

Atoms as qubits

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(16-4-2009)

Ingredients for QIPC

- **Quantum degrees of freedom**
- Local operations
- Measurements
- One of
 - Entangled state sources
 - Universal 2qb unitaries
- Error correcting schemes
- Large number of qubits

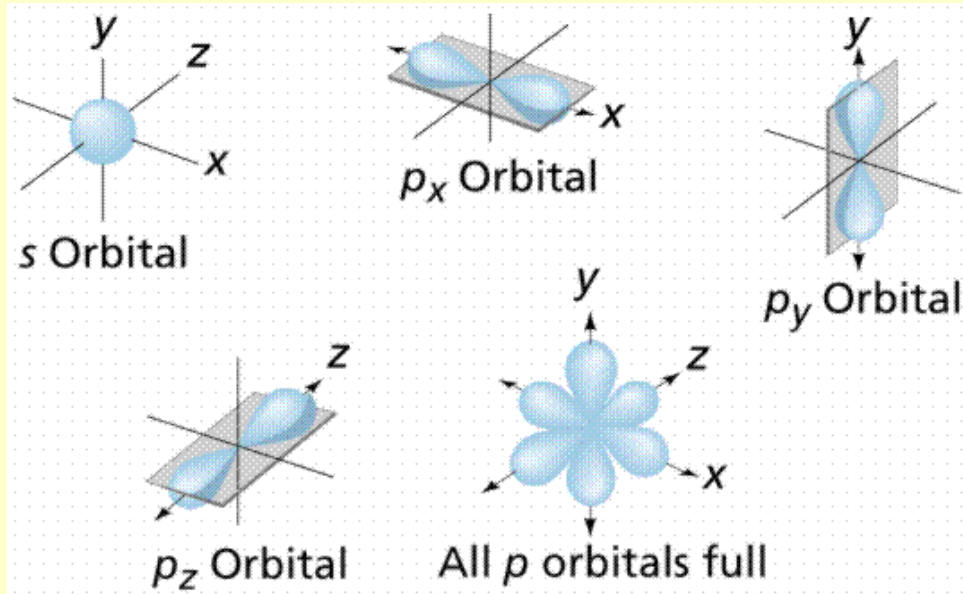
Q. Communication

Q. Cryptography

Q. Simulation

Q. Computation

Atoms



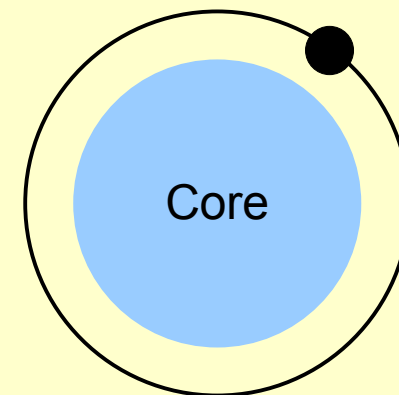
- We plan to use atoms as quantum registers.
- We have a variety of degrees of freedom to choose from
 - Electronic orbitals
 - Angular momenta
 - Electron spin
 - Nuclear angular momenta
 - Atom position/momentum
 - Collective many-atom states
- ...

Atoms

1 IA	H 1 1.008 Hydrogen	2 IIA					
2	Li 3 6.94 Lithium	Be 4 9.01 Beryllium					
3	Na 11 22.99 Sodium	Mg 12 24.31 Magnesium	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB
4	K 19 39.10 Potassium	Ca 20 40.08 Calcium	Sc 21 44.96 Scandium	Ti 22 47.88 Titanium	V 23 50.94 Vanadium	Cr 24 52.00 Chromium	Mn 25 54.94 Manganese
5	Rb 37 85.47 Rubidium	Sr 38 87.62 Strontium	Y 39 88.91 Yttrium	Zr 40 91.22 Zirconium	Nb 41 92.91 Niobium	Mo 42 95.94 Molybdenum	Tc 43 (97.9) Technetium
6	Cs 55 132.91 Cesium	Ba 56 137.33 Barium	La 57 138.91 Lanthanum	Hf 72 178.49 Hafnium	Ta 73 180.95 Tantalum	W 74 183.85 Tungsten	Re 75 186.21 Rhenium
7	Fr 87 223.02 Francium	Ra 88 226.02 Radium	Ac 89 227.03 Actinium	Rf 104 (261) Rutherfordium	Db 105 (262) Dubnium	Sg 106 (263) Seaborgium	Bh 107 (262) Bohrium

H	SYMBOL
1	ATOMIC NUMBER
1.008	ATOMIC WEIGHT
Hydrogen	NAME

- It is convenient to focus on atoms with only one valence e⁻
- This includes all hydrogen-like atoms

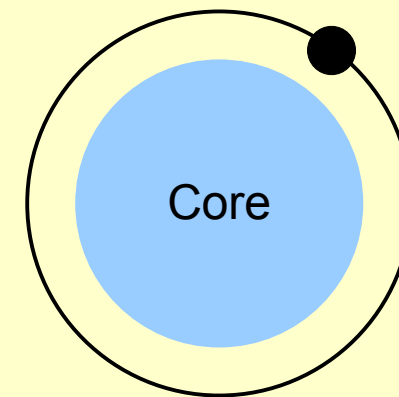


both neutral and some ions:
 ${}^{40}\text{Ca}^+$, ${}^{25}\text{Mg}^+$, ${}^9\text{Be}^+$, ...

Atoms

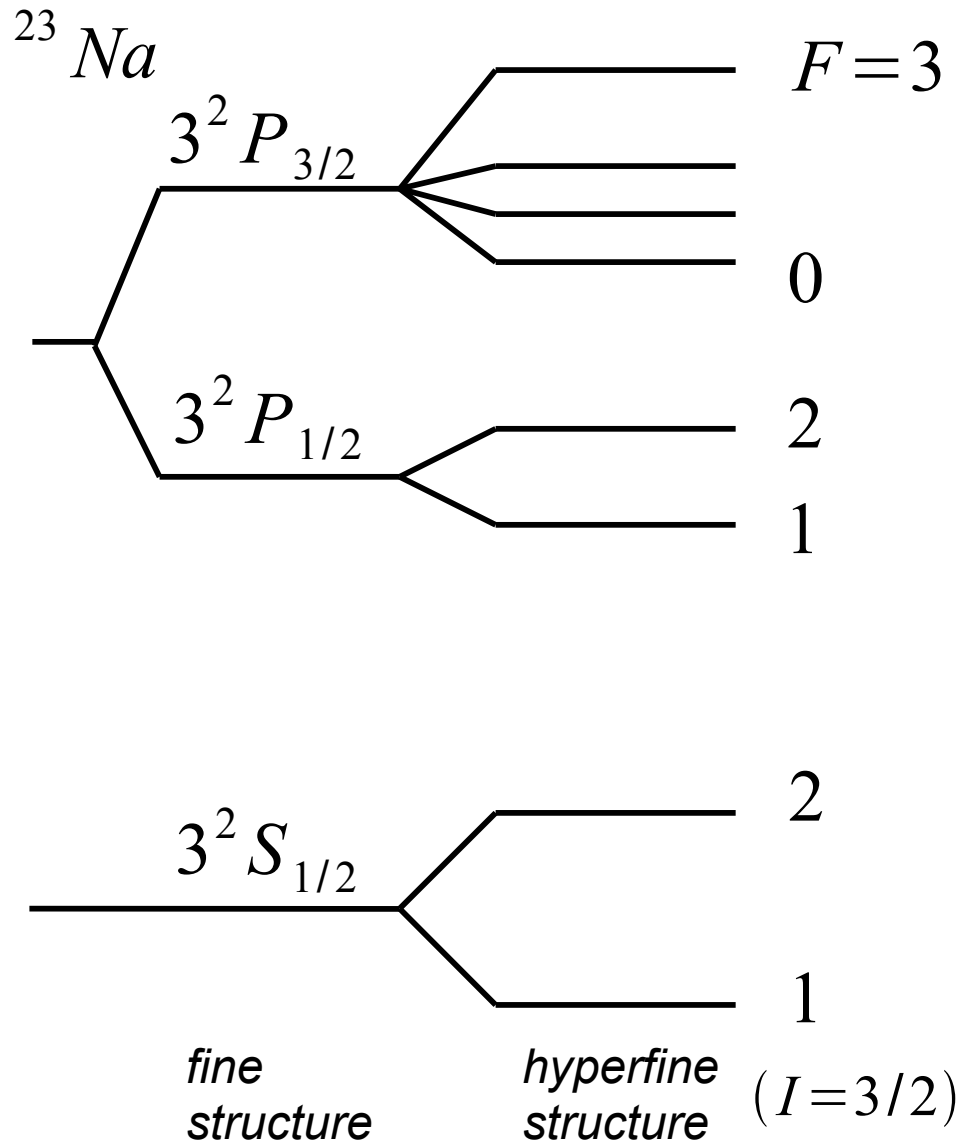
	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
	B 5 10.81 Boron	C 6 12.01 Carbon	N 7 14.01 Nitrogen	O 8 16.00 Oxygen	F 9 19.00 Fluorine	He 2 4.00 Helium
ATES	Al 13 26.98 Aluminum	Si 14 28.09 Silicon	P 15 30.97 Phosphorus	S 16 32.07 Sulfur	Cl 17 35.45 Chlorine	Ar 18 39.95 Argon
12 IIB	Zn 30 65.39 Zinc	Ga 31 69.72 Gallium	Ge 32 72.61 Germanium	As 33 74.92 Arsenic	Se 34 78.96 Selenium	Br 35 79.90 Bromine
	Cd 48 112.41 Cadmium	In 49 114.82 Indium	Sn 50 118.71 Tin	Sb 51 121.76 Antimony	Te 52 127.60 Tellurium	I 53 126.90 Iodine
	Hg 80 200.59 Mercury	Tl 81 204.38 Thallium	Pb 82 207.2 Lead	Bi 83 208.98 Bismuth	Po 84 (209) Polonium	At 85 (210) Astatine
	Unamed Discovery		Unamed Discovery		Unamed Discovery	Unamed Discovery

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Atomic levels

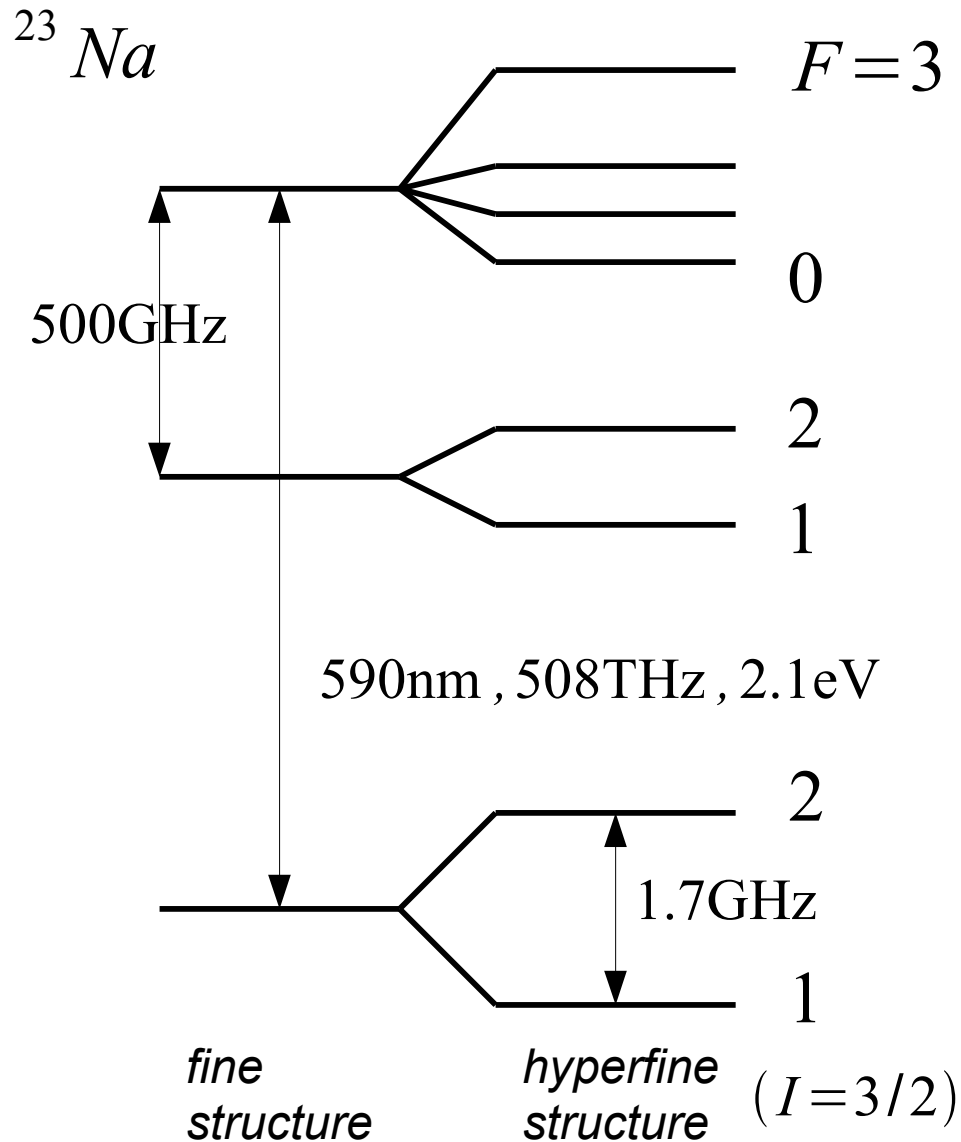


- The basic electronic levels are named

$$n^{2S+1} L_J$$

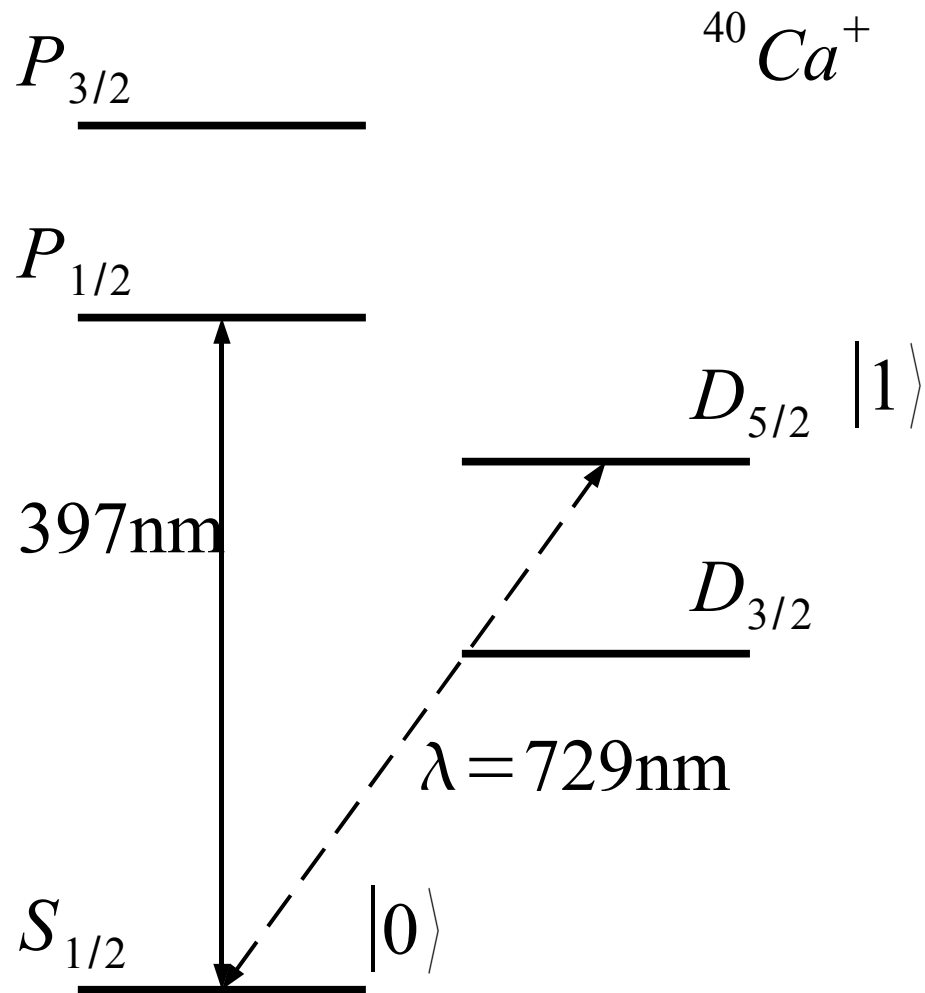
- n = principal Q number
- L = orbital angular momentum
- S = spin
- $J = L + S$
- I = nuclear ang. momt.
- $F = L + I$

Atomic levels



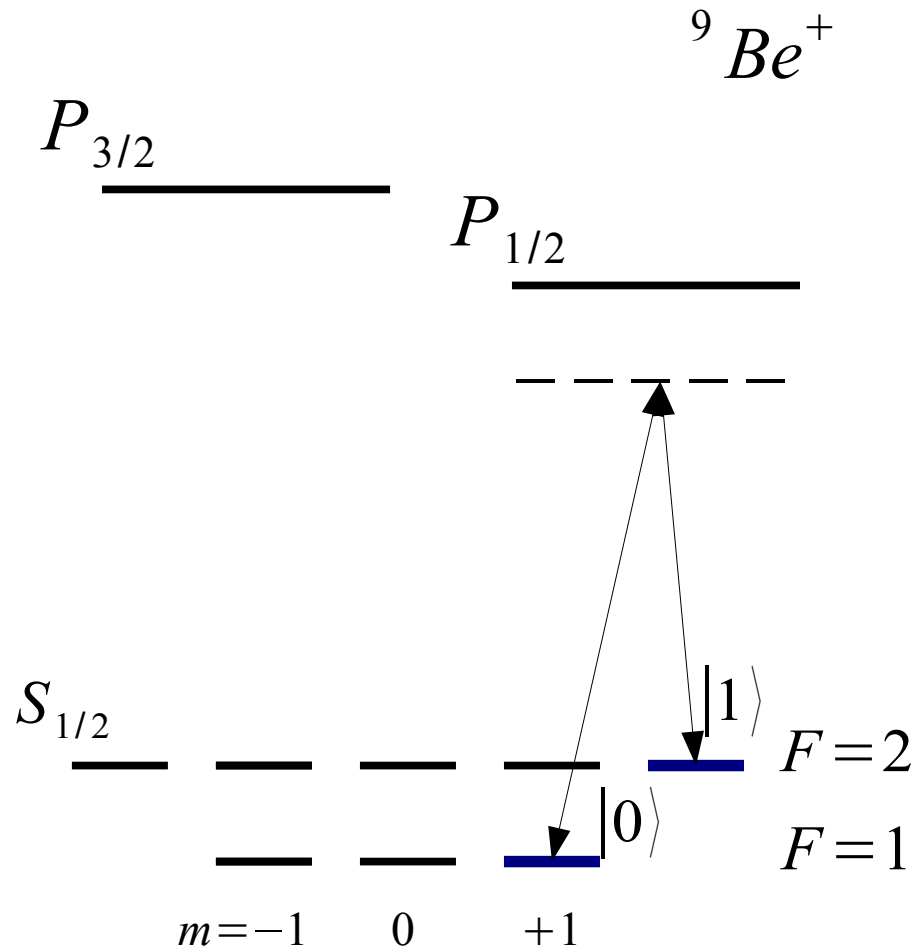
- We get a lot of different energy separations:
 - Optical
 - Energetic microwave
 - Long microwaves
- Not all transitions are permitted
 - $\Delta M = 0$ (π), ± 1 (σ^\pm)
 - $\Delta L = \pm 1$
 - $\Delta J, \Delta F = 0, \pm 1$

Excited state encoding



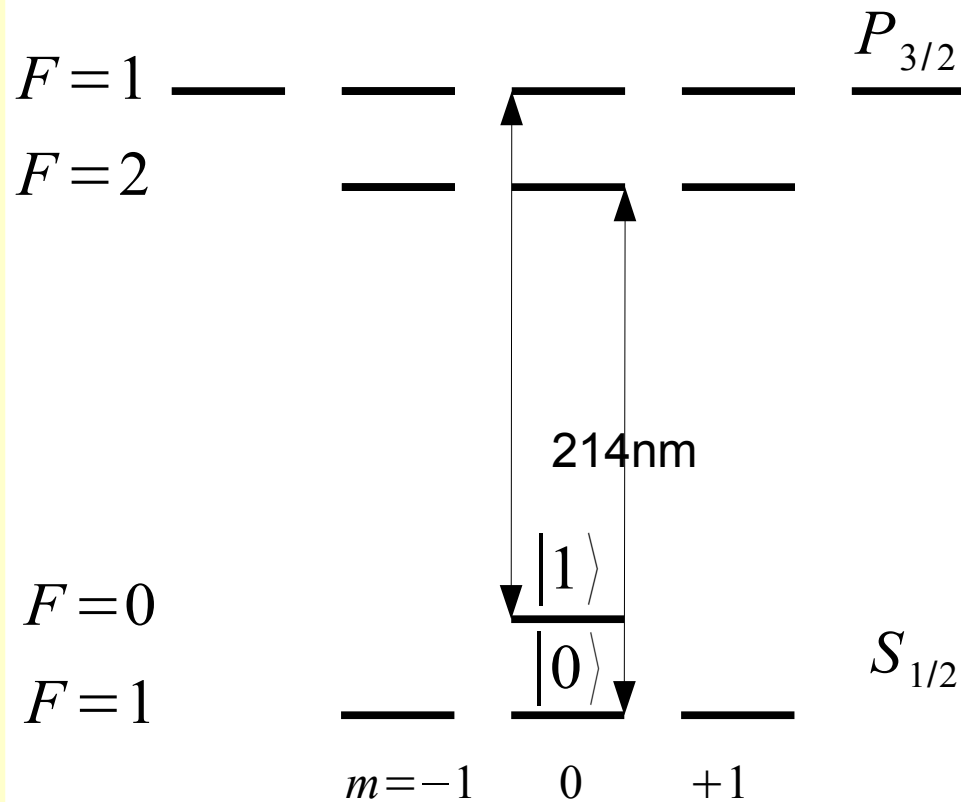
- The $^{40}\text{Ca}^+$ ion is used in the experiments from Innsbruck
- The qubit is encoded in a ground and an excited state.
- No hyperfine splitting.
- The excited state is metastable but long lived 1.5s
- For this the 0-1 transition can not be dipole-allowed
 - Second order processes
 - Strong lasers

Hyperfine encoding



- The ${}^9\text{Be}^+$ ion is used in the experiments from NIST.
- The qubit is encoded in two hyperfine ground states
 - No decay
- States have to be coupled using either
 - Microwave (delicate)
 - Raman transitions
- The choice of states is still sensitive to magnetic fields.

Hyperfine encoding



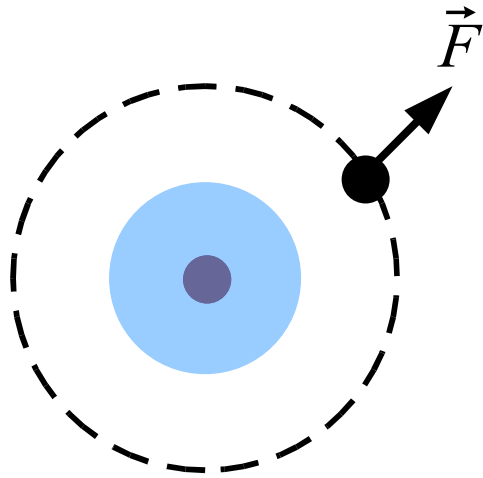
$^{111}\text{Cd}^+$ ($I=1/2$)

- A variant being used in Maryland (Monroe).
- The qubit is stored in hyperfine states.
- Now $m_F=0$, so that coupling to magnetic fields can be prevented.
- The advantage is ultrashort transitions to excited states

Atom manipulation

Atom-light interaction

$$\vec{E}(x, t) \sim e^{i(kx - \omega t)}$$

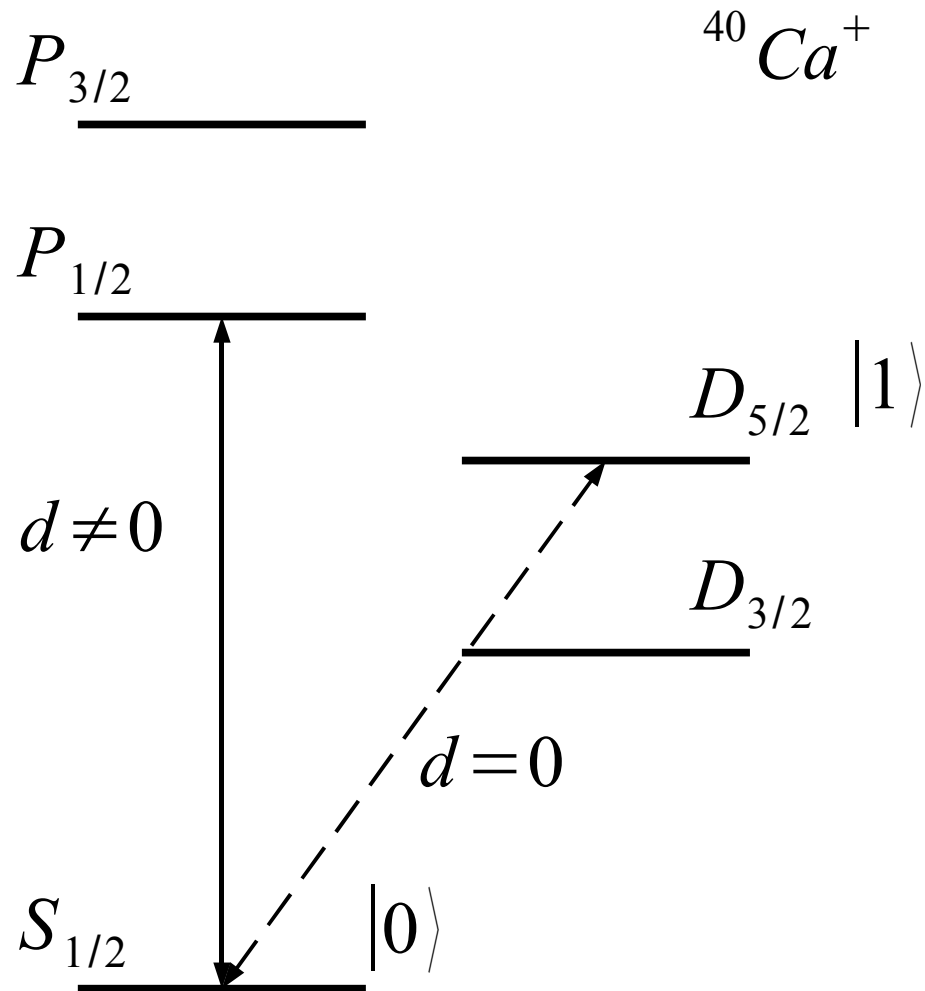


$$\lambda \gg a_0, E(x, t) \sim E(R_{CM}, t)$$

- The atom sees the wave as an oscillating potential.
- We can treat the problem as
 - A light electron
 - trapped by a heavy core
 - subject to a force
- Dipole coupling Hamiltonian

$$\begin{aligned} H = & H_{core} + H_{light} \\ & + \frac{\vec{p}_e^2}{2m} + V_e(\vec{x}) \\ & + \vec{d} \cdot \vec{E} \end{aligned}$$

Atom-light interaction



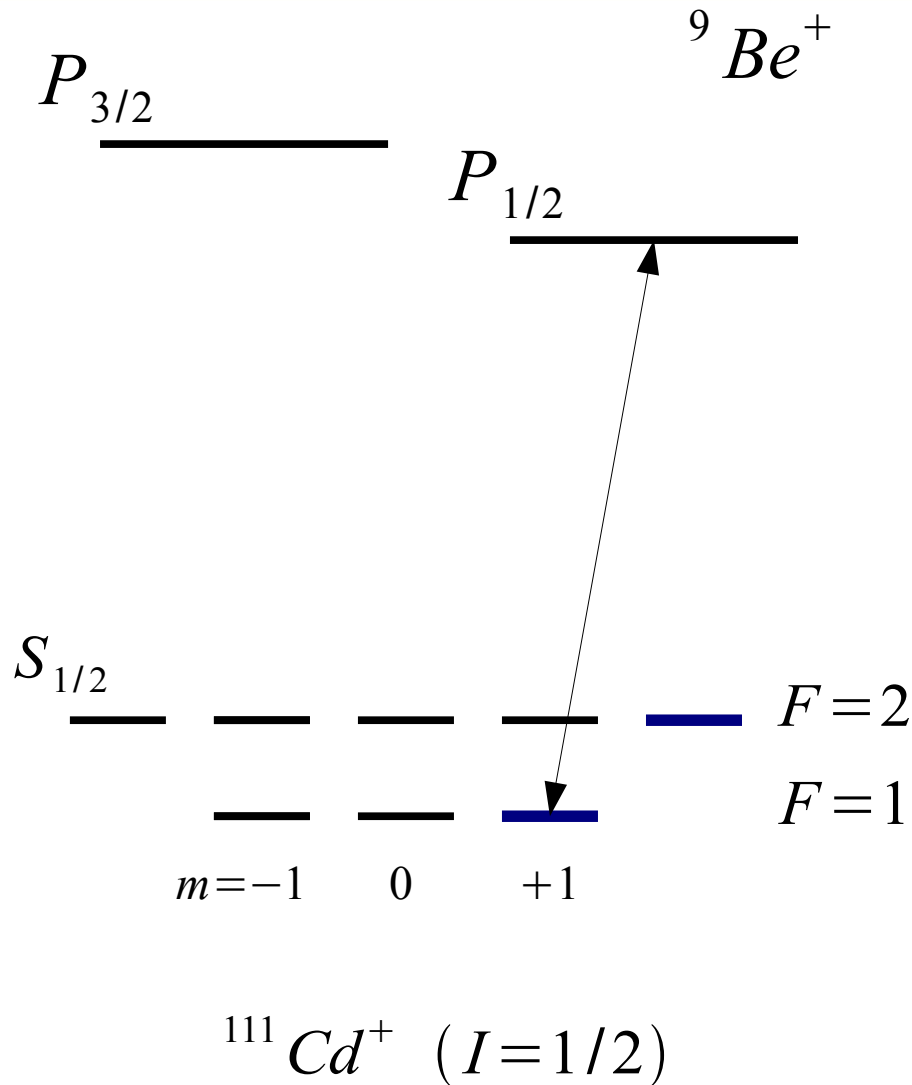
- First we diagonalize everything but the coupling

$$H = \sum_{nFm_F} E_{nFm_F} |nFm_F\rangle \langle nFm_F| + \sum_{\omega} \hbar \omega a_{\omega}^{\dagger} a_{\omega} + \vec{d} \cdot \vec{E}$$

- The dipole moment only has elements between different states

$$\vec{d} = \sum d_{ge} (|e\rangle \langle g| + |g\rangle \langle e|)$$

Atom-light interaction



- The coupling ends up in the form

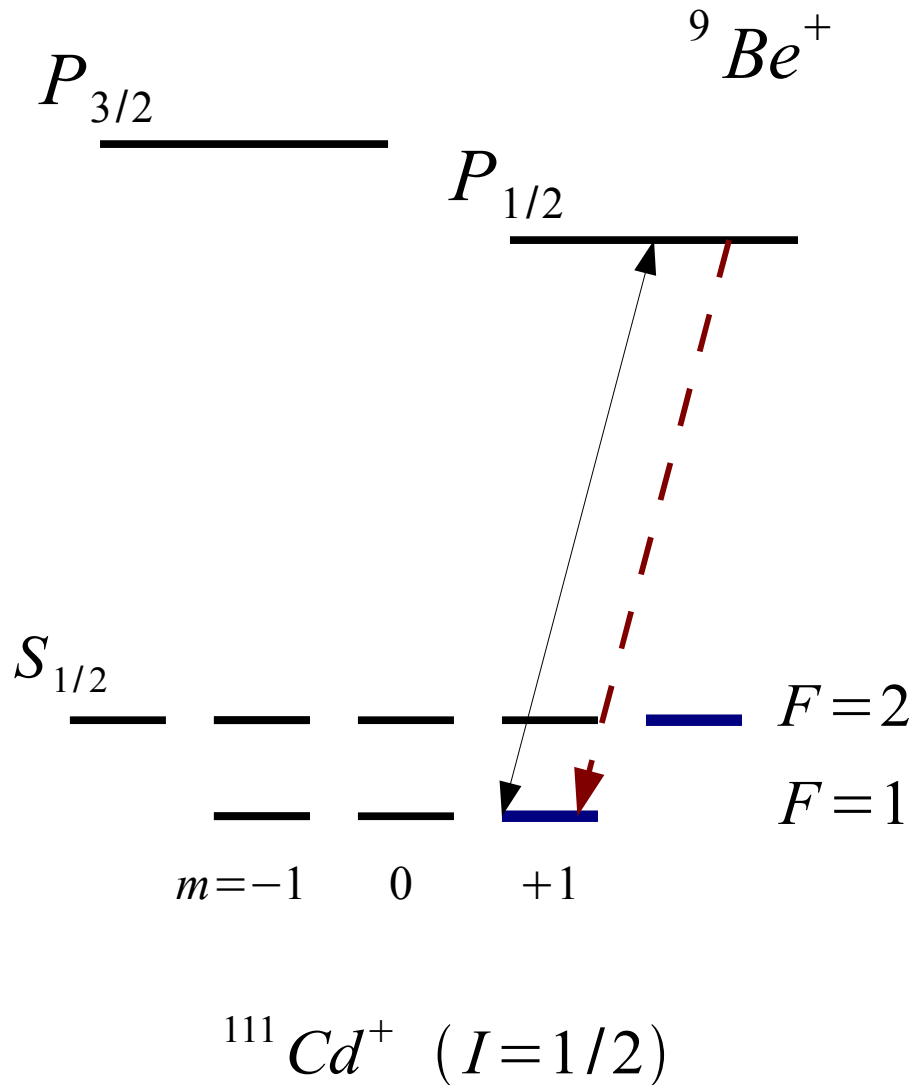
$$H_{dip} = \Omega (|e\rangle\langle g| + |g\rangle\langle e|) \times (a^+ + a)$$

- For long times, the energy nonconserving terms can be neglected

$$H_{RWA} = \Omega (|e\rangle\langle g| a + a^+ |g\rangle\langle e|)$$

RWA = rotating wave approximation

Spontaneous emission



- There are other coupling channels

$$H_{dip} = \sum_{k \neq k'} \Omega (|e\rangle\langle g| + |g\rangle\langle e|) \times \times (a_k^+ + a_k)$$

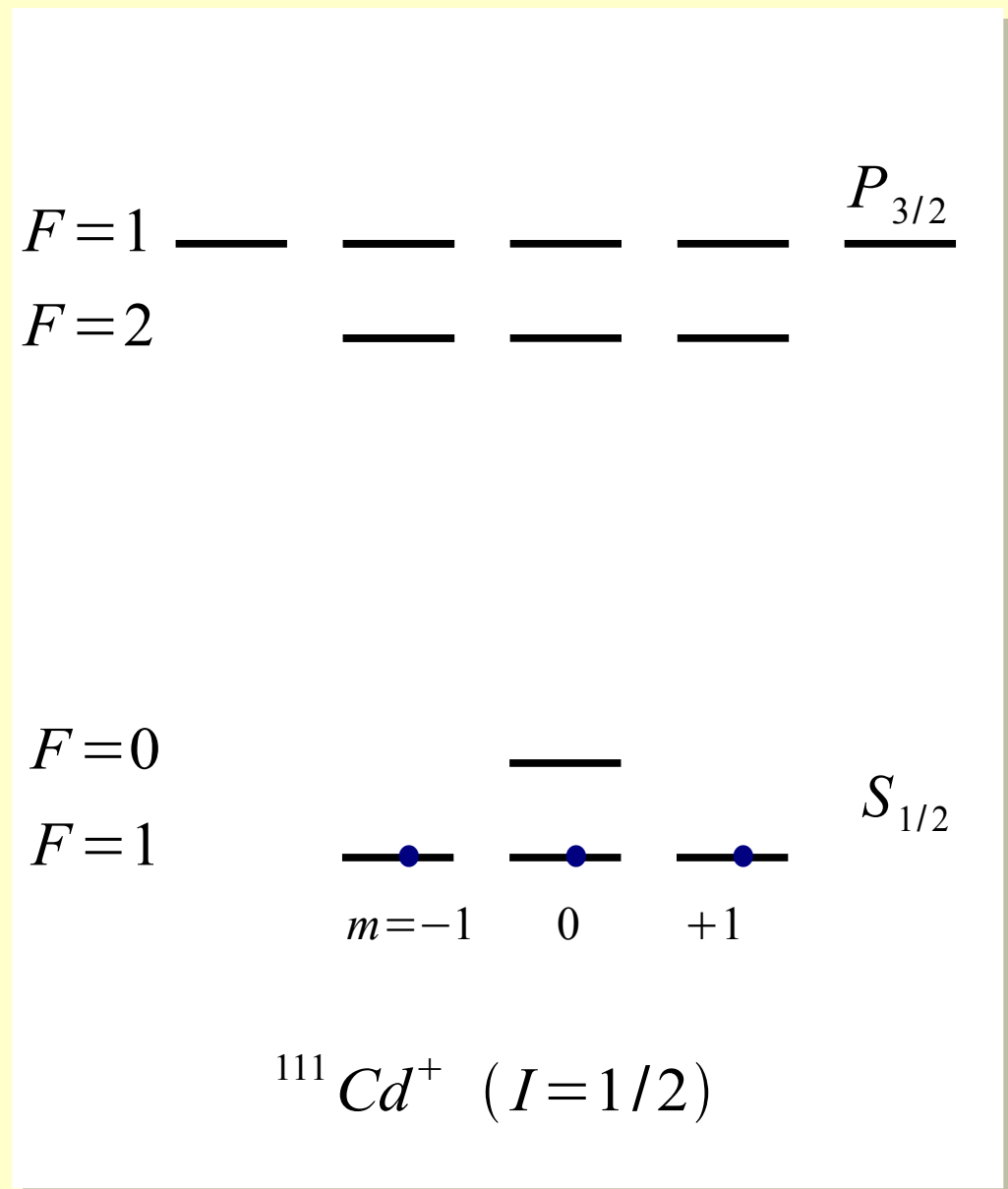
- Atom emit into **different** modes from absorbed photon
- When tracing out the lost photons, decoherence

$$\rho \rightarrow (1 - \varepsilon) \rho + \varepsilon \sigma^- \rho \sigma^+$$

Quantum register preparation

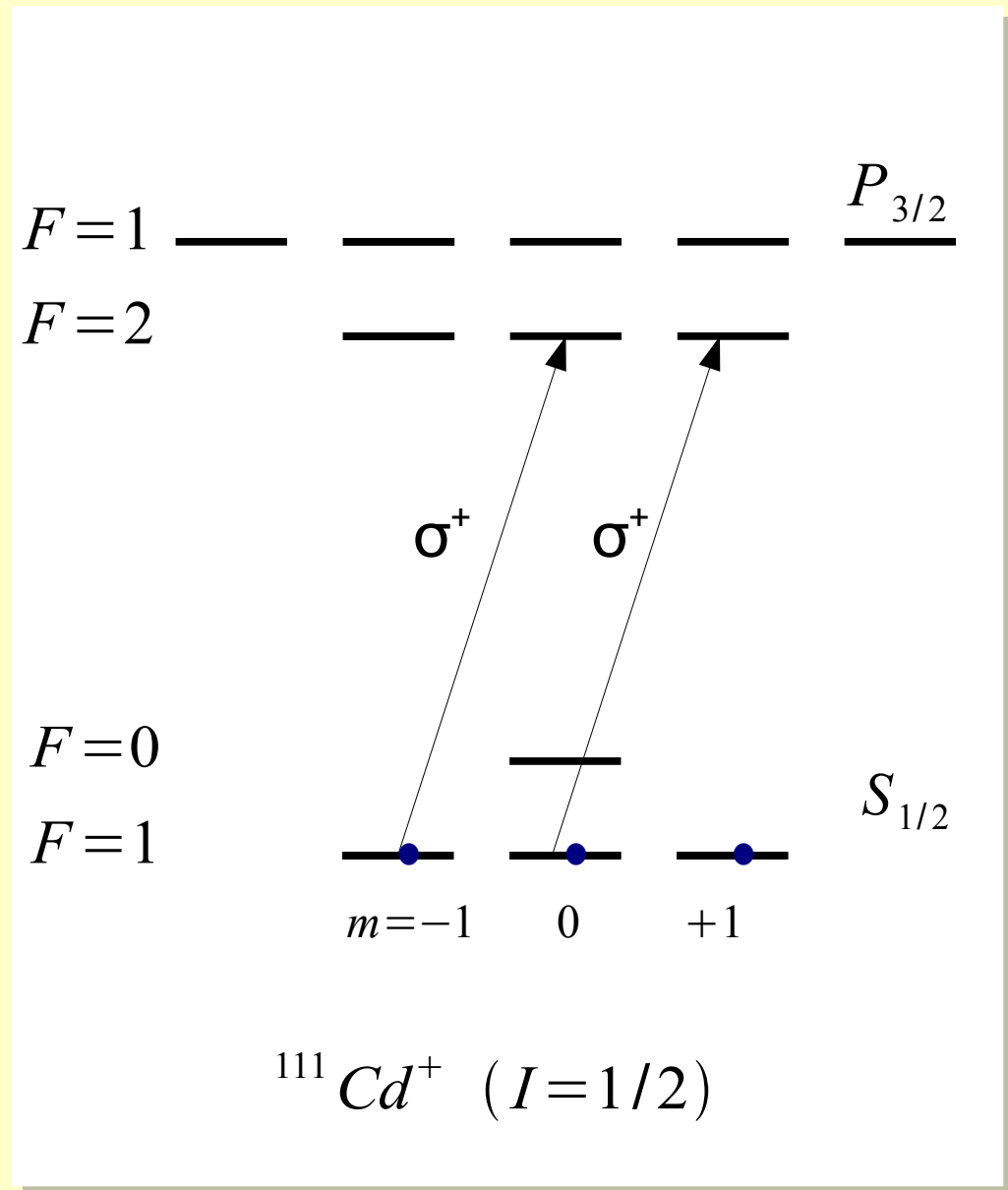
Quantum register preparation

- An essential step in the quantum computation.
- We do not need to prepare an arbitrary state.
- We just need to **reset** the ions to the **same state** and use unitaries.



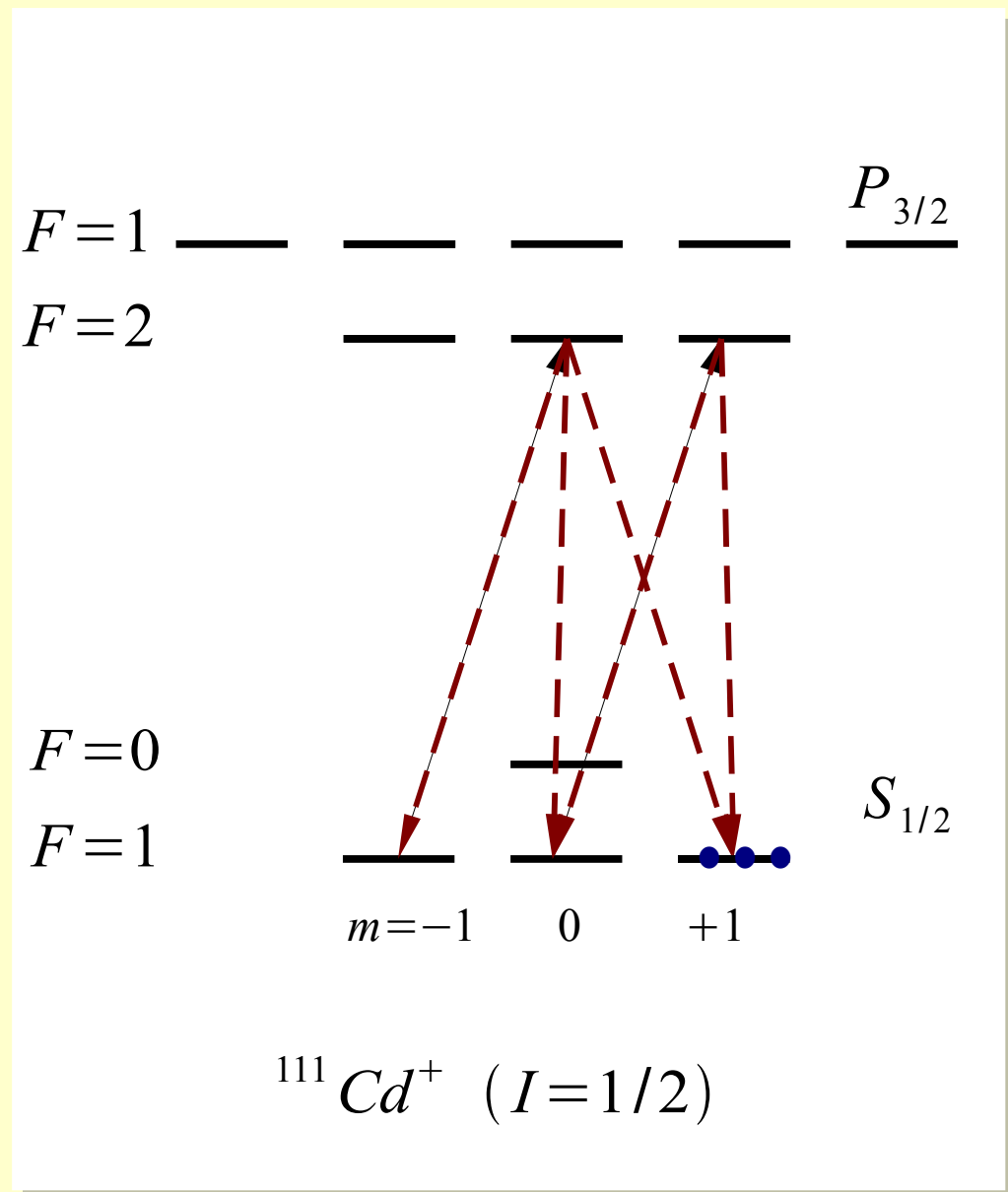
Optical pumping

- An essential step in the quantum computation.
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- We pump atoms with the same polarization, σ^+



Optical pumping

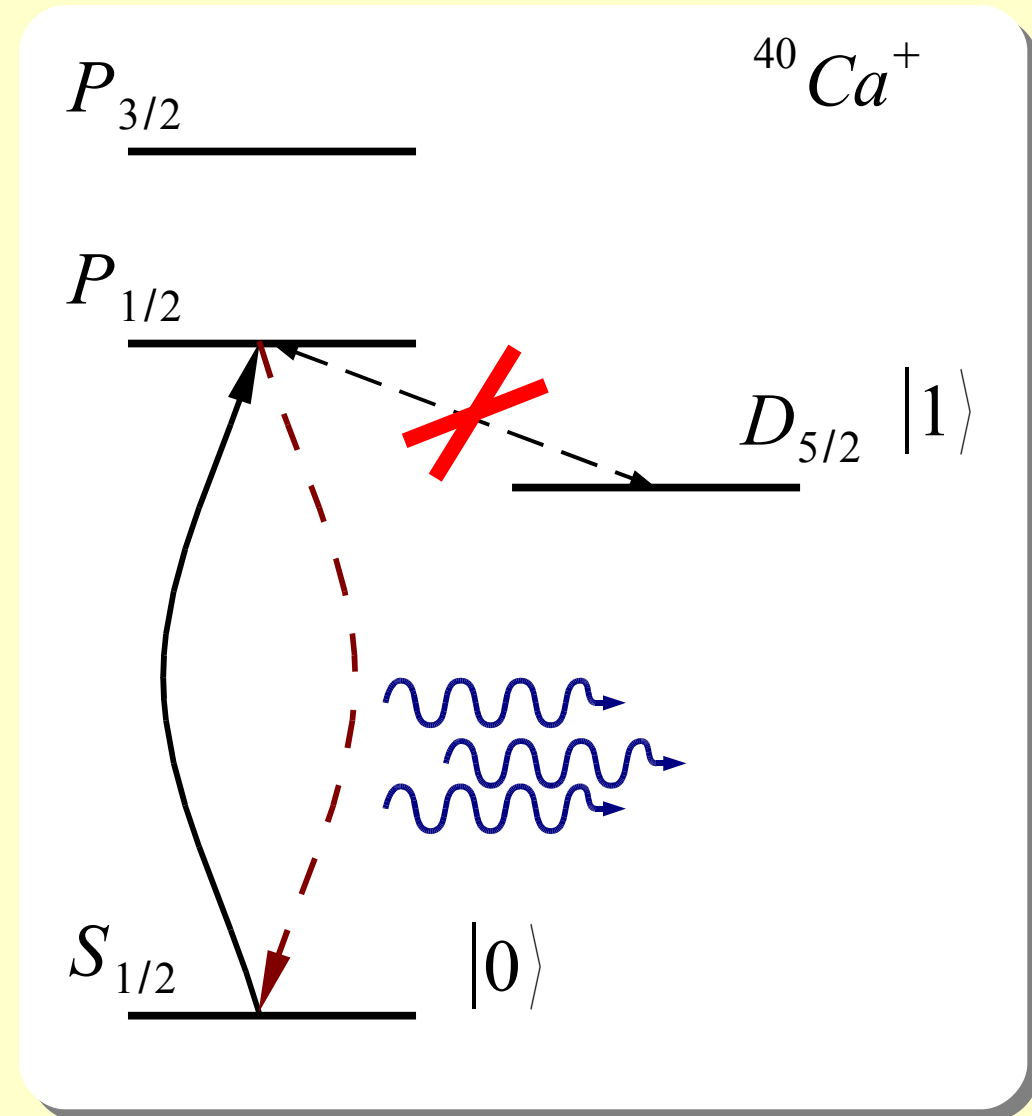
- An essential step in the quantum computation.
- We do not need to prepare an arbitrary state.
- We just need to **reset** the ions to the **same state** and use unitaries.
- We pump atoms with the same polarization, σ^+
- A fraction of the atoms decays with smaller m_F
- Total net flow towards $m_F = +1$



Measurements

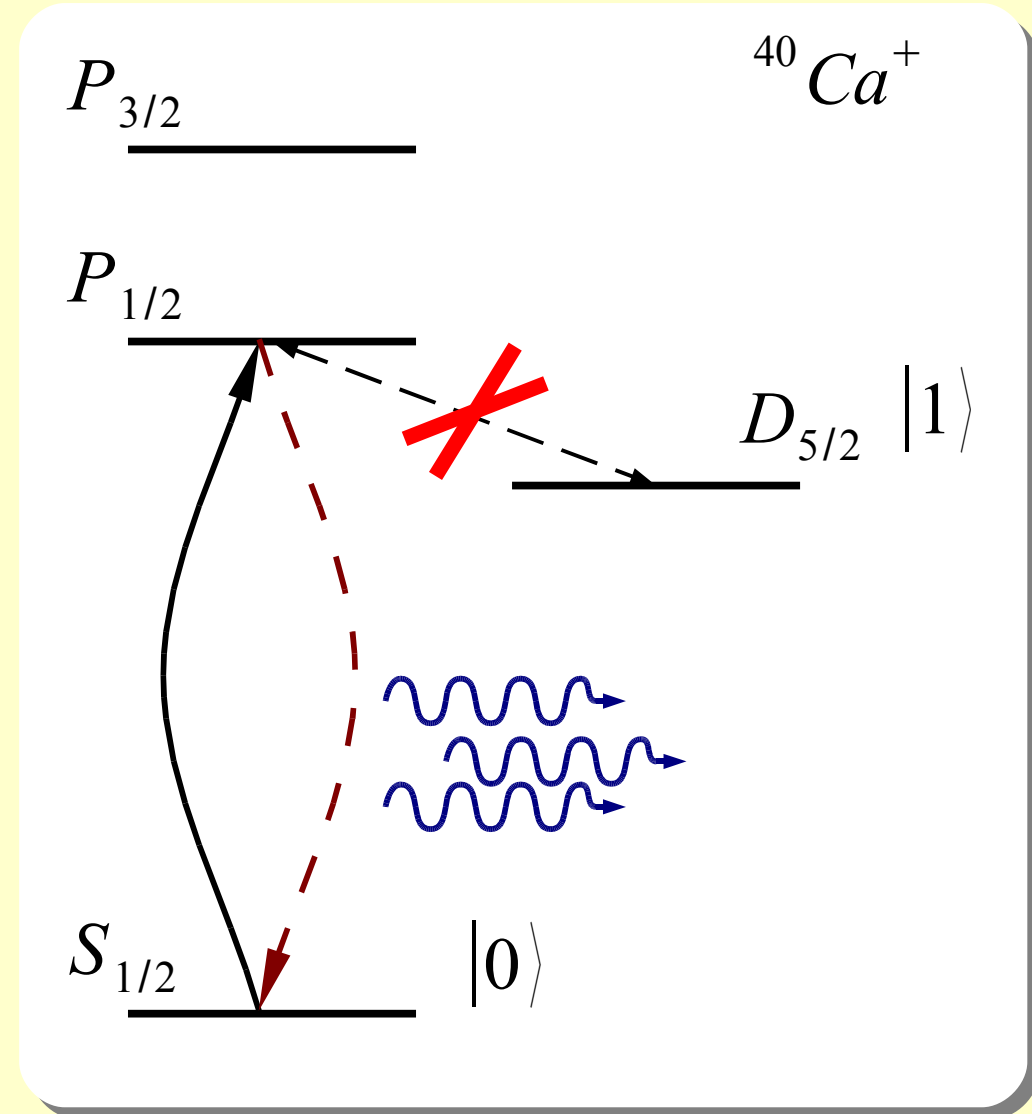
Measurement

- A process to accurately distinguish the qubit 0 and 1.
- **Electron shelving**
 - Pump the 0 state to an auxiliary state.
 - The transition is forbidden for the 1 state.
 - The excited state decays emitting photon
 - Repeat until enough photons are gathered.



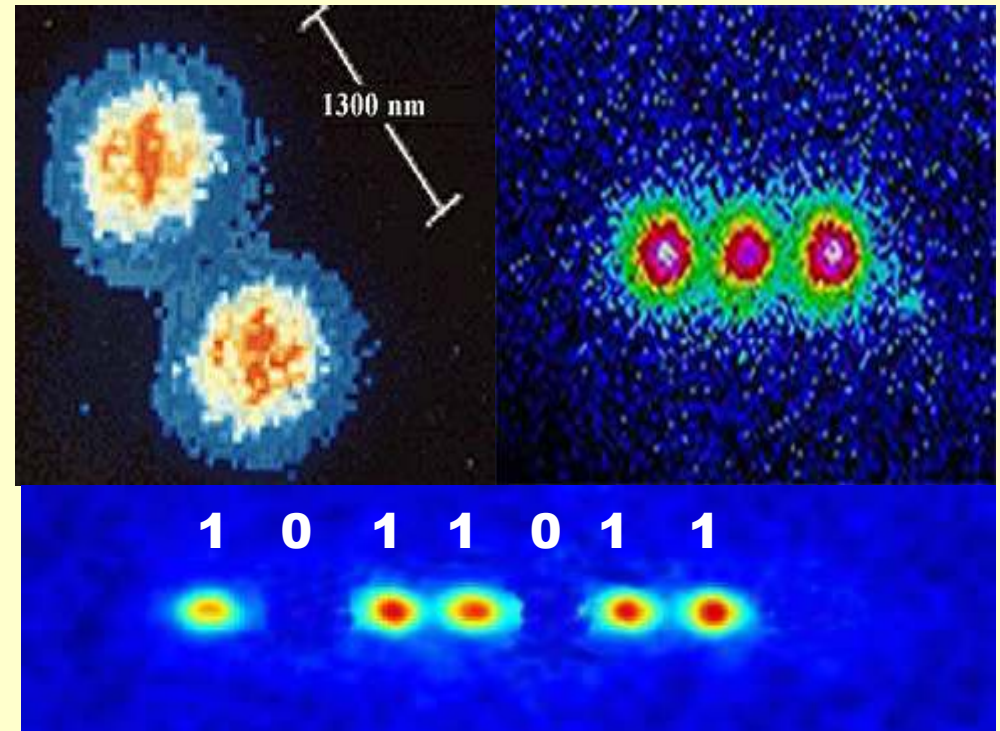
Measurement

- Projective measurement
 - Atom ends up in 0 or 1
 - It can be repeated indefinitely.
- Lots of photons compensate for bad detector efficiency
 - Almost 100% accuracy
 - Best qubit measurements ever!
- Better done at the end
 - Scattered photons may affect neighboring atoms.



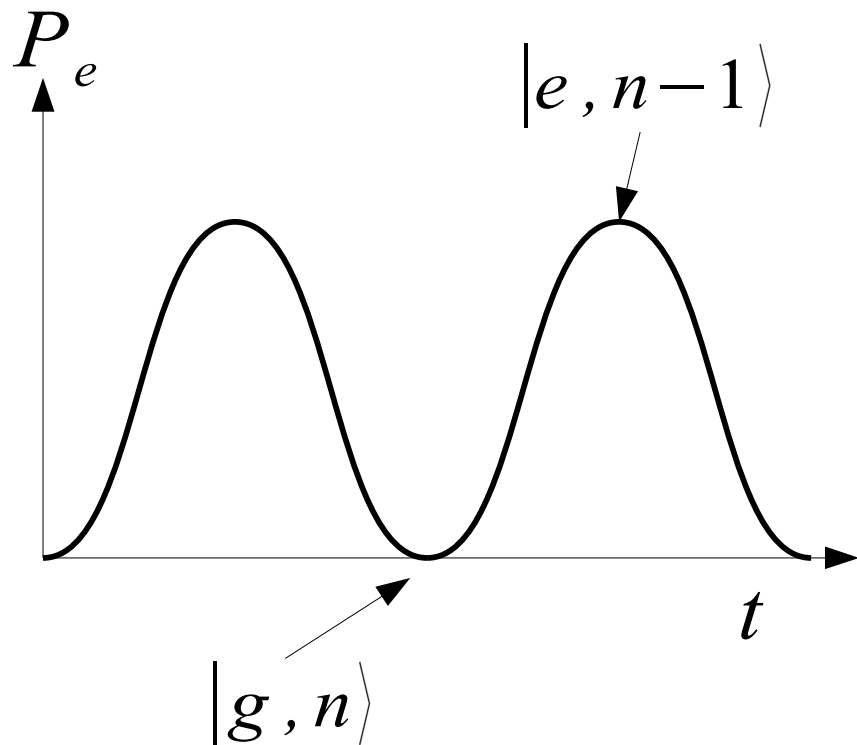
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Single qubit rotations

Rabi oscillations

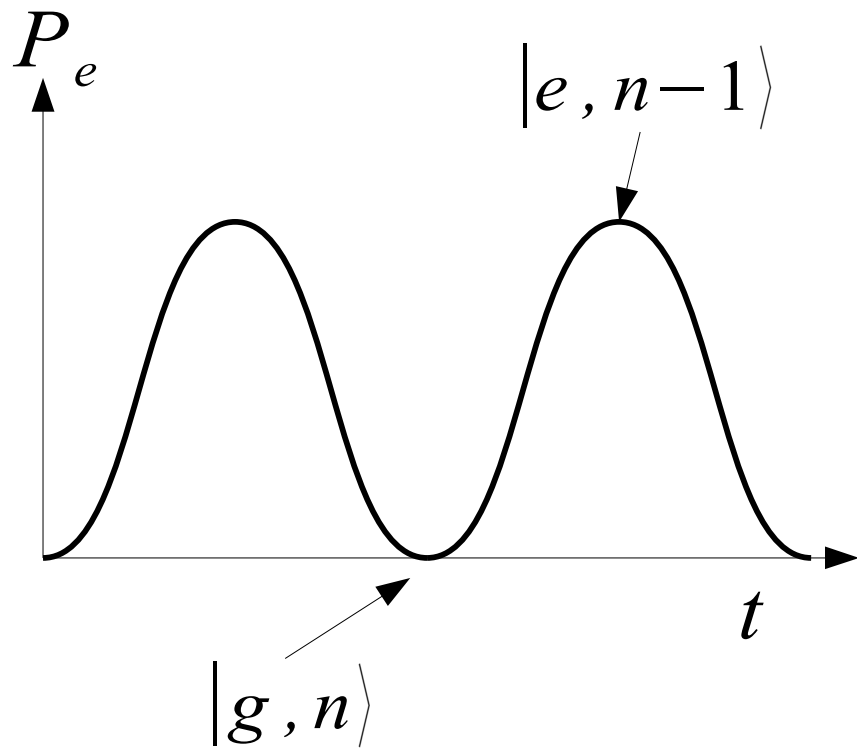


- An ideal but realistic case is when only two levels are coupled:

$$H_{RWA} = \omega_a \sigma_z + \omega_l a^\dagger a + \Omega (|e\rangle\langle g| a + a^\dagger |g\rangle\langle e|)$$

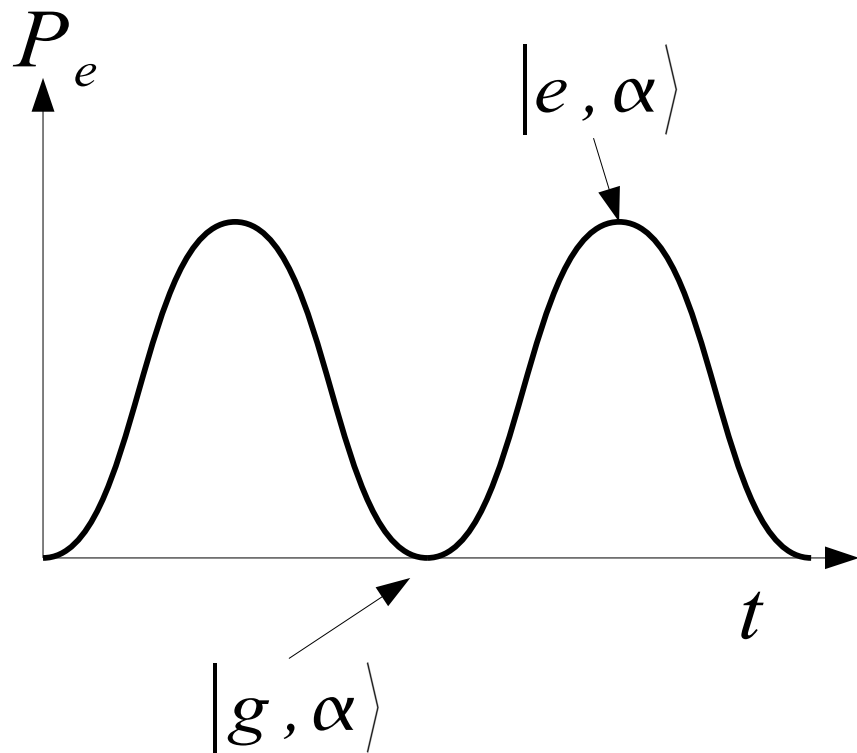
- Physical parameters
 - Detuning $\delta = \omega_l - \omega_a$
 - Rabi frequency Ω
- Integrable, oscillations between ground and excited state.

Rabi oscillations



- Not useful for doing qubit gates: Light and atom are entangled.

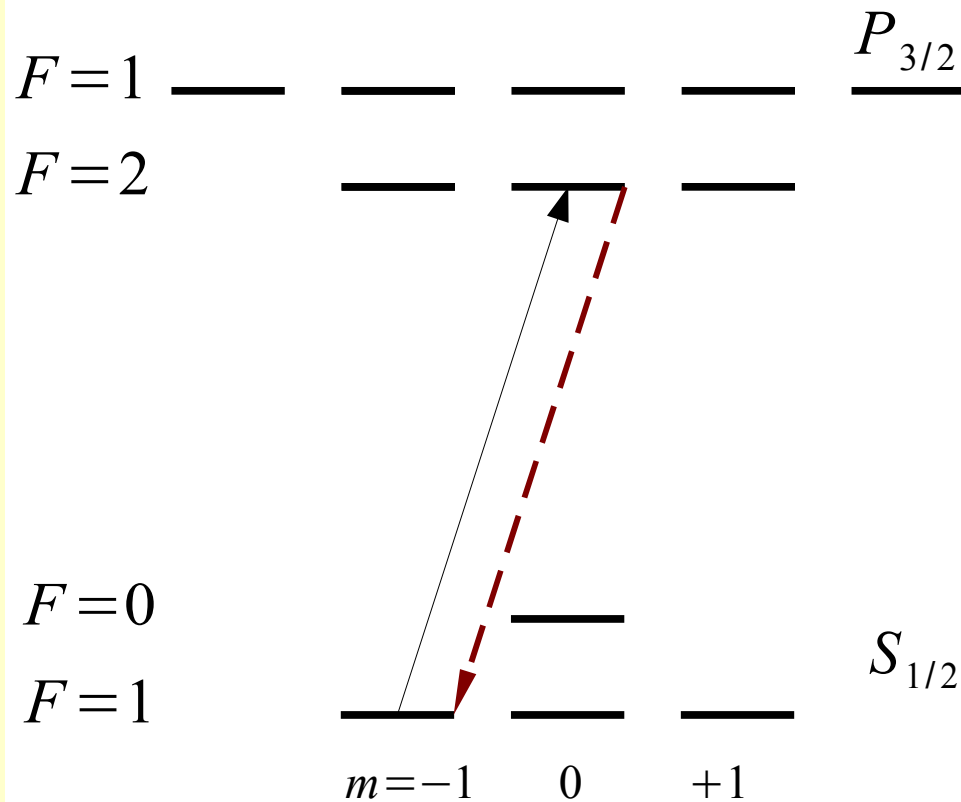
Rabi oscillations



- Not useful for doing qubit gates: Light and atom are **entangled**.
- We have to consider the incoming light as “classical” coherent beam.

$$a|\alpha\rangle \propto |\alpha\rangle$$

Rabi oscillations



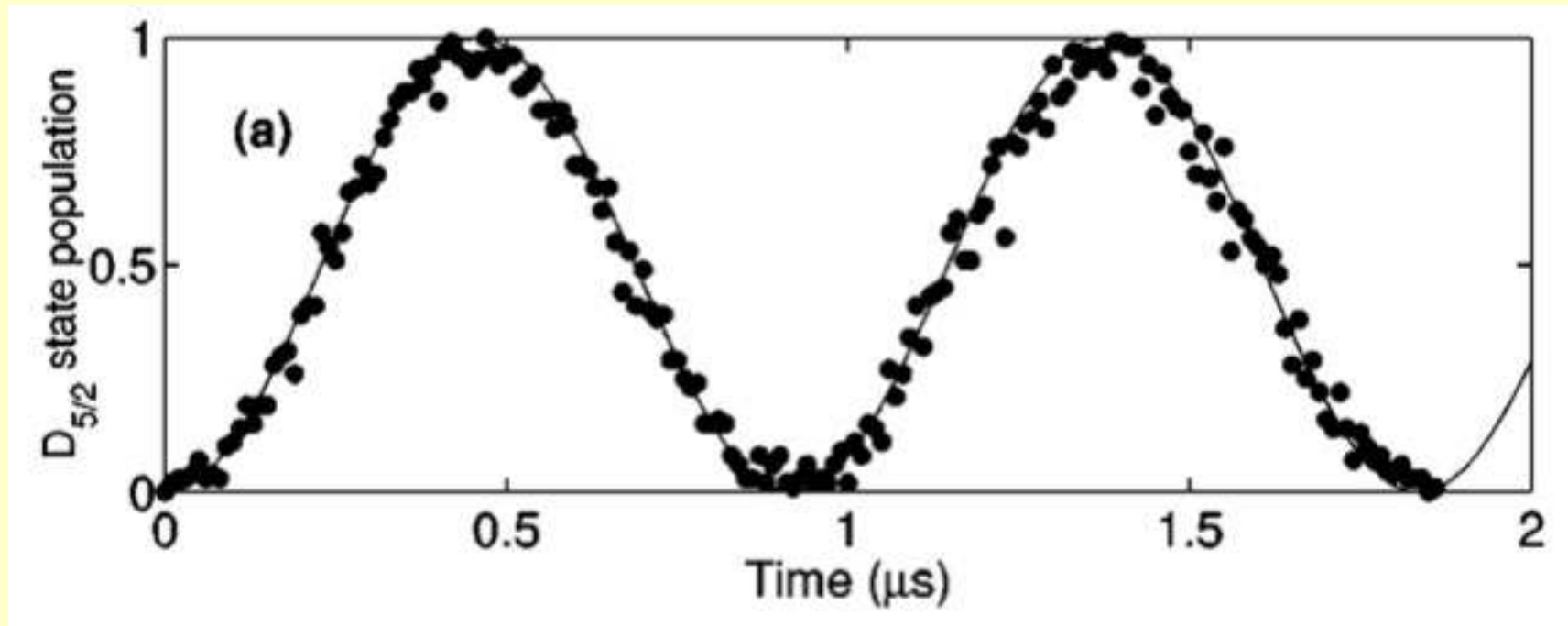
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$$a|\alpha\rangle \propto |\alpha\rangle$$

- But using resonant processes means the atom can **decay!**

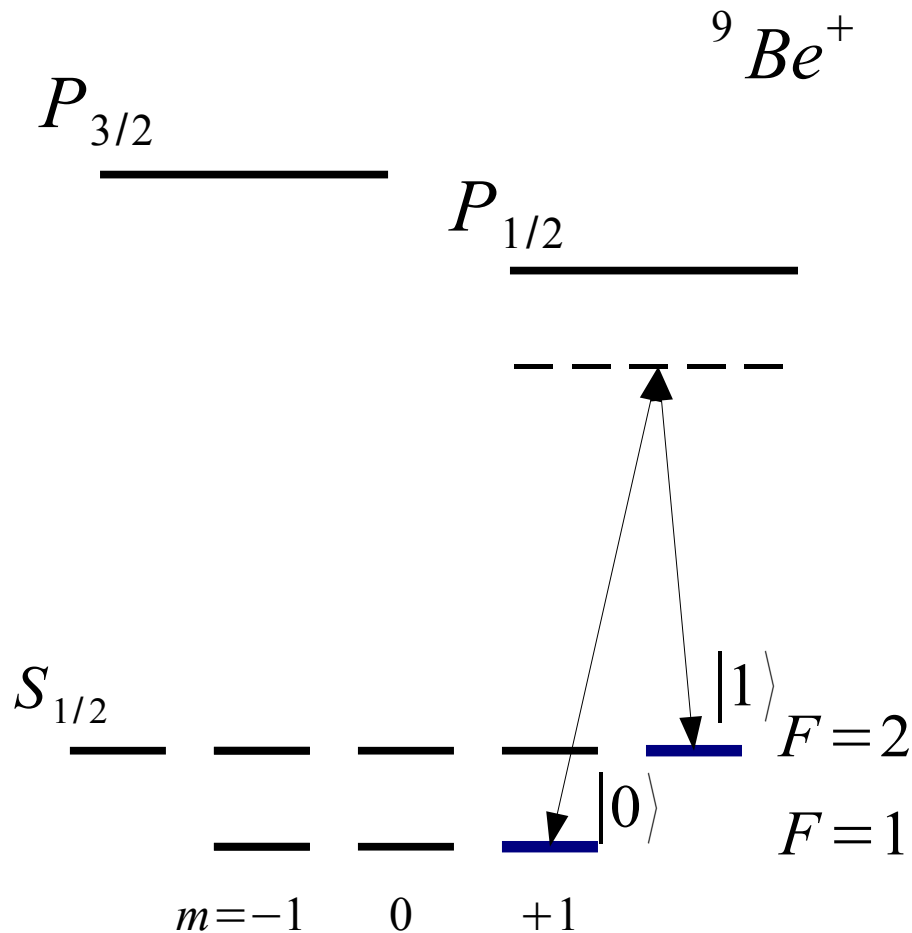
Rabi oscillations



- Rabi oscillations for a trapped Ca ion

A. Steane et al, *Speed of ion-trap quantum-information processors*,
Phys. Rev. A **62**, 042305 (2000).

Raman transitions



- When the light is largely detuned (**off-resonance**)

$$\delta = \omega_l - \omega_a \gg \Omega$$

we have to consider second order processes.

- The effective Hamiltonian may include coupling between hyperfine levels

$$H_{eff} \sim \hbar \frac{\Omega^2}{\Delta} (|1\rangle\langle 0| + |0\rangle\langle 1|) + \frac{E_{hfs}}{2} (|1\rangle\langle 1| - |0\rangle\langle 0|)$$

Arbitrary unitaries

- With this Hamiltonian we have enough to perform **any** single-qubit rotation

$$H_{eff} = \hbar \frac{\Omega^2}{\Delta} (|1\rangle\langle 0| + |0\rangle\langle 1|) + \frac{E_{hfs}}{2} (|1\rangle\langle 1| - |0\rangle\langle 0|)$$

identify Pauli operators

$$H_{eff} = \hbar \frac{\Omega^2}{\Delta} \sigma_x + \frac{E_{hfs}}{2} \sigma_z$$

Both Ω and E_{hfs} can be tuned by changing the laser intensity and applying magnetic fields.

- H, S, Z are direct. T requires combining X and Z rotations.