Photons: entangling operations

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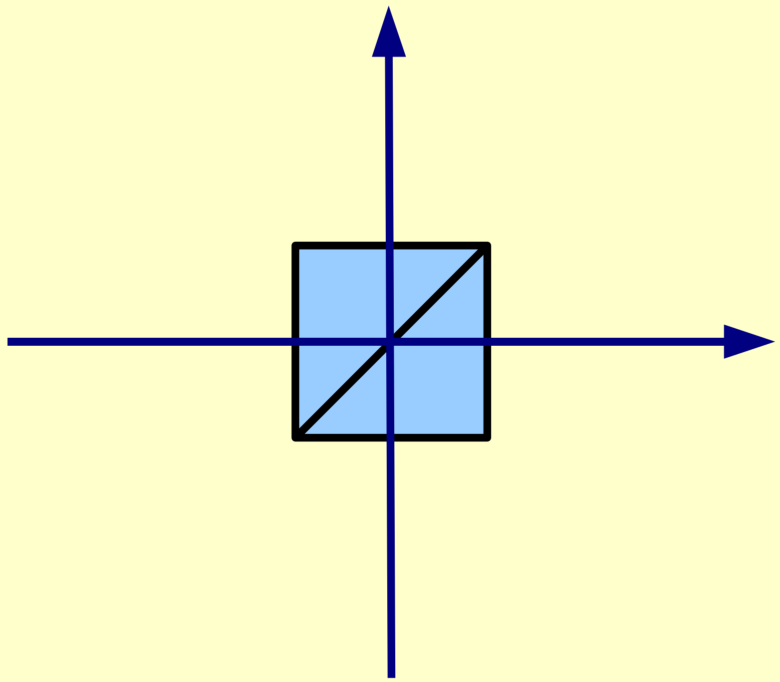
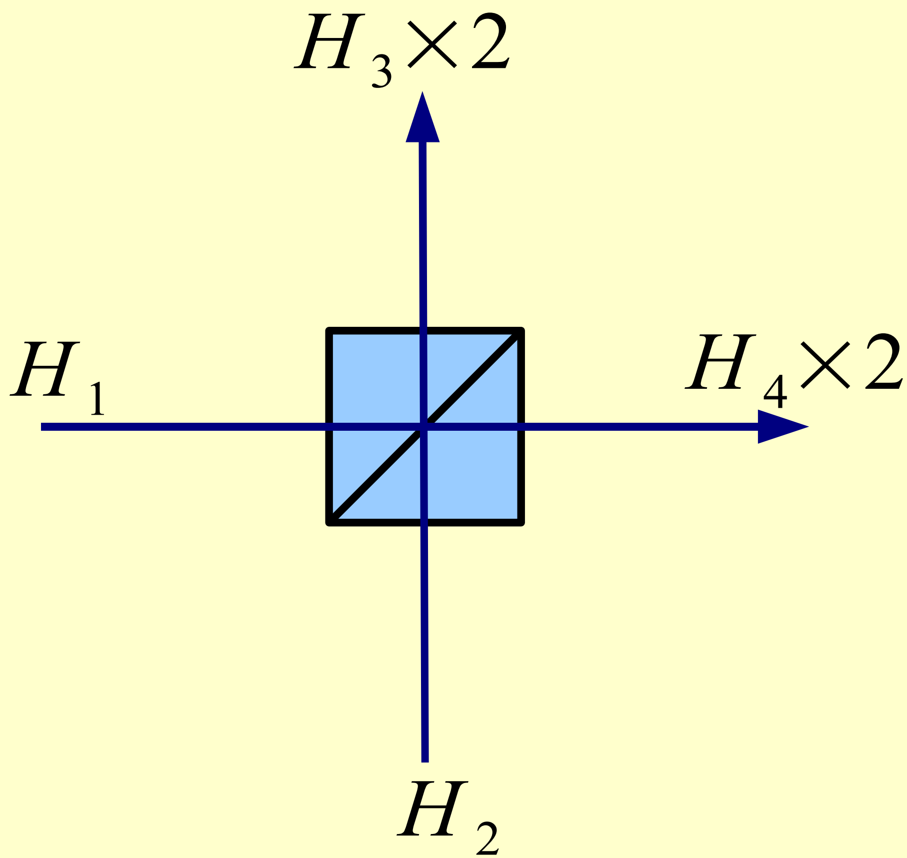
Bell basis measurement



Bell basis measurement



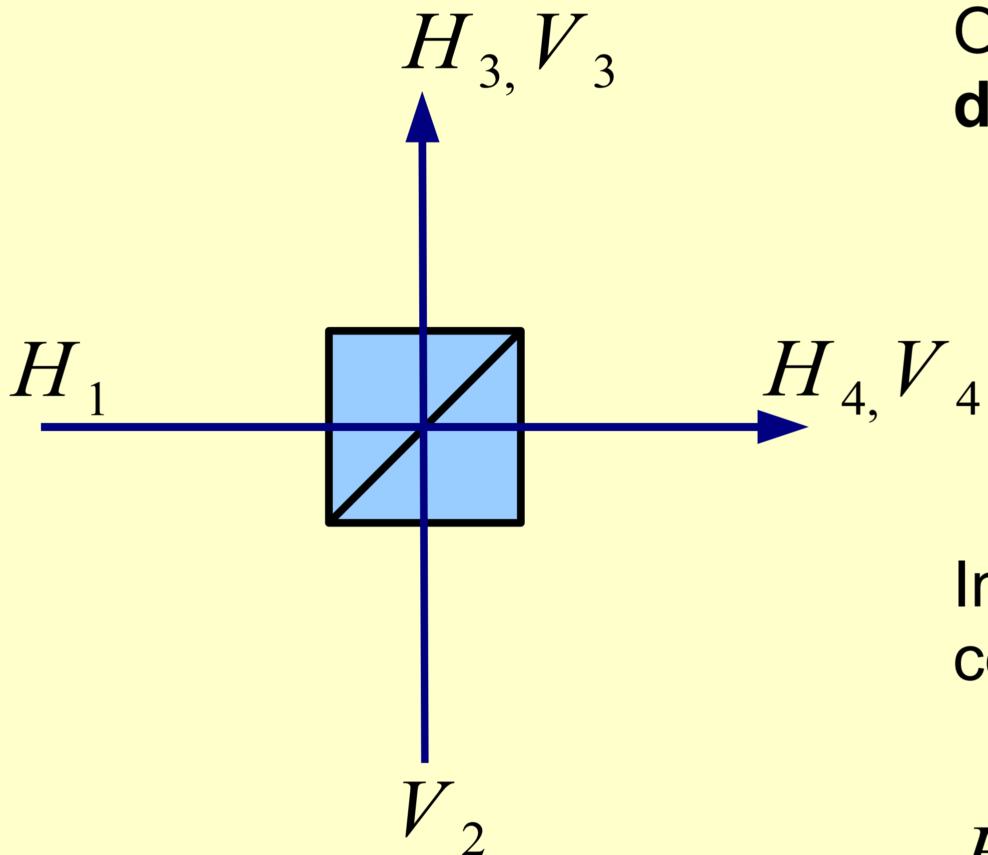
Ordinary beam splitter, with undistinguishable photons

$$H_1 \rightarrow \frac{1}{\sqrt{2}}(H_3 + i\,H_4)$$
$$H_2 \rightarrow \frac{1}{\sqrt{2}}(H_3 - i\,H_4)$$

We obtain both photons on the same phort: **bunching**

$$a_1^+ a_2^+ \rightarrow \frac{1}{2}(a_3^{+2} + a_4^{+2})$$
$$H_1 H_2 \rightarrow \frac{1}{\sqrt{2}}(H_3 \otimes H_3 + H_4 \otimes H_4)$$

Bell basis measurement



Ordinary beam splitter, but with **distinguishable** photons

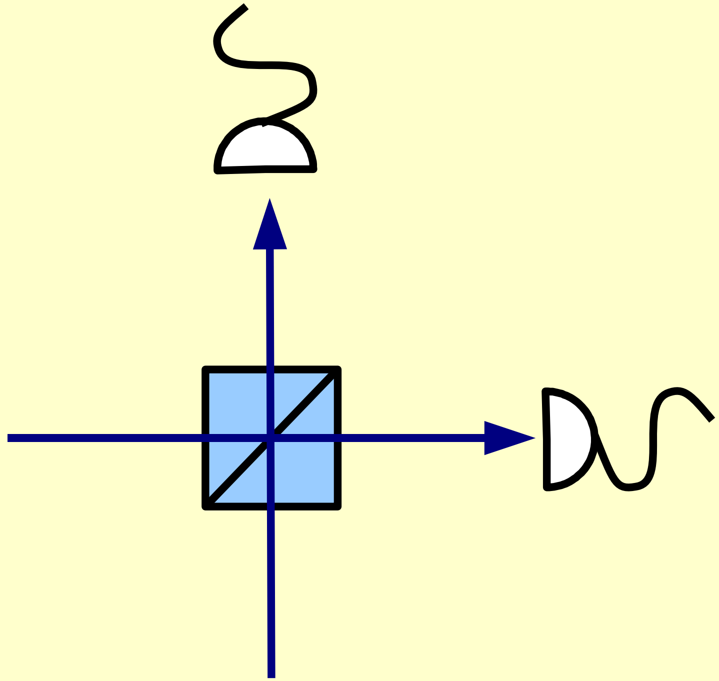
$$H_1 \rightarrow \frac{1}{\sqrt{2}}(H_3 + i H_4)$$

$$V_2 \rightarrow \frac{1}{\sqrt{2}}(V_3 - i V_4)$$

In this case, we can have coincidences

$$H_1 V_2 \rightarrow \frac{1}{2}(H_3 V_3 + H_4 V_4) + \frac{i}{2}(H_4 V_3 - H_3 V_4)$$

Bell basis measurement



D1	D2	
C	C	← $\left|\psi^{-}\right\rangle$
C	0	← Unknown
0	C	← Unknown
0	0	← Unknown

For an arbitrary input, a double-detector coincidence implies a projection onto

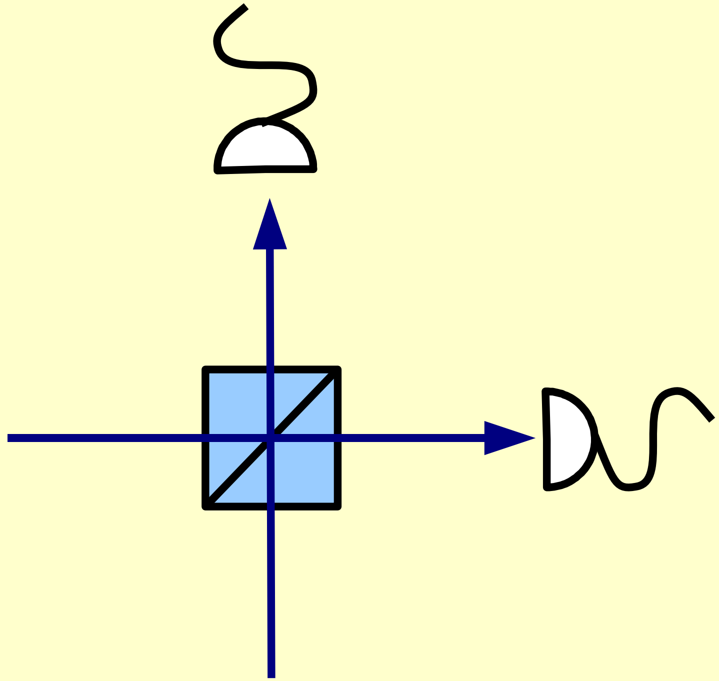
$$\left|\psi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|H_1 V_2\right\rangle - \left|V_1 H_2\right\rangle\right)$$

In other situations, we cannot say much about the input state.

May be any other Bell state

$$\left|\psi^{+}\right\rangle, \left|\phi^{+}\right\rangle, \left|\phi^{-}\right\rangle$$

Post-selection



D1	D2	
C	C	← $\lvert\psi^-\rangle$
C	0	← Unknown
0	C	← Unknown
0	0	← Unknown

Post-select: measure something at the end of the experiment and decide whether it worked

$$\rho_{PS} = tr(P_{ok}\rho) P_{ok}\rho P_{ok} + tr(P_{no}\rho) P_{no}\rho P_{no}$$

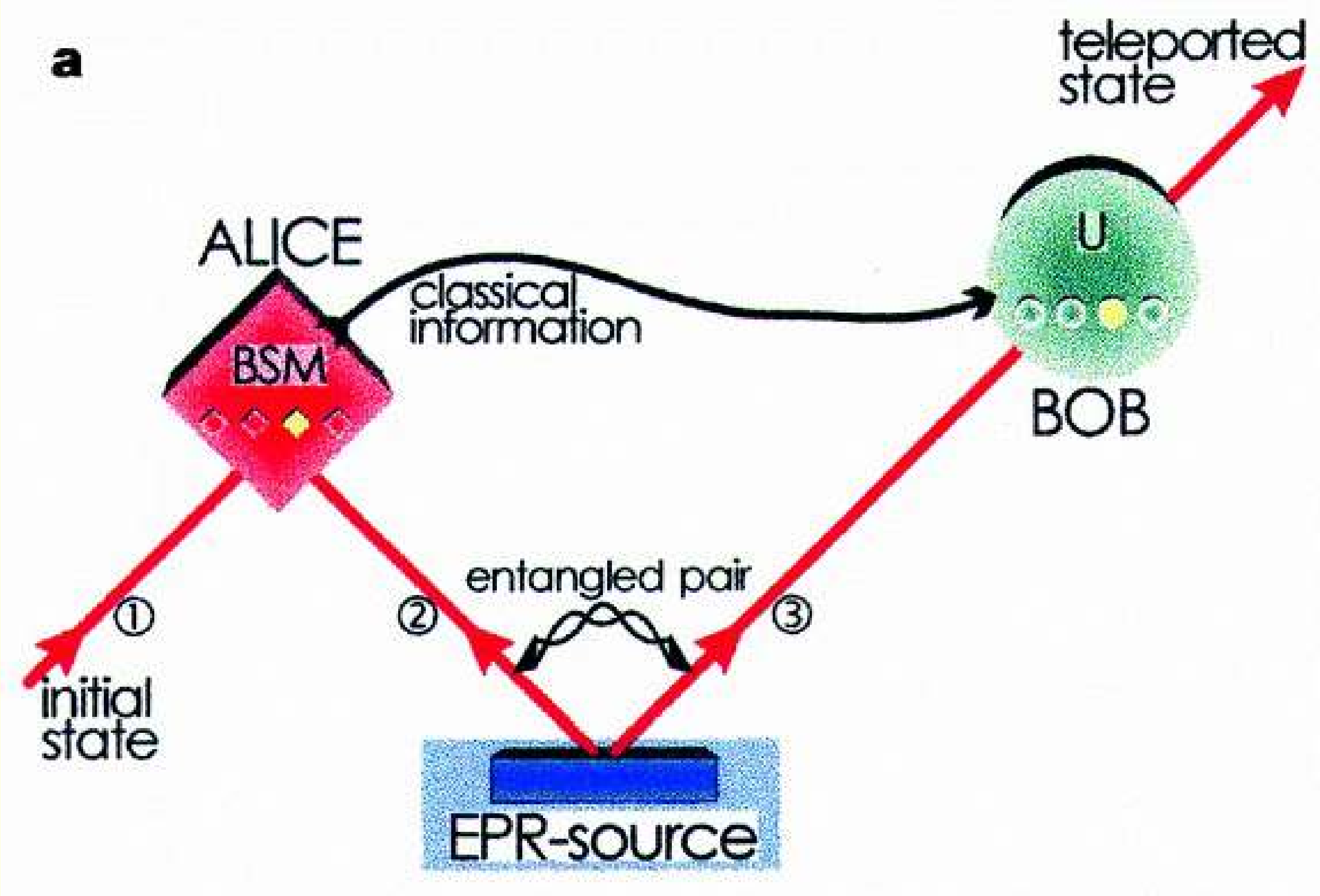
The experiment becomes probabilistic, not deterministic.

Success probability

$$tr(P_{ok}\rho)$$

Quantum teleportation

Quantum teleportation



Experimental quantum teleportation
Dik Bouwmeester et al, Nature **390**, 575-579 (1997)

Quantum teleportation

- Reminder: a qubit and an entangled state

$$\left|\psi\right\rangle \sim \left(a\left|0\right\rangle + b\left|1\right\rangle\right) \otimes \left(\left|0\,1\right\rangle - \left|1\,0\right\rangle\right)$$

- Also written as

$$\left|\psi\right\rangle = a\left(\left|0\,0\,1\right\rangle - \left|0\,1\,0\right\rangle\right) + b\left(\left|1\,0\,1\right\rangle - \left|1\,1\,0\right\rangle\right)$$

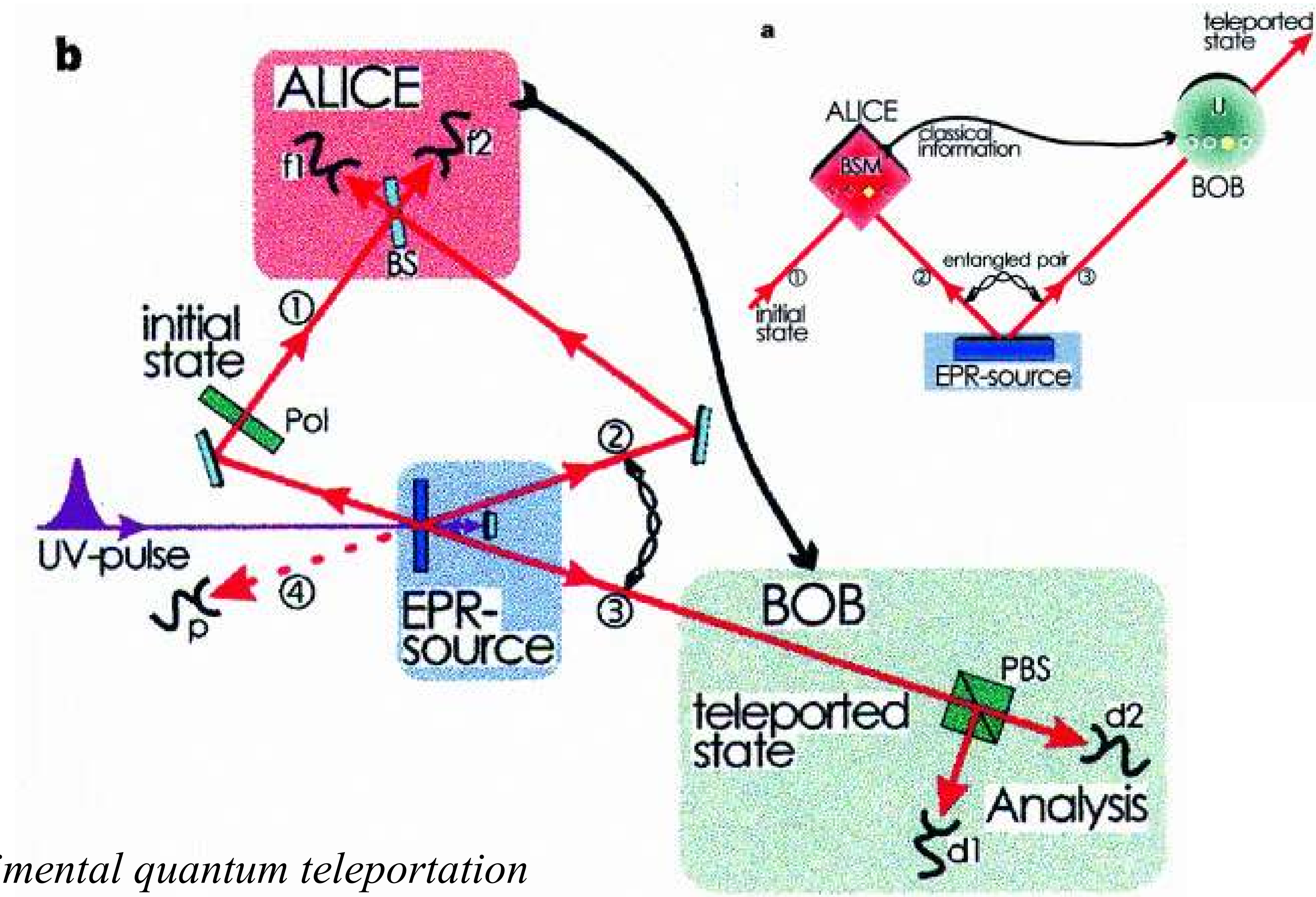
- In the Bell basis

$$\begin{aligned}\left|\psi\right\rangle \sim\; & a\left(\phi^{+} + \phi^{-}\right)\left|1\right\rangle + a\left(\psi^{-} - \psi^{+}\right)\left|0\right\rangle \\ & + b\left(\psi^{-} + \psi^{+}\right)\left|1\right\rangle + b\left(\phi^{-} - \phi^{+}\right)\left|0\right\rangle\end{aligned}$$

- We only need to post-select one Bell state

$$\left|\psi\right\rangle \sim \psi^{-}\left(a\left|0\right\rangle + b\left|1\right\rangle\right) + \ldots$$

Actual experiment



Experimental quantum teleportation
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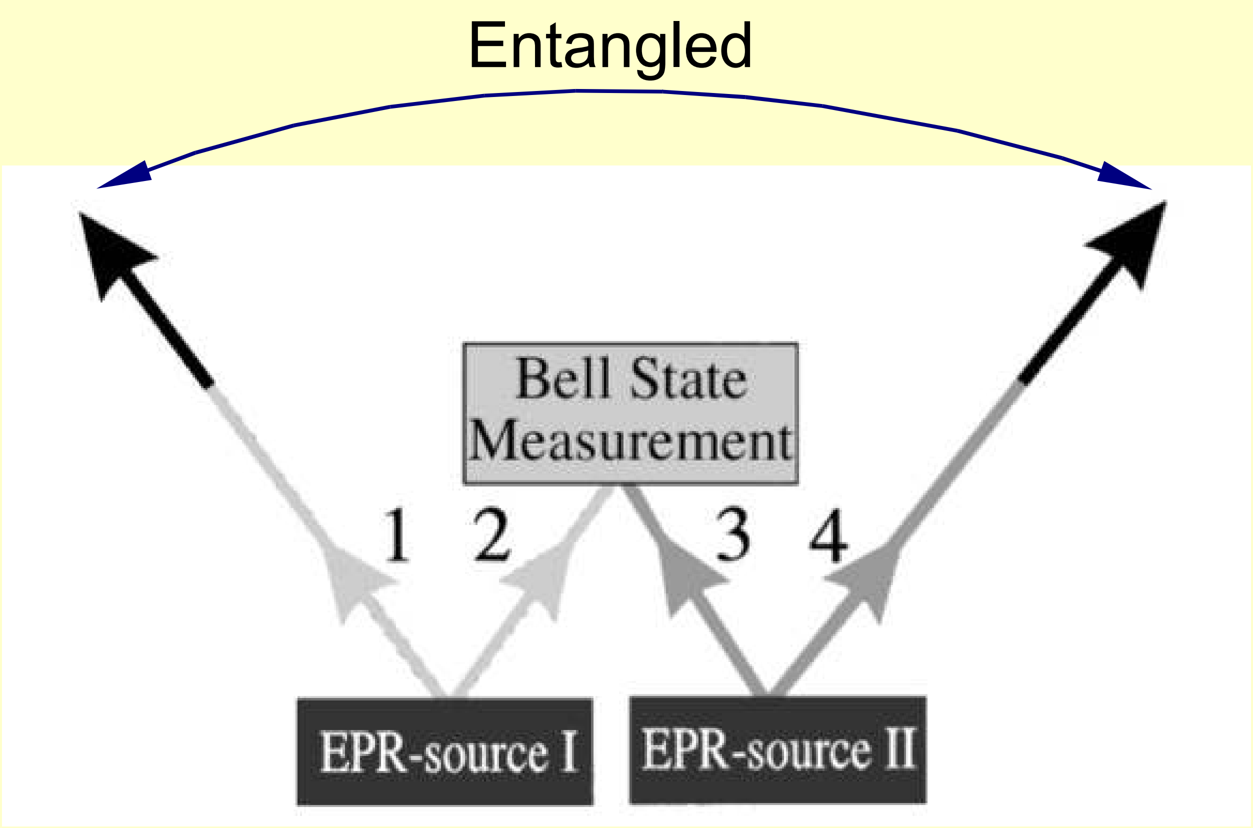
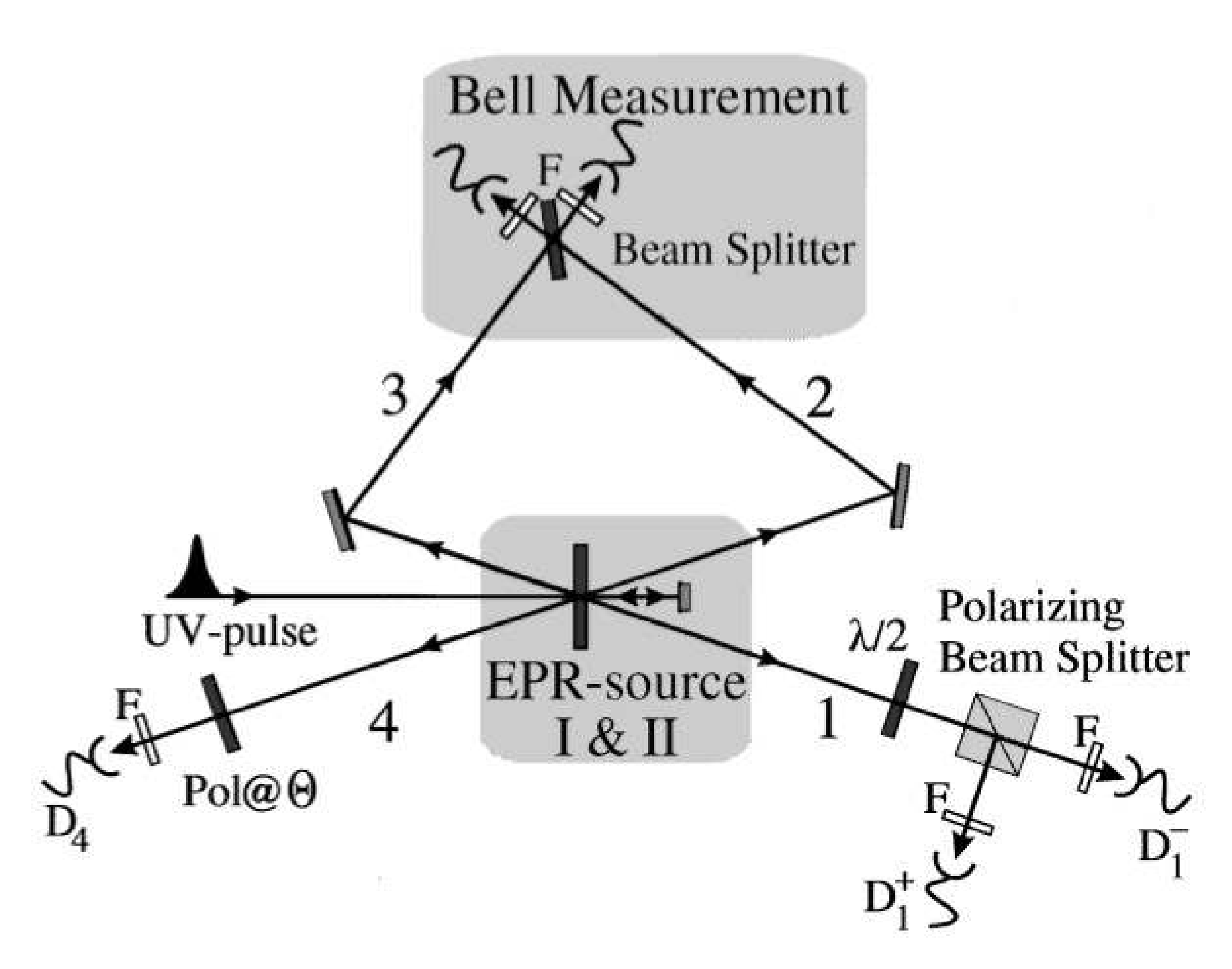
Entanglement swap

Entanglement swap



- A generalization of teleportation protocol.
- Entangles distant particles that never interacted.
- Basis for quantum repeaters.

Entanglement swap



Experimental quantum teleportation
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Entanglement swap

- Input states: two entangled pairs

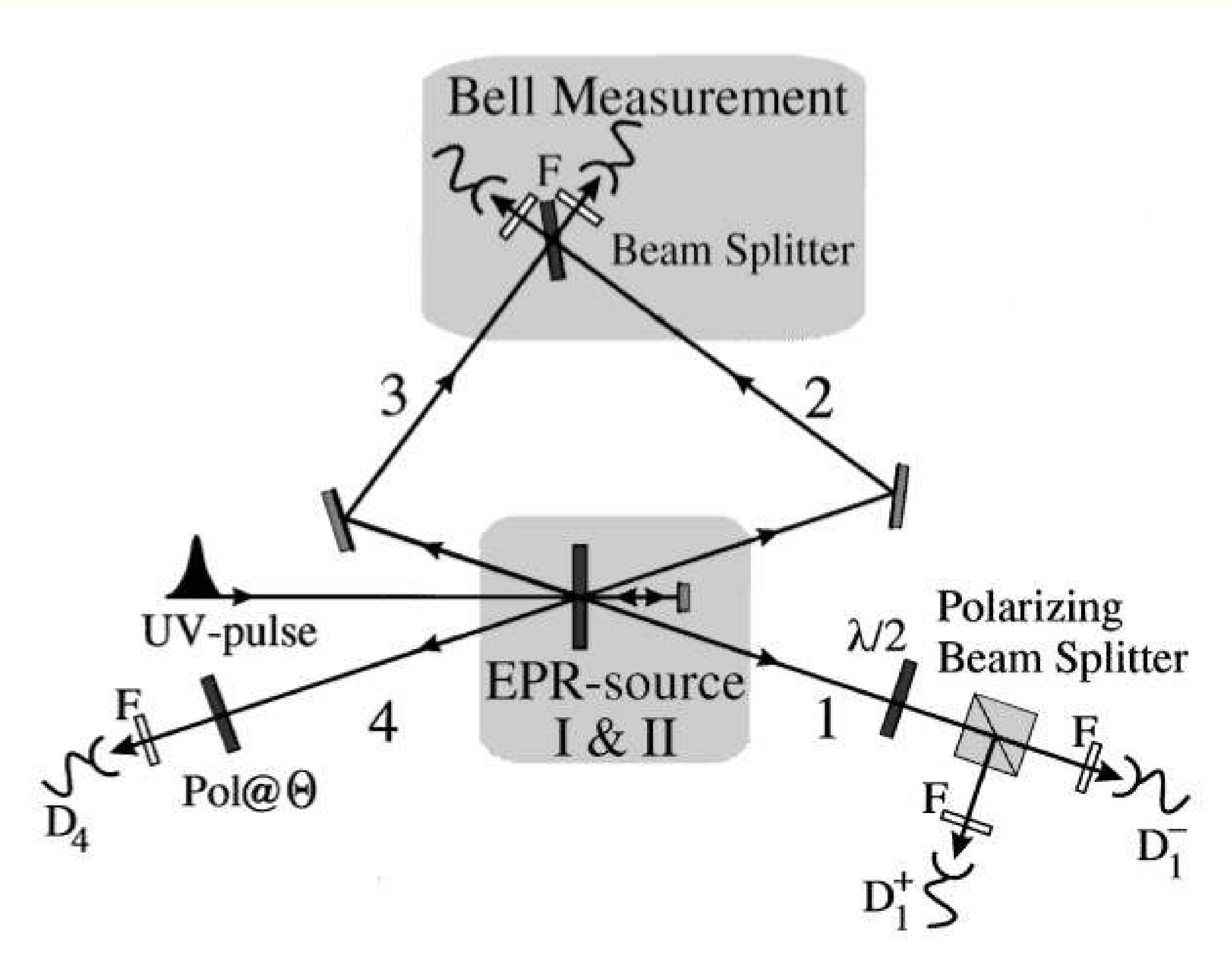
$$\left|\Psi_{1234}\right\rangle \sim \left(\left|H_1 V_2\right\rangle - \left|V_1 H_2\right\rangle\right)\left(\left|H_3 V_4\right\rangle - \left|V_3 H_4\right\rangle\right)$$

- May be rewritten in the Bell basis of the paired ports

$$\begin{aligned}\left|\Psi_{1234}\right\rangle \sim &\left|\psi_{14}^{+}\right\rangle\left|\psi_{23}^{+}\right\rangle + \left|\psi_{14}^{-}\right\rangle\left|\psi_{23}^{-}\right\rangle \\ &+ \left|\phi_{14}^{+}\right\rangle\left|\phi_{23}^{+}\right\rangle + \left|\phi_{14}^{-}\right\rangle\left|\phi_{23}^{-}\right\rangle\end{aligned}$$

- We can post-select, depending on the Bell measurement of ports 2 and 3

Entanglement swap



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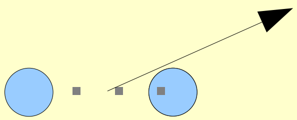
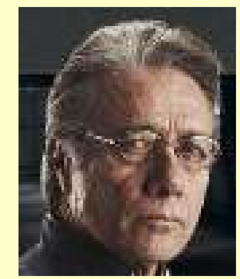
Quantum repeater model

Quantum repeater model

- **Quantum channel**

$$\frac{1}{\sqrt{2}}|0\,1\rangle - \frac{1}{\sqrt{2}}|1\,0\rangle$$

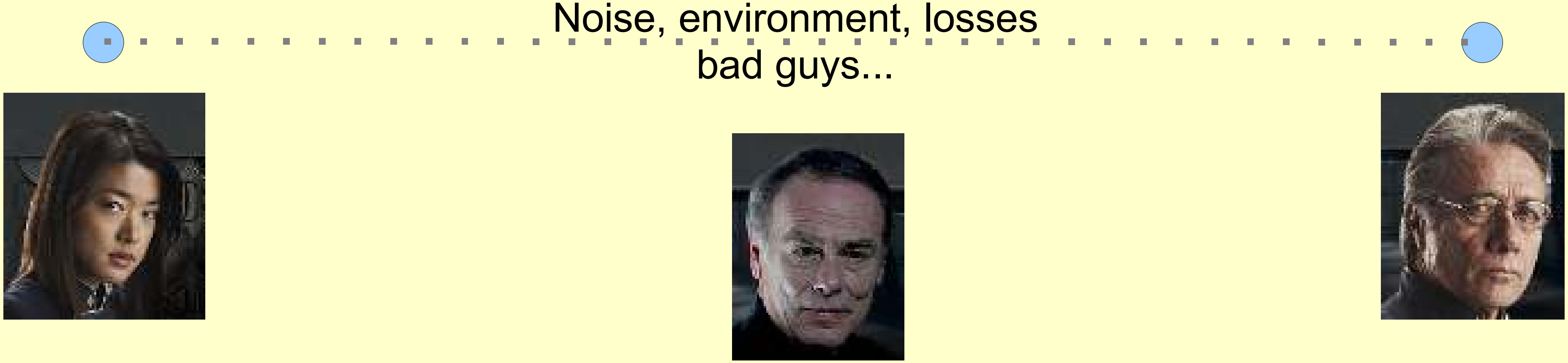




Quantum repeater model

- **Quantum channel**

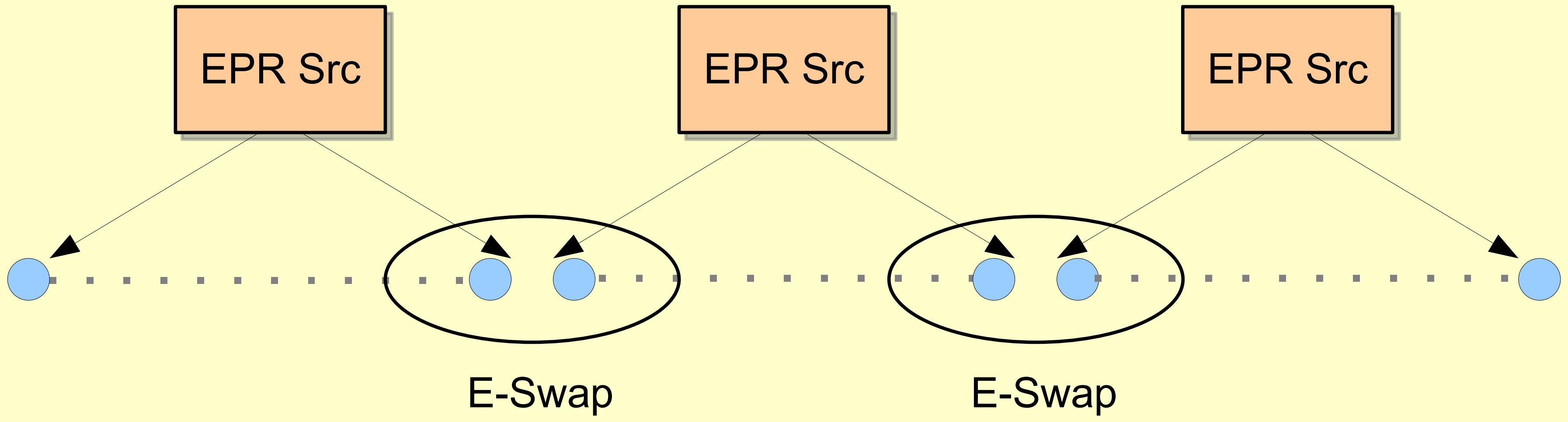
$$\frac{1}{\sqrt{2}}|0\,1\rangle - \frac{1}{\sqrt{2}}|1\,0\rangle\ ???$$



- We can detect errors or interceptions, by computing for instance Bell inequalities.
- But how to limit them?

Quantum repeater model

- **Quantum repeaters**



- Entangled pair sources distributed along the channel.
- Detection of incoming particles and entanglement swap.
- Result: long distance entangled pair.
- Problems: on-demand sources, storage of photons, synchronization...

Large entangled states

Four-photon entanglement

- Assume we have two entangled pairs out of a BBO

$$\left|\Psi_{1234}\right\rangle \sim \left(\left|H_1 V_2\right\rangle - \left|V_1 H_2\right\rangle\right)\left(\left|H_3 V_4\right\rangle - \left|V_3 H_4\right\rangle\right)$$

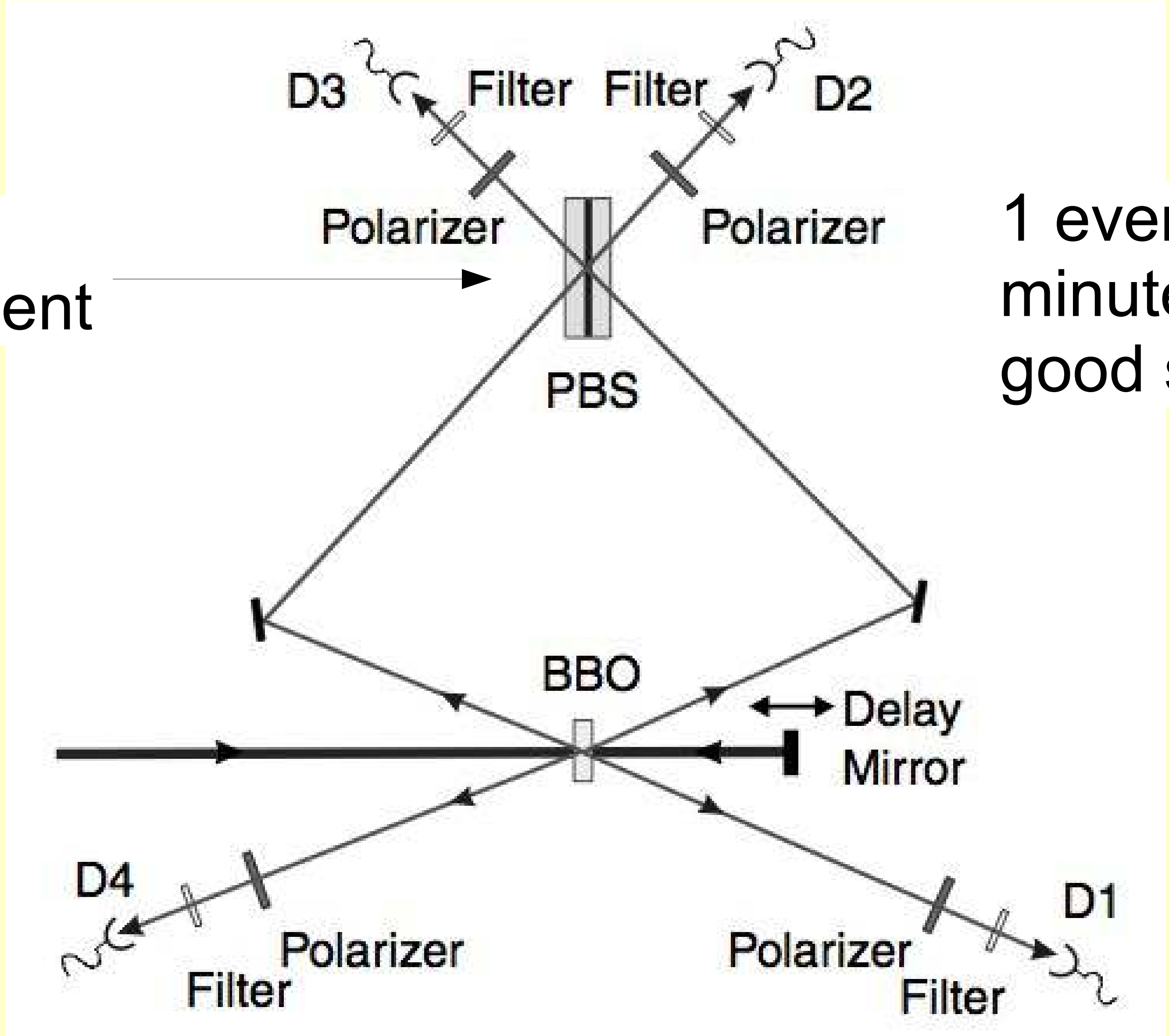
- We could perform a post-selection, projective measurement such tha the 2 and 3 photons have the same state.

$$\left|\Psi_{post}\right\rangle \sim \left|H_1 V_2 V_3 H_4\right\rangle - \left|V_1 H_2 H_3 V_4\right\rangle$$

- How to measure equality without measuring polarization?

Four photon entanglement

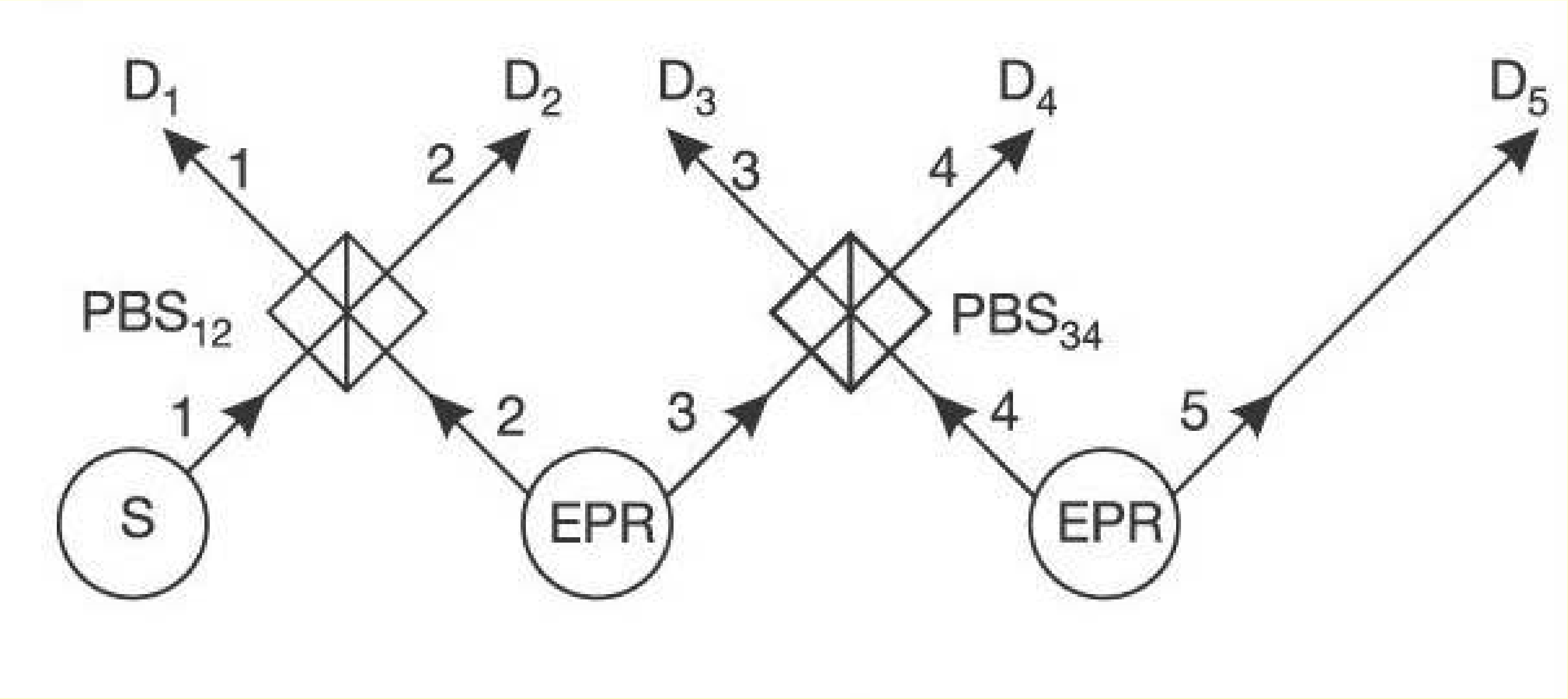
Entangling measurement



1 event per minute, slow for good statistics

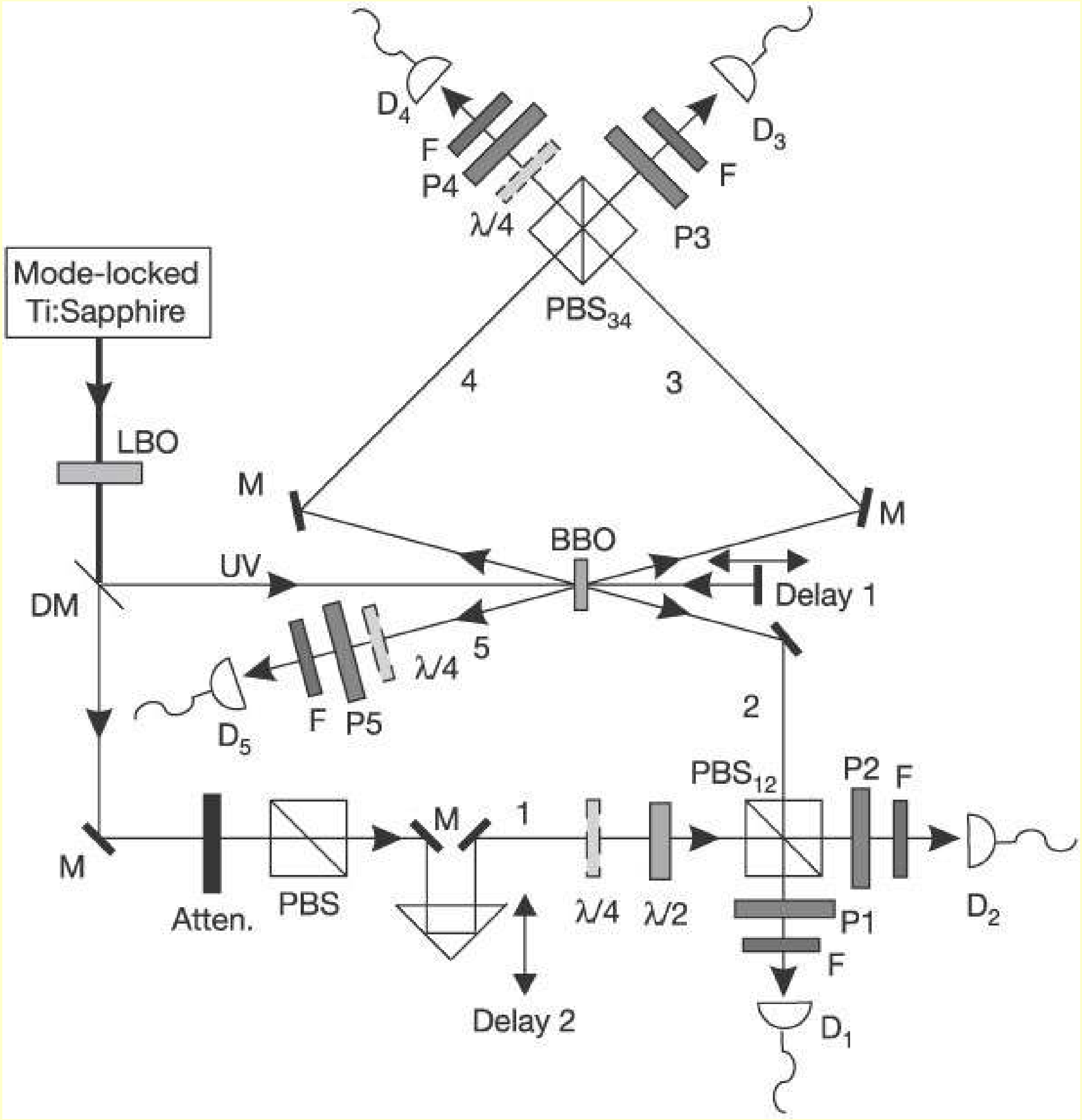
Experimental demonstration of four-photon entanglement [...]
Jian-Wei Pan, PRL **86**, 4435 (2001)

Five-photon entanglement



Experimental demonstration of five-photon entanglement [...]
Zhi Zhao et al Nature **430** 54-58 (2004)

Five-photon entanglement



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Entanglement witness

- The state is too large for tomography

- We would need a large number of measurements

$$\sim 2^{10} \sim 1024$$

- At a rate of 1 experiment / minute, quite a long time to gather statistics.

- Problem of accuracy: too many measurements, errors in the density matrix.

Entanglement witness

- An observable, or function of them, that checks whether an state is entangled

$$\langle W \rangle_\rho < 0 \;\rightarrow\; \rho \;\; entangled$$

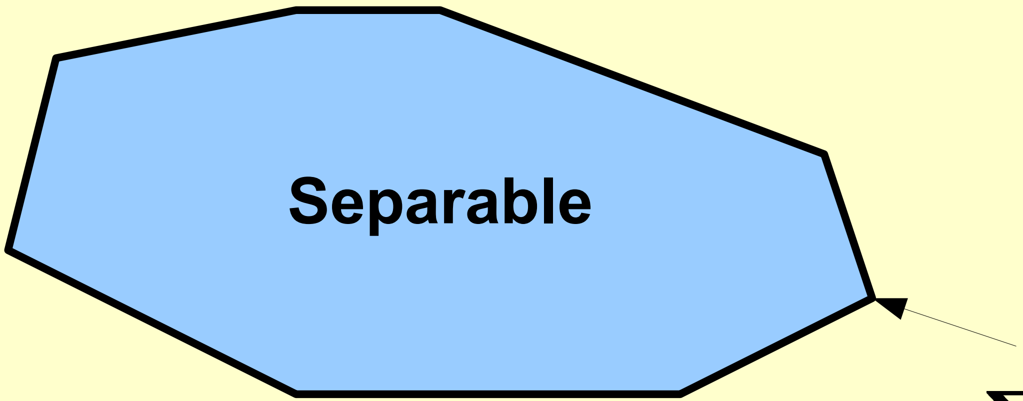
- We expect the operator to be simple to check

$$W_{C^4} = 3 - \frac{1}{2}(\sigma_z^1 \sigma_z^2 + 1)(\sigma_z^2 \sigma_z^3 \sigma_z^4 + 1) - \frac{1}{2}(\sigma_x^1 \sigma_x^2 \sigma_z^3 + 1)(1 + \sigma_z^3 \sigma_z^5)$$

- Only three measurement setups, no Bell measurements.

Entanglement witness

- The set of separable states is **convex**: stable under linear combinations with positive coefficients

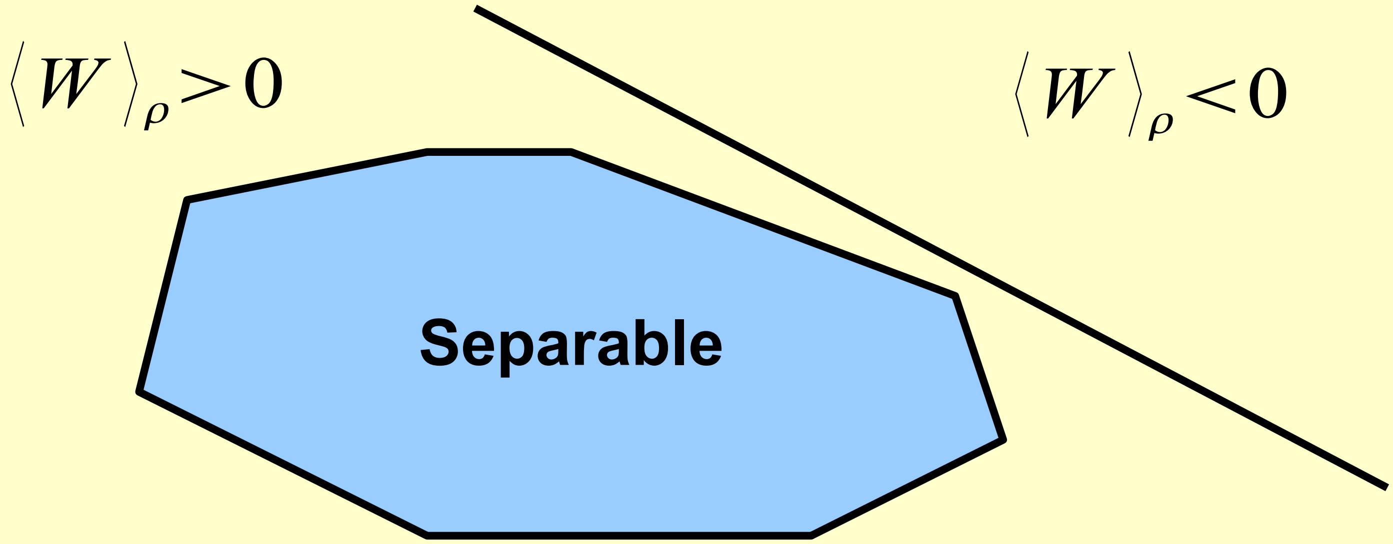


$$\rho_{sep} = \sum_k c_k \rho_{1,k} \otimes \rho_{2,k}$$

$$c_k \geq 0$$

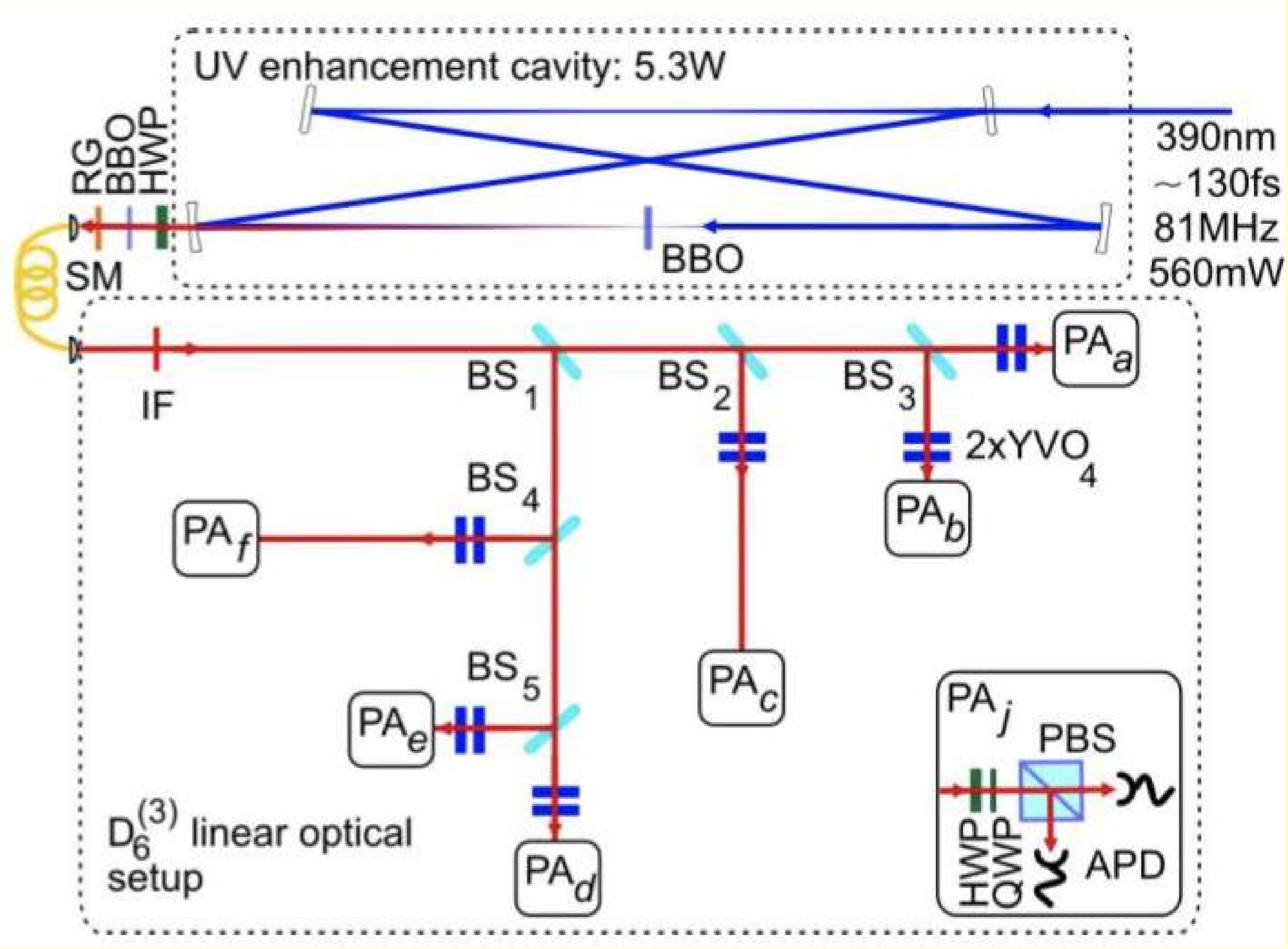
Entanglement witness

- The set of separable states is **convex**: stable under linear combinations with positive coefficients



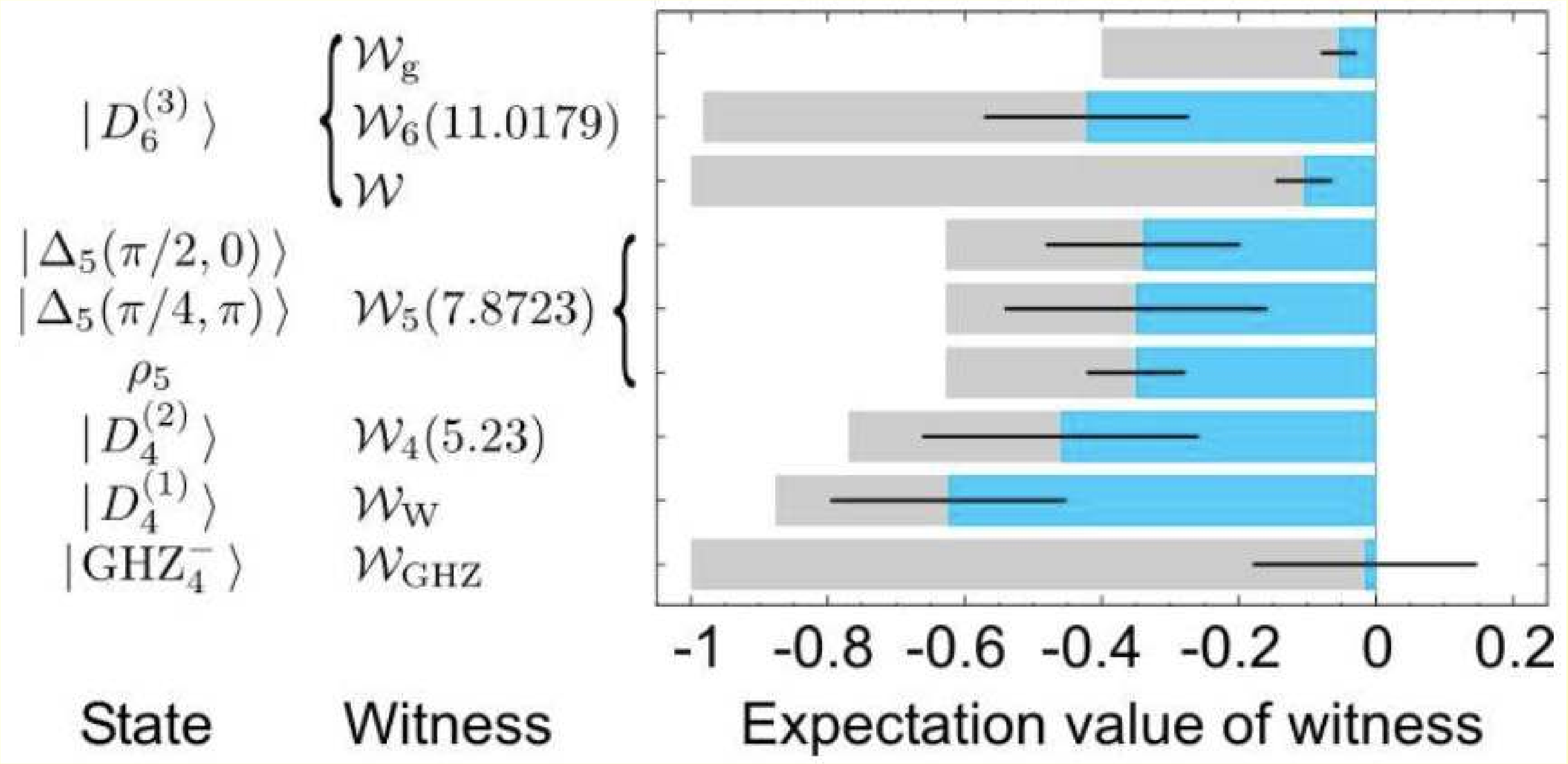
- The entanglement witness is like a plane that divides the Hilbert space into two: entangled and unknowns.

Entanglement witness



Experimental entanglement of a six-photon symmetric Dicke state
Witlef Wieczorek et al, arXiv:0903.2213

Entanglement witness



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