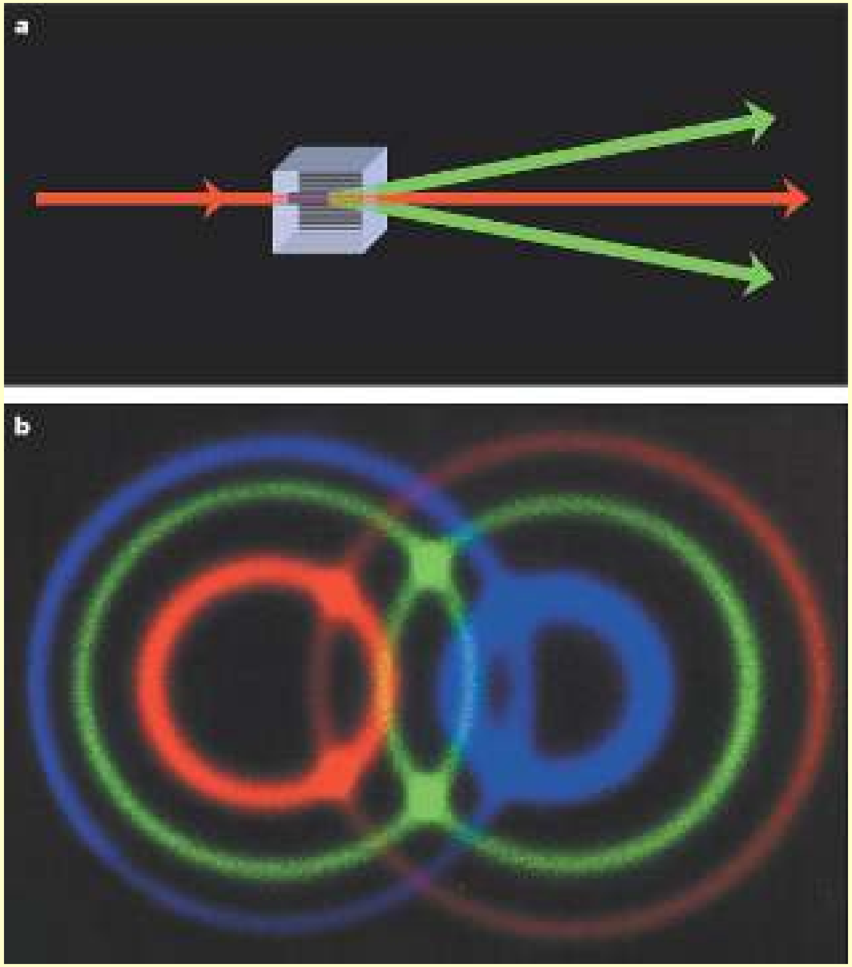

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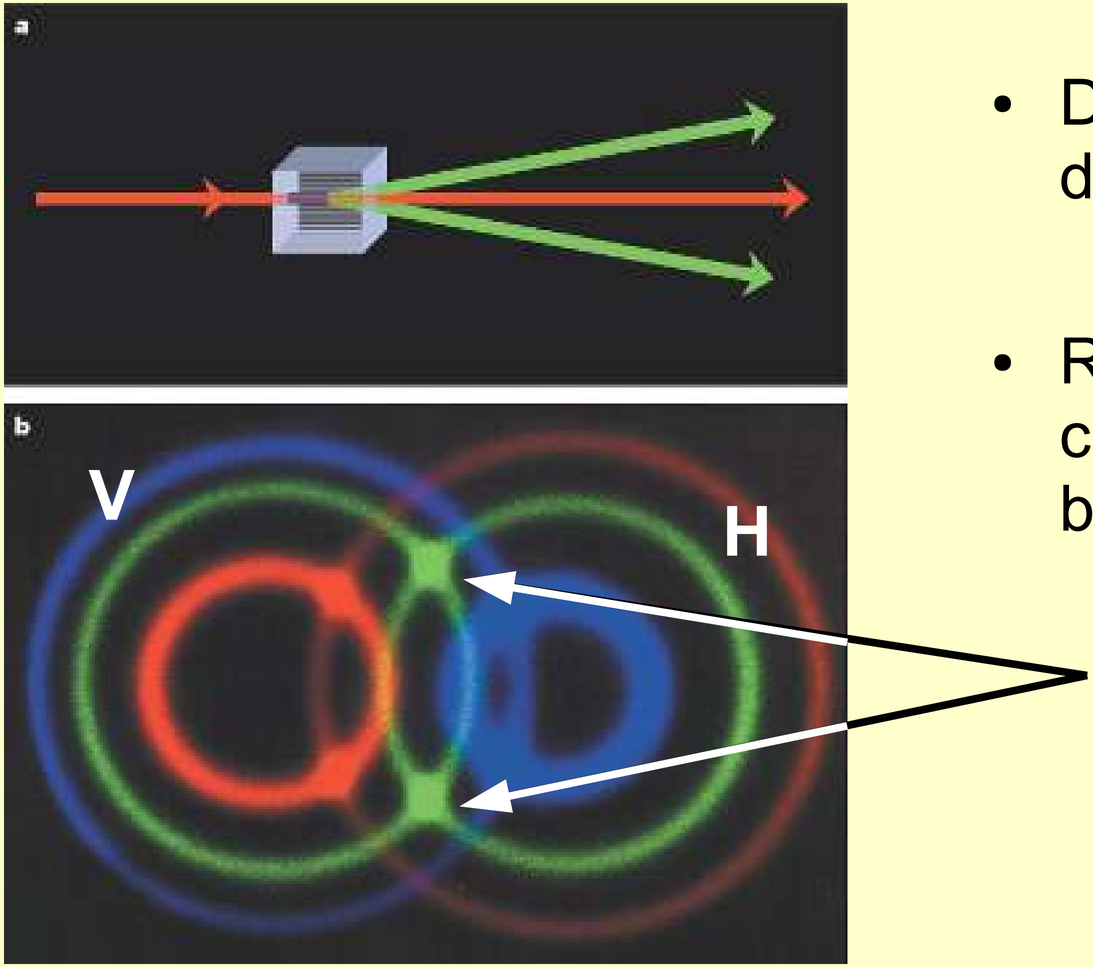


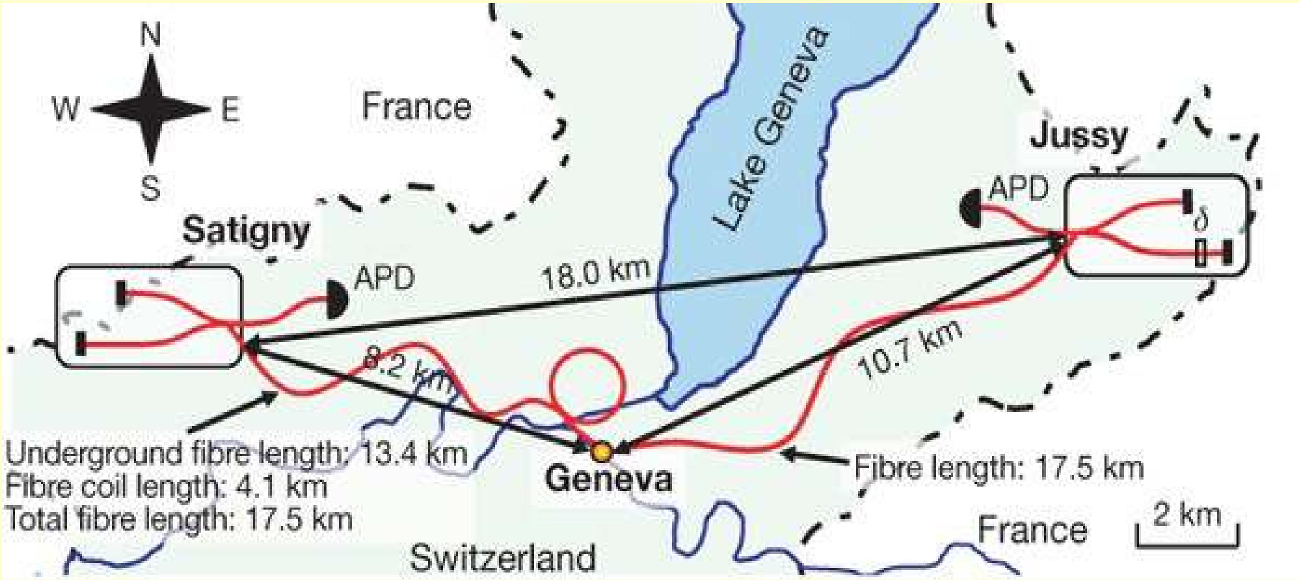
- Different polarizations along different outcoming beams

- Right on the crossing, coherent superposition of both choices:

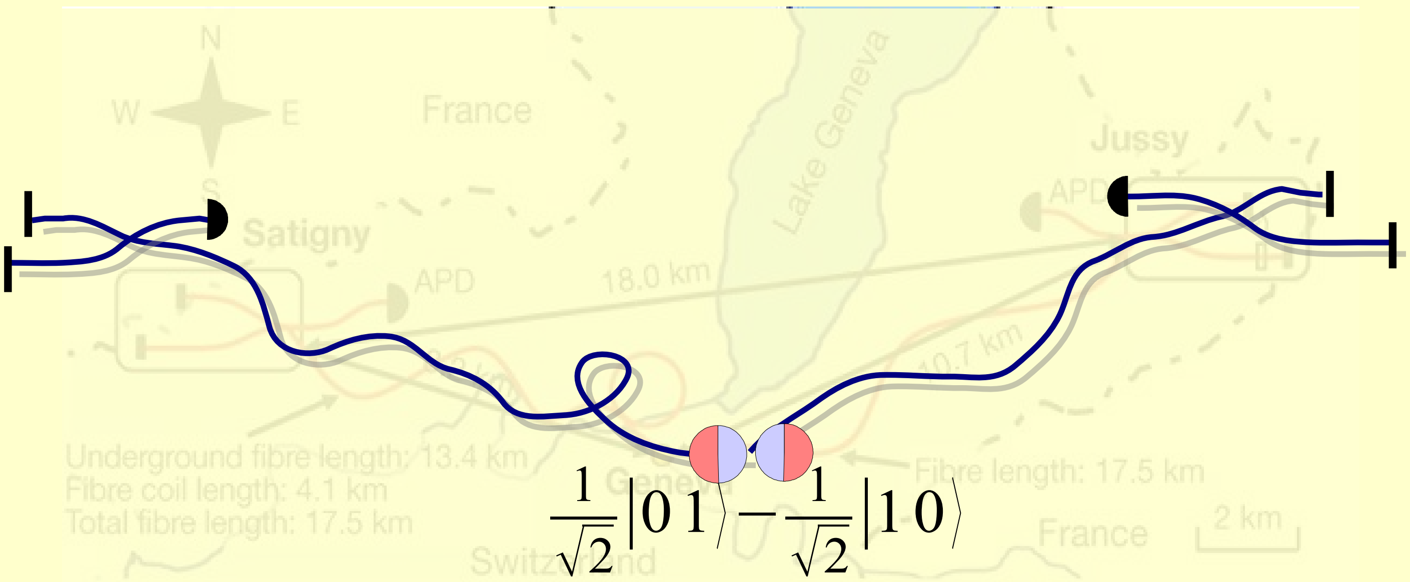
$$\frac{1}{\sqrt{2}}\left|H,V\right\rangle-\frac{1}{\sqrt{2}}\left|V,H\right\rangle$$

Application: EPR experiments

Bell tests of nonlocality



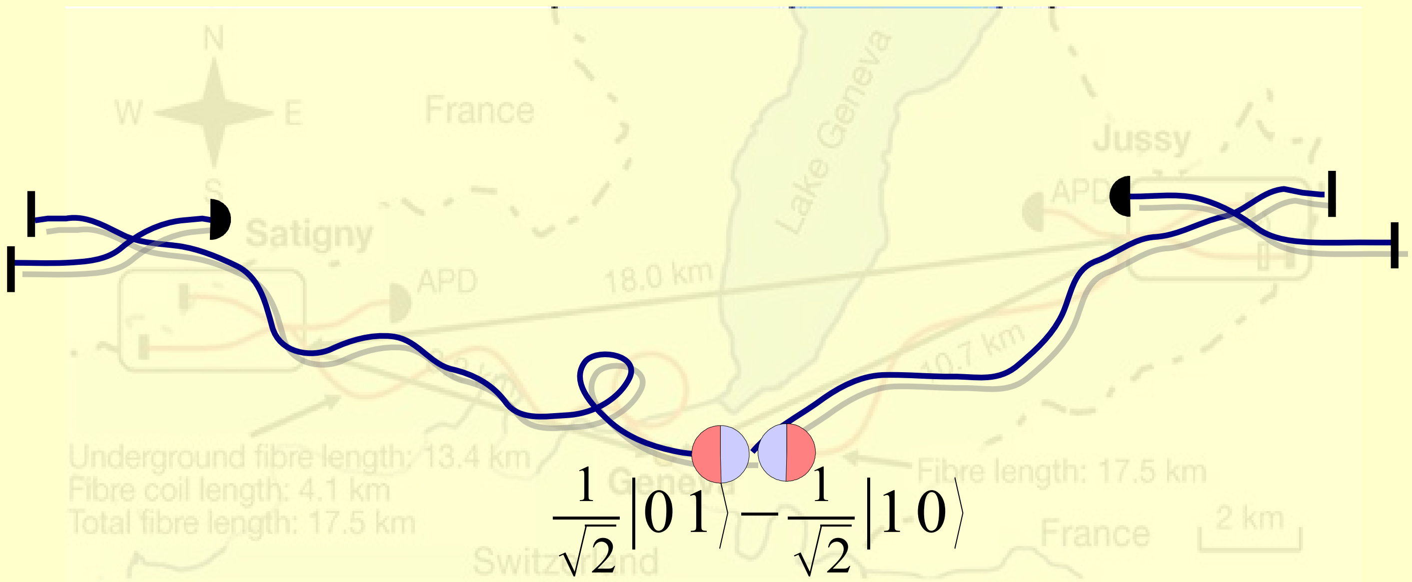
Bell tests of nonlocality



$$\frac{1}{\sqrt{2}}|0\,1\rangle - \frac{1}{\sqrt{2}}|1\,0\rangle$$

- Create an entangled Bell pair.

Bell tests of nonlocality



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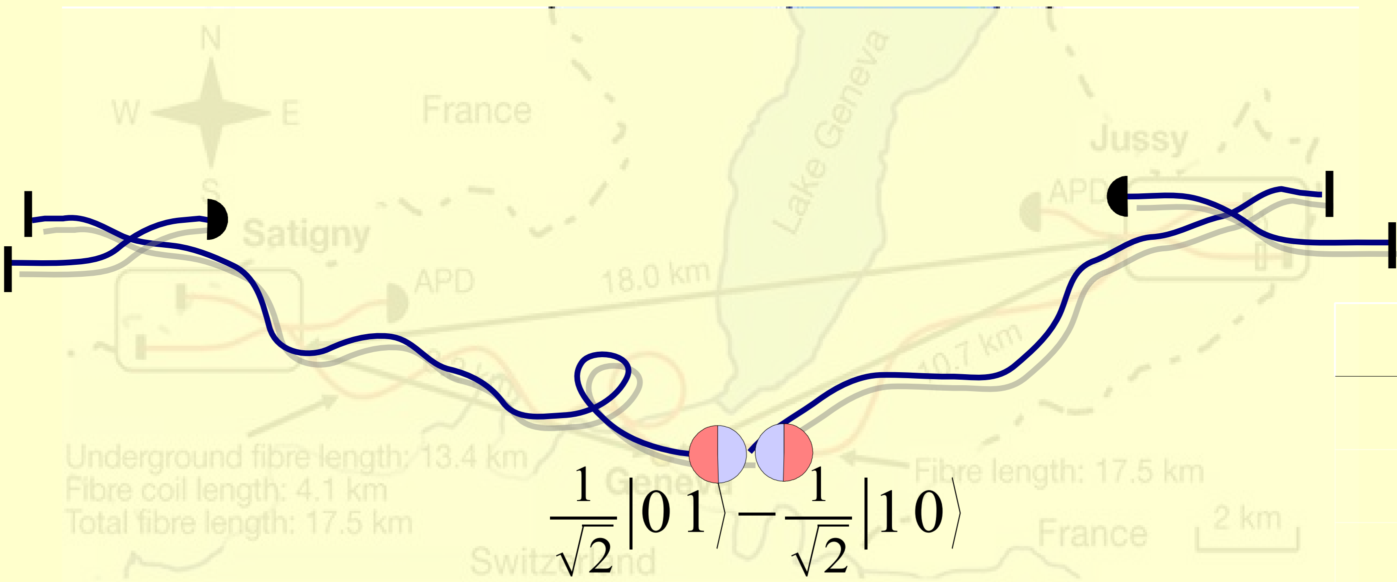
- Distribute the two parts and perform measurements on random basis, kilometers apart.

Bell tests of nonlocality



$$\frac{1}{\sqrt{2}}|0\,1\rangle - \frac{1}{\sqrt{2}}|1\,0\rangle$$

A	B
0	1
-	+
0	+
-	0
+	-

- Create an entangled Bell pair
- Distribute the two parts and perform measurements on random basis, kilometers apart.
- Measurements are more strongly correlated than any classical theory could allow.

Bell inequalities

Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[a] and Gérard Roger
Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France

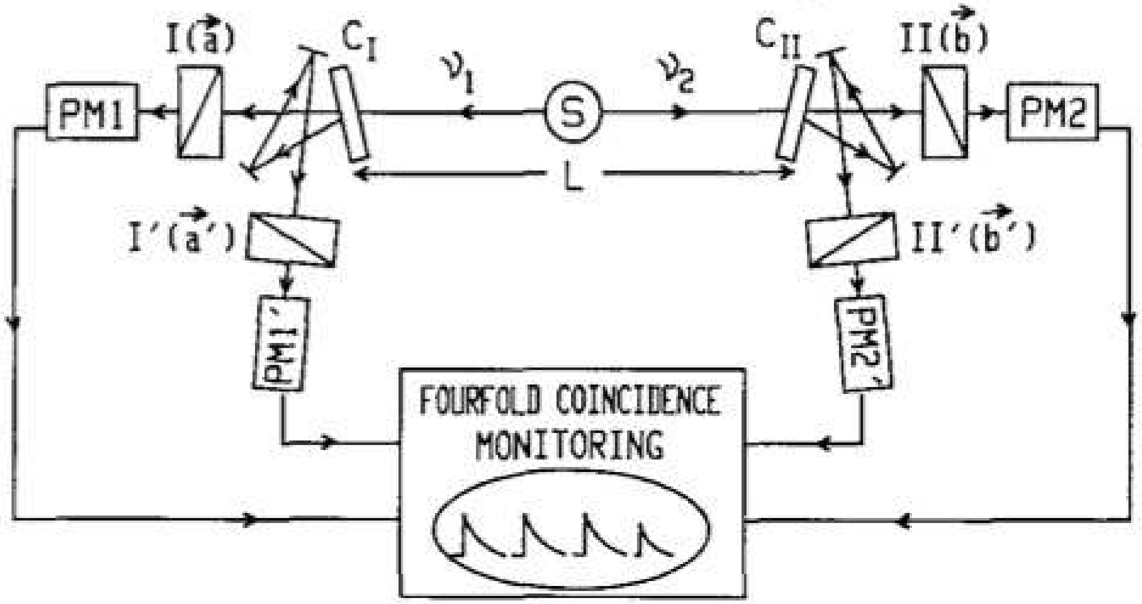
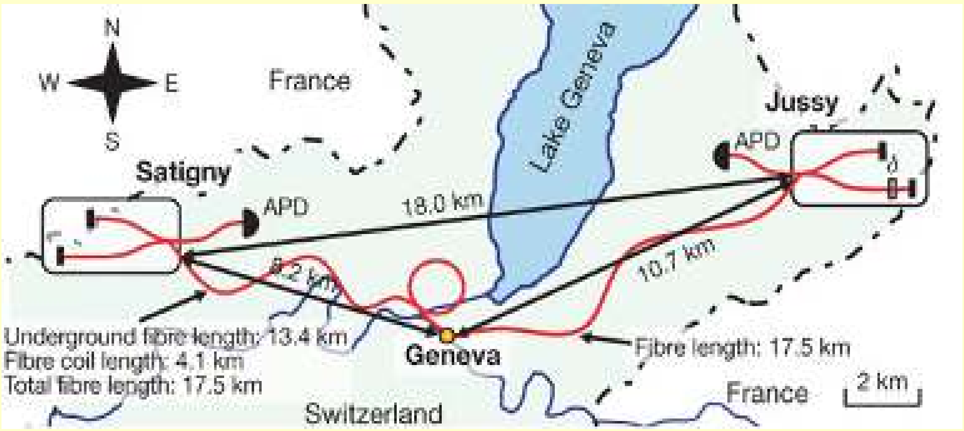
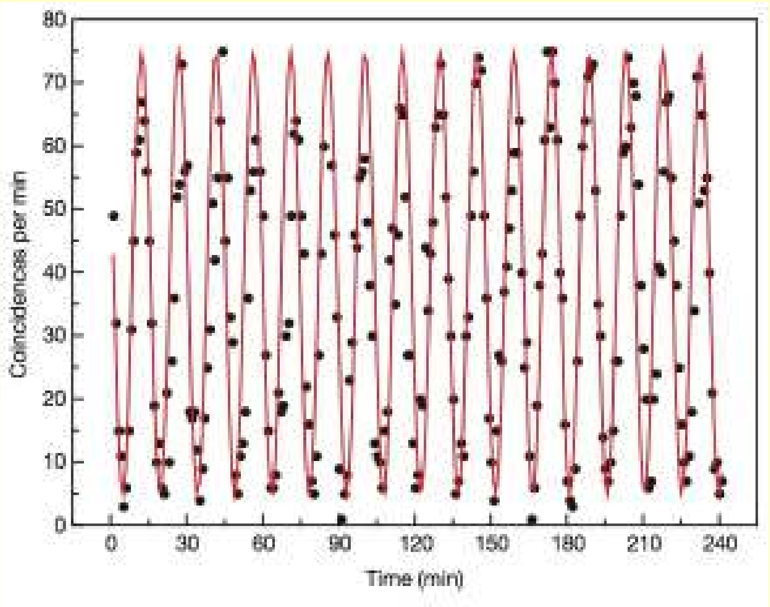
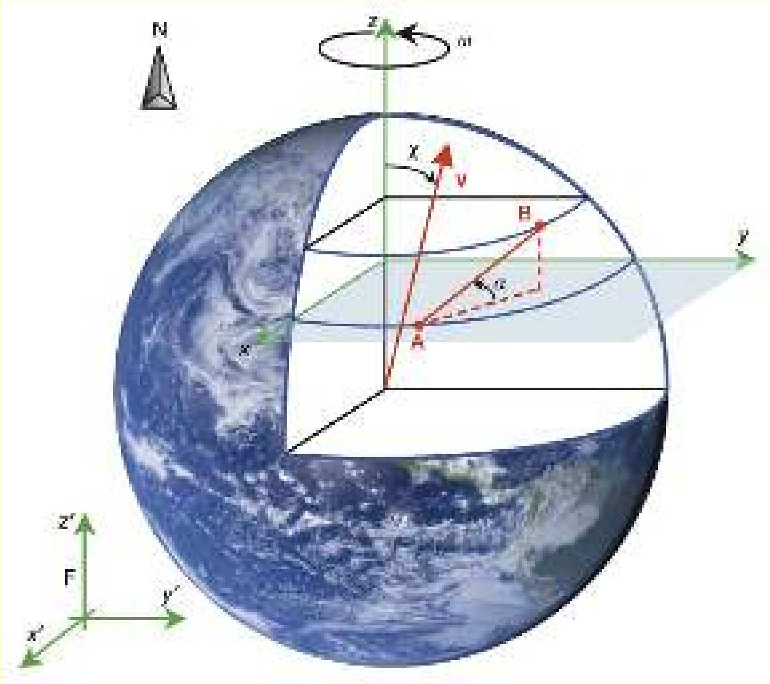


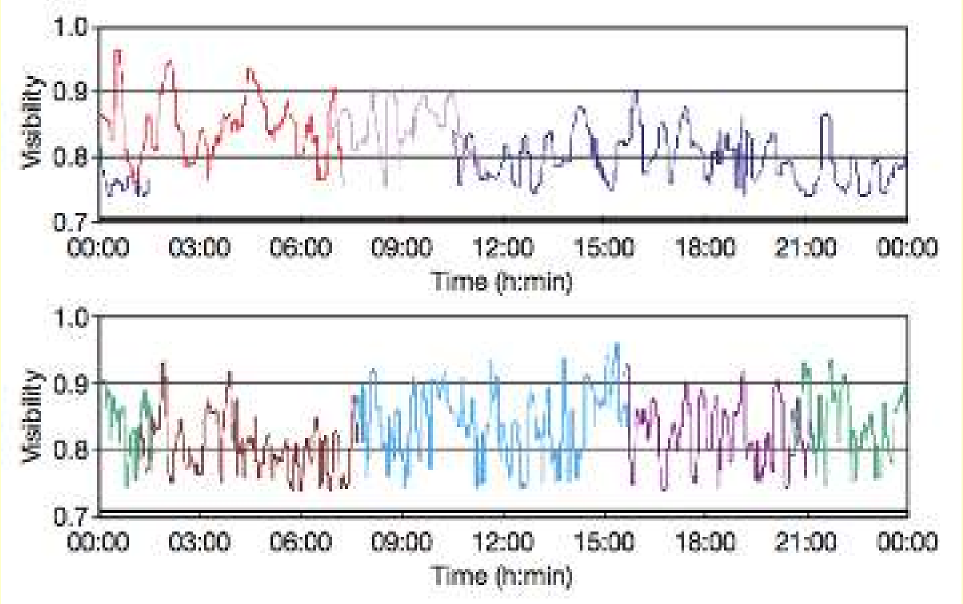
FIG. 2. Timing experiment with optical switches. Each switching device (C_I, C_{II}) is followed by two polarizers in two different orientations. Each combination is equivalent to a polarizer switched fast between two orientations.

Bell inequalities









Bell tests = entanglement check

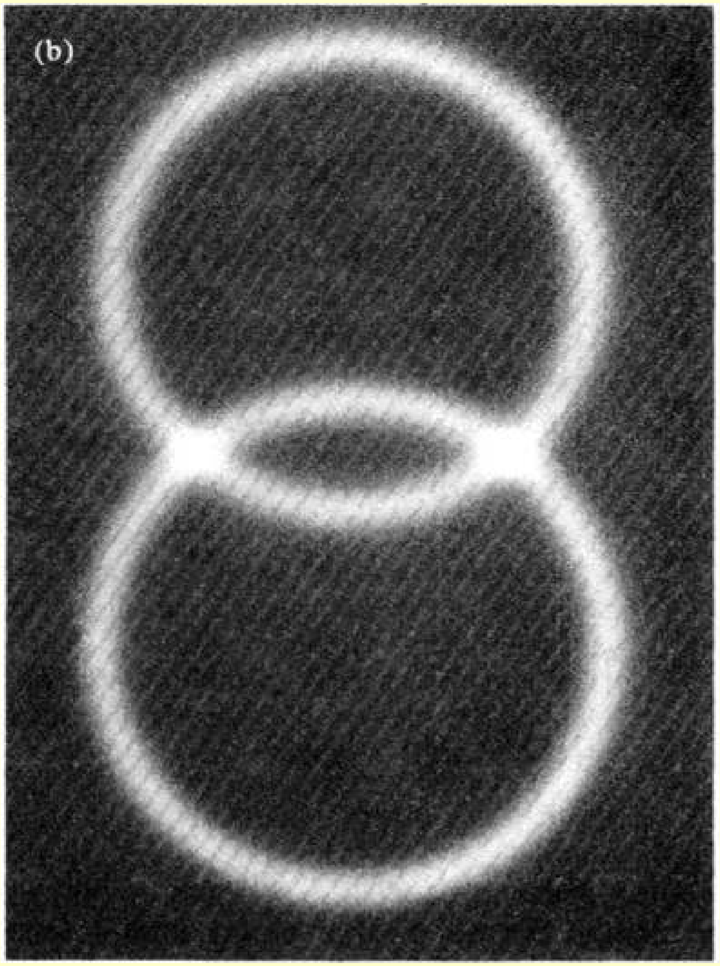


TABLE I. The four EPR-Bell states, the associated coincidence rate predictions, and the measured value of the parameter S.

EPR-Bell state	$C(\theta_1, \theta_2)$	S [a]
$\|\psi^+\rangle$	$\sin^2(\theta_1 + \theta_2)$	-2.6489 ± 0.0064
$\|\psi^-\rangle$	$\sin^2(\theta_1 - \theta_2)$	-2.6900 ± 0.0066
$\|\phi^+\rangle$	$\cos^2(\theta_1 - \theta_2)$	2.557 ± 0.014
$\|\phi^-\rangle$	$\cos^2(\theta_1 + \theta_2)$	2.529 ± 0.013

[a]Data for the $|\phi^\pm\rangle$ states were taken with a single compensating crystal, data for the $|\psi^\pm\rangle$ states with a compensating crystal in each path (see text).

$$S = E(\theta_1, \theta_2) + E(\theta_1', \theta_2) + E(\theta_1, \theta_2') - E(\theta_1', \theta_2'), \tag{3a}$$

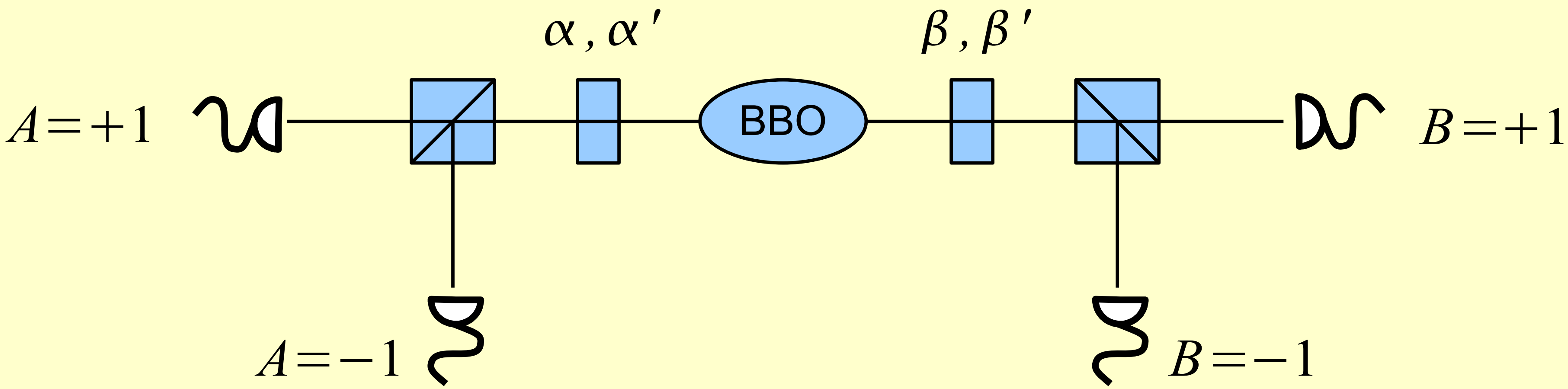
and $E(\theta_1, \theta_2)$ is given by [28]

$$\frac{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) - C(\theta_1, \theta_2^\perp) - C(\theta_1^\perp, \theta_2)}{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) + C(\theta_1, \theta_2^\perp) + C(\theta_1^\perp, \theta_2)}. \tag{3b}$$

New High-Intensity Source of Polarization-Entangled Photon Pairs
P. G. Kwiat et al Physical Review Letters **75**, 4337 (1995)

Bell inequality

- A bipartite system, which we measure separately on two different basis, {α,α'} for A and {β,β'}.



- In general, the measurement outcomes A, B, will depend on the choice of measurement basis, {α,α',β,β'} which is random.
- Can we explain the correlations depending on some additional **hidden** degree of freedom, λ

Bell inequality

- We introduce a correlated variable depending on the probability distribution of this hidden variable, λ

$$E(\alpha,\beta)=\int d\lambda\, A(\alpha,\lambda)B(\beta,\lambda)$$

- We introduce a difference

$$E(\alpha,\beta)-E(\alpha,\beta')=\int d\lambda\, p(\lambda)[A(\alpha,\lambda)B(\beta,\lambda)-A(\alpha,\lambda)B(\beta',\lambda)]$$

- Let us add and subtract the same quantity

$$E(\alpha,\beta)-E(\alpha,\beta') \quad +\int d\lambda\, p(\lambda)A(\alpha,\lambda)B(\beta',\lambda)A(\alpha',\lambda)B(\beta,\lambda)$$
$$-\int d\lambda\, p(\lambda)A(\alpha,\lambda)B(\beta',\lambda)A(\alpha',\lambda)B(\beta,\lambda)$$

Bell inequality

- We can now choose different groupings

$$E(\alpha,\beta)-E(\alpha,\beta') = \int d\lambda\, p(\lambda) A(\alpha,\lambda) B(\beta,\lambda)[1 \pm A(\alpha',\lambda) B(\beta',\lambda)]$$
$$- \int d\lambda\, p(\lambda) A(\alpha,\lambda) B(\beta',\lambda)[1 \pm A(\alpha',\lambda) B(\beta,\lambda)]$$

- Take absolute values and use the fact that |A|,|B| <= 1

$$E(\alpha,\beta)-E(\alpha,\beta') \le \int d\lambda\, p(\lambda)[1 \pm A(\alpha',\lambda) B(\beta',\lambda)]$$
$$+ \int d\lambda\, p(\lambda)[1 \pm A(\alpha',\lambda) B(\beta,\lambda)]$$
$$= 2 \pm E(\alpha',\beta') \pm E(\alpha',\beta)$$

- This leads to the following **classical** inequality

$$S = E(\alpha,\beta) - E(\alpha,\beta') \pm E(\alpha',\beta) \pm E(\alpha',\beta') \le 2$$

Bell inequality

- This inequality is violated by some entangled states

$$\left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\,1\right\rangle - \left|1\,0\right\rangle\right)$$

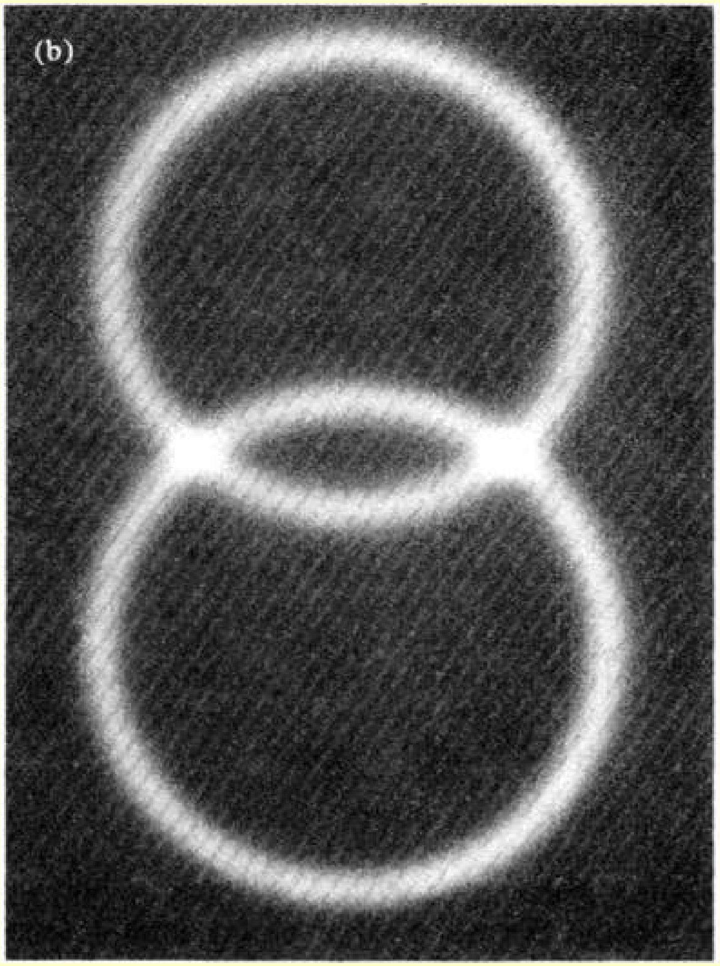
- Take four pairwise orthogonal directions

$$A(\alpha)=\sigma_z \quad B(\beta)=(-\sigma_z-\sigma_x)/\sqrt{2}$$
$$A(\alpha')=\sigma_x \quad B(\beta')=(\sigma_z-\sigma_x)/\sqrt{2}$$

- The inequality becomes

$$S=E(\alpha,\beta)-E(\alpha,\beta')+E(\alpha',\beta)+E(\alpha',\beta')$$
$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=2\sqrt{2}$$

Bell tests = entanglement check



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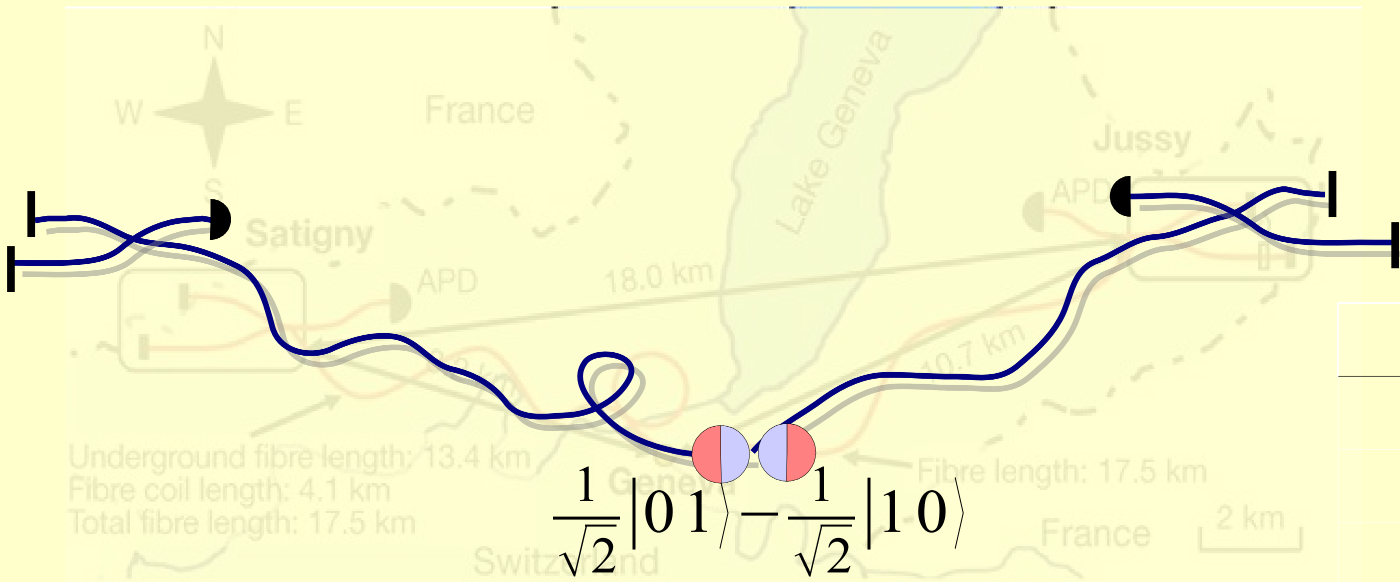
Appl. 2: EPR QKD (Ekert 91)

EPR based QKD (Ekert 91)



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A	B
0	1
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- Create a entangled Bell pair
- Perform random measurements on both parts.
- Communicate basis.
- Create the key with the outcomes that shared the same measurement basis.

EPR based QKD