Photons and Quantum Channels

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Quantum channels

Quantum channels

- A system interacting with the environment gets mixed

$$\rho_{out} = tr_{env}\left(U\left[|\psi\rangle\langle\psi| \otimes \rho_{env}\right]U^{+}\right)$$

- In general we have to consider more general processes, or positive maps

$$\varepsilon : B(H) \rightarrow B(H)$$

which preserve positivity $\varepsilon(A^{+}A) \geq 0$

trace $tr\{\varepsilon(A)\} = tr\{A\}$

and can be composed $\varepsilon \otimes I_{any} \geq 0$

Quantum channels

- Completely depolarizing channel: all information erased

$$\rho_{out} = (1-p)\rho + p\frac{1}{d}I$$

which for qubits can also be written

$$\rho_{out} = \left(1-\frac{3}{4}p\right)\rho + \frac{1}{4}\left(\sigma_x \rho \sigma_x + \sigma_y \rho \sigma_y + \sigma_z \rho \sigma_z\right)$$

- Dephasing channel

$$\rho_{out} = (1-p)\rho + p\sum_s |s\rangle\langle s|\rho|s\rangle\langle s|$$

equivalent to randomizing the phase of the s=0 and s=1 states

$$\rho \rightarrow (1-p)\rho + p\frac{1}{2\pi}\int e^{-i\sigma_z \phi} \rho e^{+i\sigma_z \phi} d\phi$$

Quantum channels

- Completely depolarizing channel

$$\rho_{out} = \left(1 - \frac{3}{4} p\right) \rho + \frac{1}{4} \left(\sigma_x \rho \sigma_x + \sigma_y \rho \sigma_y + \sigma_z \rho \sigma_z\right)$$

- Dephasing channel

$$\rho_{out} = (1 - p) \rho + p \sum_s |s\rangle\langle s|\rho|s\rangle\langle s|$$

- Just examples of the general Kraus decomposition

$$\varepsilon(\rho) = \sum_k A_k^+ \rho A_k$$

$$\sum_k A_k^+ A_k = I$$

Testing quantum channels

Long distance Q Comm

High-fidelity transmission of polarization encoded qubits from an entangled source over 100 km of fiber

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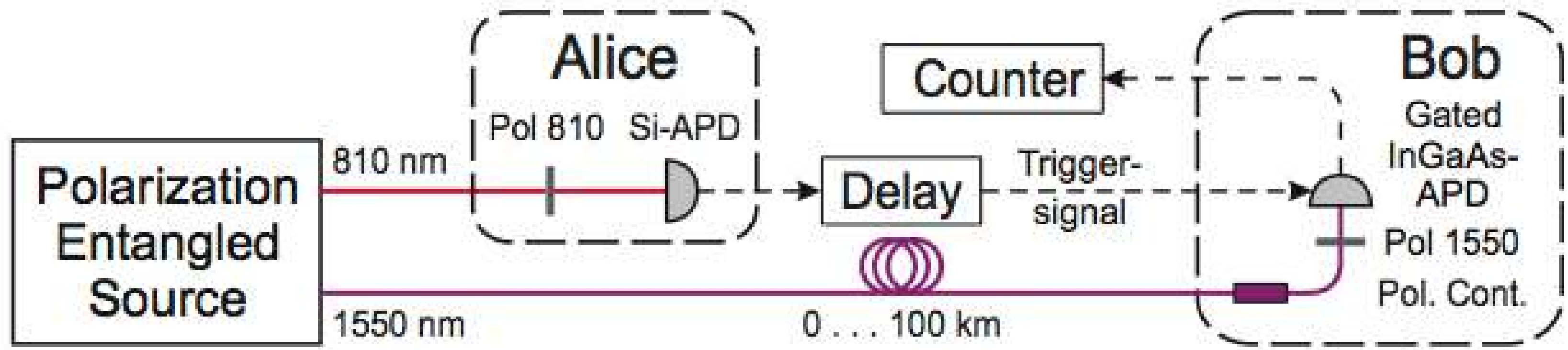
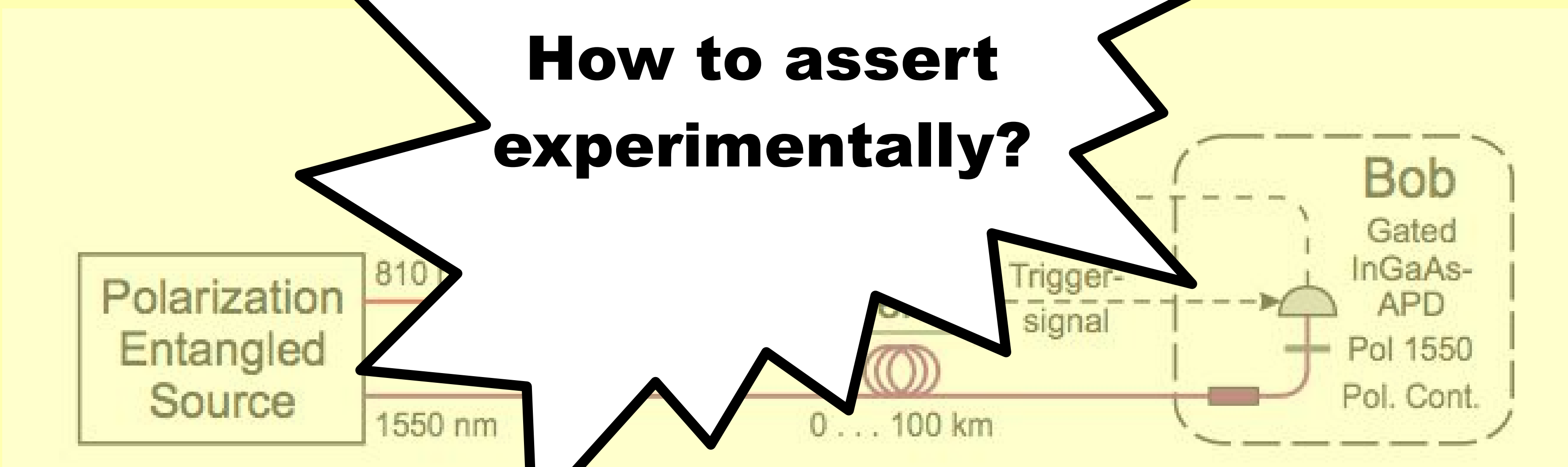
Long distance Q Comm

High-fidelity transmission of polarization encoded qubits from an entangled source over 100 km of fiber



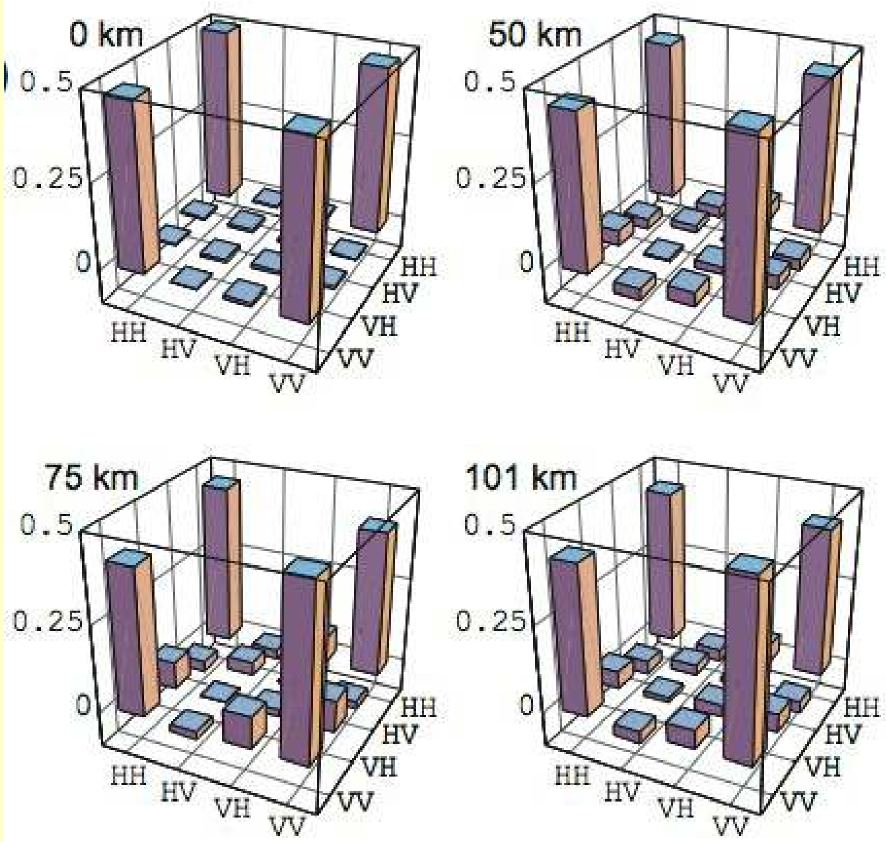
Long distance Q Comm

High-fidelity transmission of
polarization encoded q bits from an
entangled source 00 k n of fiber



How to assert experimentally?

Long distance Q Comm



Quantum tomography: measuring elements of density matrix,

$$\rho = \sum_{ijkl \in \{H, V\}} \rho_{ij,kl} |ij\rangle\langle kl|$$

Quantum tomography

- Reconstructing a density matrix from measurements

$$\rho = \sum_i tr(\rho A_i) A_i$$

- For a single qubit we can use the Bloch basis

$$\rho = \frac{1}{4}\left(1 + \langle \sigma_x \rangle \sigma_x + \langle \sigma_y \rangle \sigma_y + \langle \sigma_z \rangle \sigma_z\right)$$

- Alternatively, measure states

$$|+\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle, \qquad |H\rangle$$
$$|R\rangle = \frac{1}{\sqrt{2}}|H\rangle + i\frac{1}{\sqrt{2}}|V\rangle, \qquad |V\rangle$$

Quantum tomography

- For more components, use a combination of Pauli matrices

$$\rho = 2^{-N} \sum_i tr(\rho\, \sigma_{\alpha_1} \cdots \sigma_{\alpha_N})\, \sigma_{\alpha_1} \cdots \sigma_{\alpha_N}$$

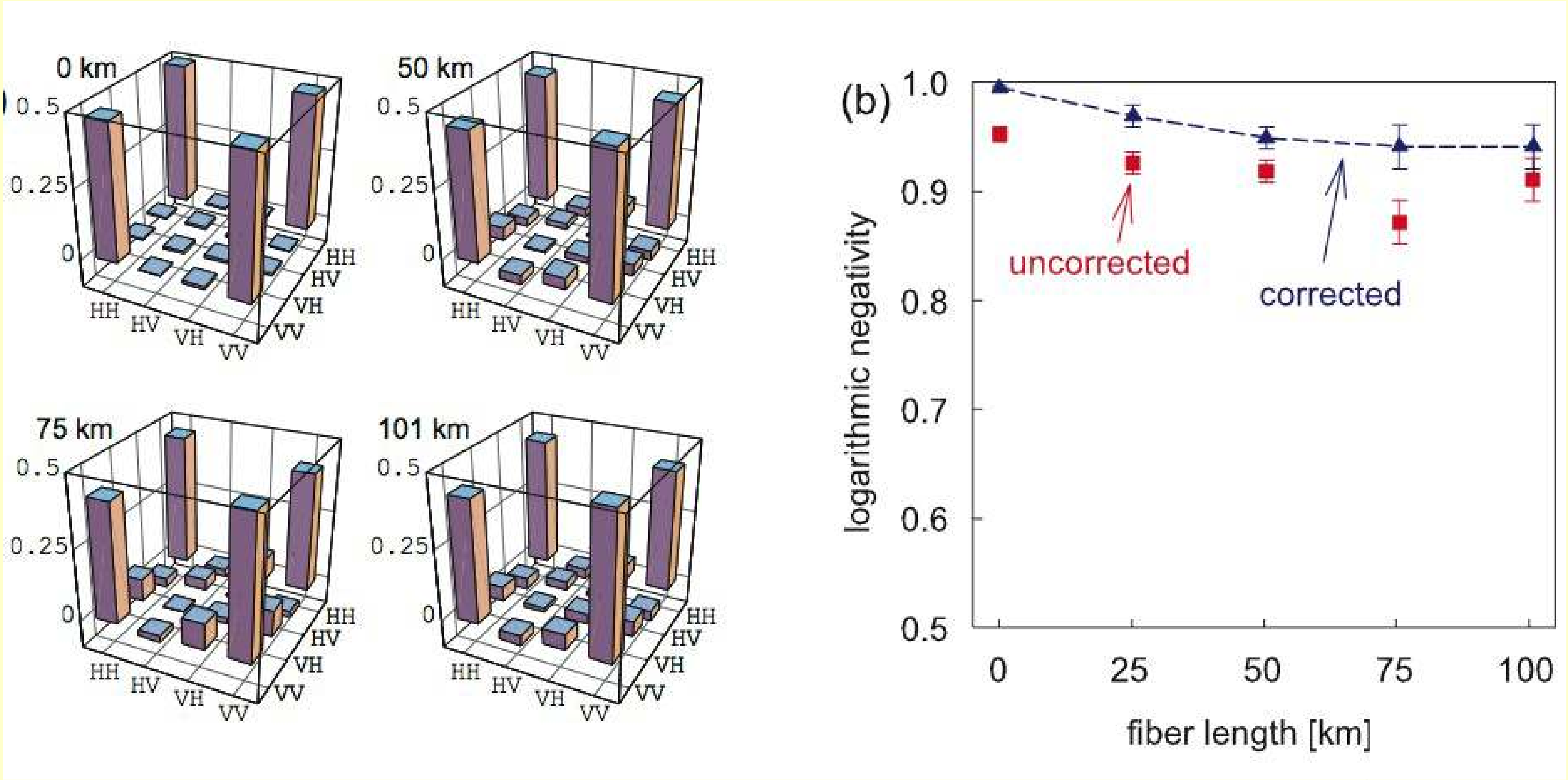
- This is based on orthogonality relations

$$tr(\sigma_\alpha \sigma_\beta) = 2\,\delta_{\alpha\beta}$$

$$\sigma_\alpha \in \{1, \sigma_x, \sigma_y, \sigma_z\}$$

- Problem: unless we have some extra information, 2N measurement basis required!

Long distance Q Comm



Measure of entanglement,
logarithmic negativity

Entanglement measures

- Some quantity which cannot grow under LOCC and which defines an order for entangled states
- Number of coefficients? E(P) < E(A) < E(B)

$$|P\rangle = |0,0\rangle$$

$$|A\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle)$$

$$|B\rangle = \frac{1}{\sqrt{3}}(|0,0\rangle + |1,1\rangle + |2,2\rangle)$$

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- One way to quantify this: von Neuman entropy

$$|\psi_{12}\rangle \rightarrow \rho_1 = tr_2\left(|\psi_{12}\rangle\langle\psi_{12}|\right) \rightarrow S = -tr\left(\rho_1 \log(\rho_1)\right)$$

Entanglement measures

- Previous example:

$$\rho_{P1} = |0\rangle\langle 0| \qquad \rightarrow S_P = \log(1) = 0$$

$$\rho_{A1} = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) \qquad \rightarrow S_A = \log(2)$$

$$\rho_{B1} = \frac{1}{3}(|0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|) \rightarrow S_B = \log(3)$$

- But only a good measure for pure states

$$\rho_{12} = II \otimes II = \frac{1}{4}(|0\rangle\langle 0| + |1\rangle\langle 1|)^{\otimes 2}$$

$$\rho_1 = II = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) \qquad \rightarrow S = \log(2)???$$

Entanglement measures

- Partial transposition: transpose only one subsystem

$$\rho = \rho_{ij,kl} |i,j\rangle\langle k,l| \rightarrow \rho^{PT} = \rho_{ij,kl} |k,j\rangle\langle i,l|$$

- Product states remain valid density matrices

$$(\rho_1 \otimes \rho_2)^{PT} \rightarrow \tilde{\rho}_1 \otimes \tilde{\rho}_2$$

- **Negativity**

$$N = \sum_{\lambda_i < 0} |\lambda_i|, \quad \lambda_i \in \sigma(\rho^{PT})$$

- **Logarithmic negativity**

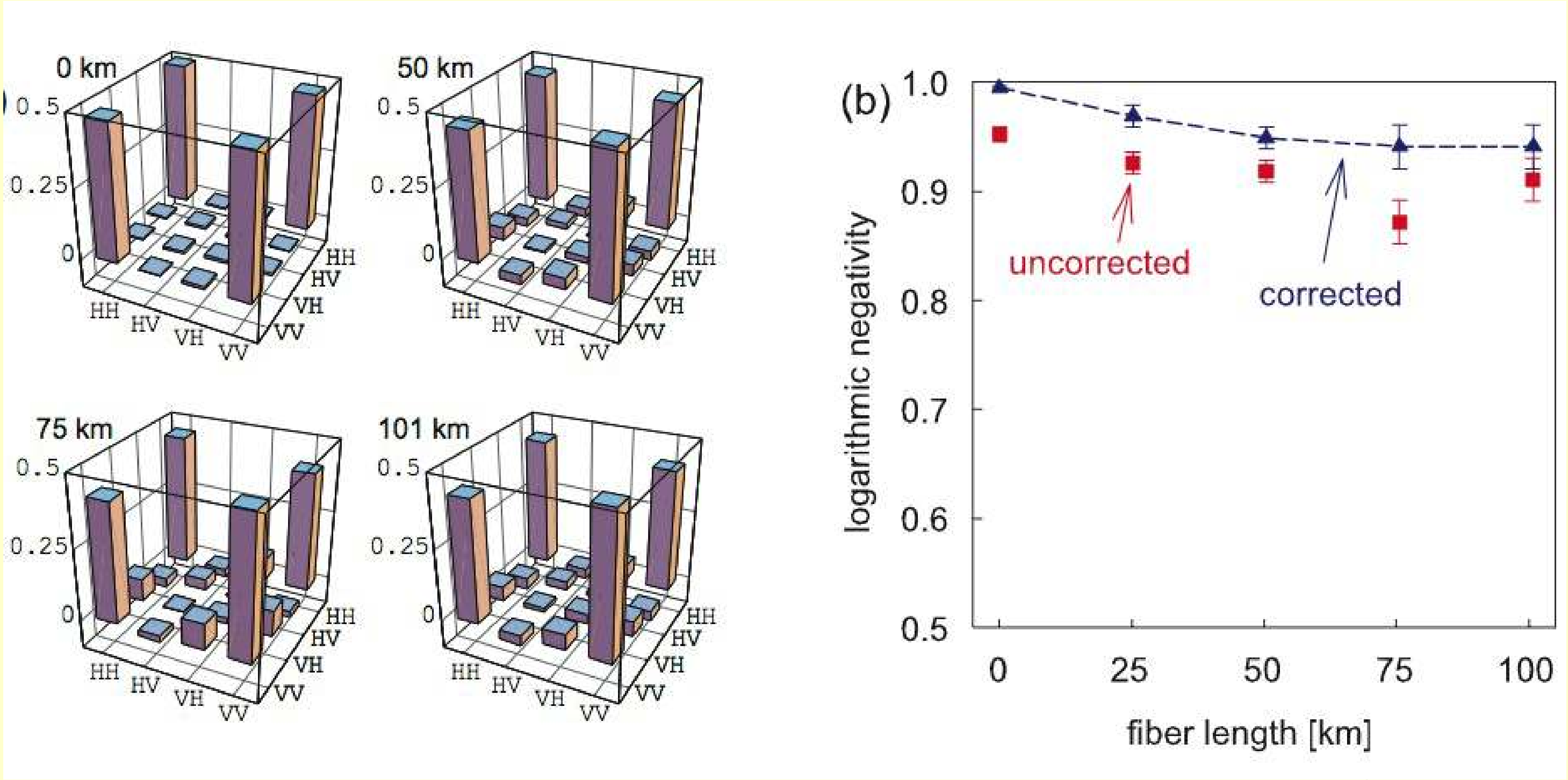
$$E_N(\rho) = \log_2(2\mathrm{N}(\rho) + 1)$$

Entanglement measures

- There are **operational measures**:
  - Distillable entanglement: how many singlets can we produce from copies of the state.
  - Entanglement cost: how many singlets do we need to recreate the state.

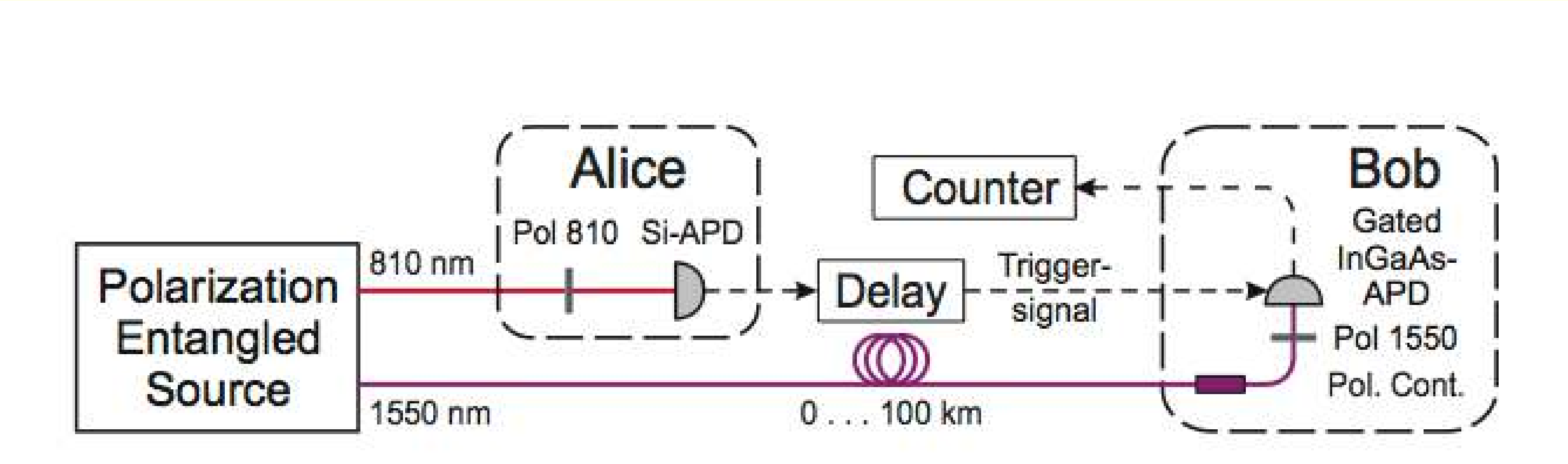
  but are sometimes difficult to compute.
- **Violation of Bell inequalities**
  - Entanglement monotone
  - Like negativity, it may be zero!
  - A good practical indicator for protocols -> information about probability distributions.

Long distance Q Comm



Measure of entanglement, logarithmic negativity

Process tomography



- In this particular setup, only one of the photons would be subject to errors

$$\rho_{out} = (I \otimes \varepsilon)|\psi\rangle\langle\psi|$$

- For a maximally entangled input, the output characterizes ε

$$\psi = \sum_i |i, i\rangle \qquad \rho_{out} = \sum_{ij} |i\rangle\langle j| \otimes \varepsilon(|i\rangle\langle j|)$$