Ions: trapping an interactions

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Ingredients for QIPC

- **Quantum degrees of freedom**
- Local operations
- Measurements
- One of
  - Entangled state sources
  - Universa 2qb unitaries
- Error correcting schemes
- Large number of qubits

Atomic internal states, light – matter interaction

Q. Computation

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Q. Computation

How shall we TRAP and MANIPULATE the atoms?

Ions

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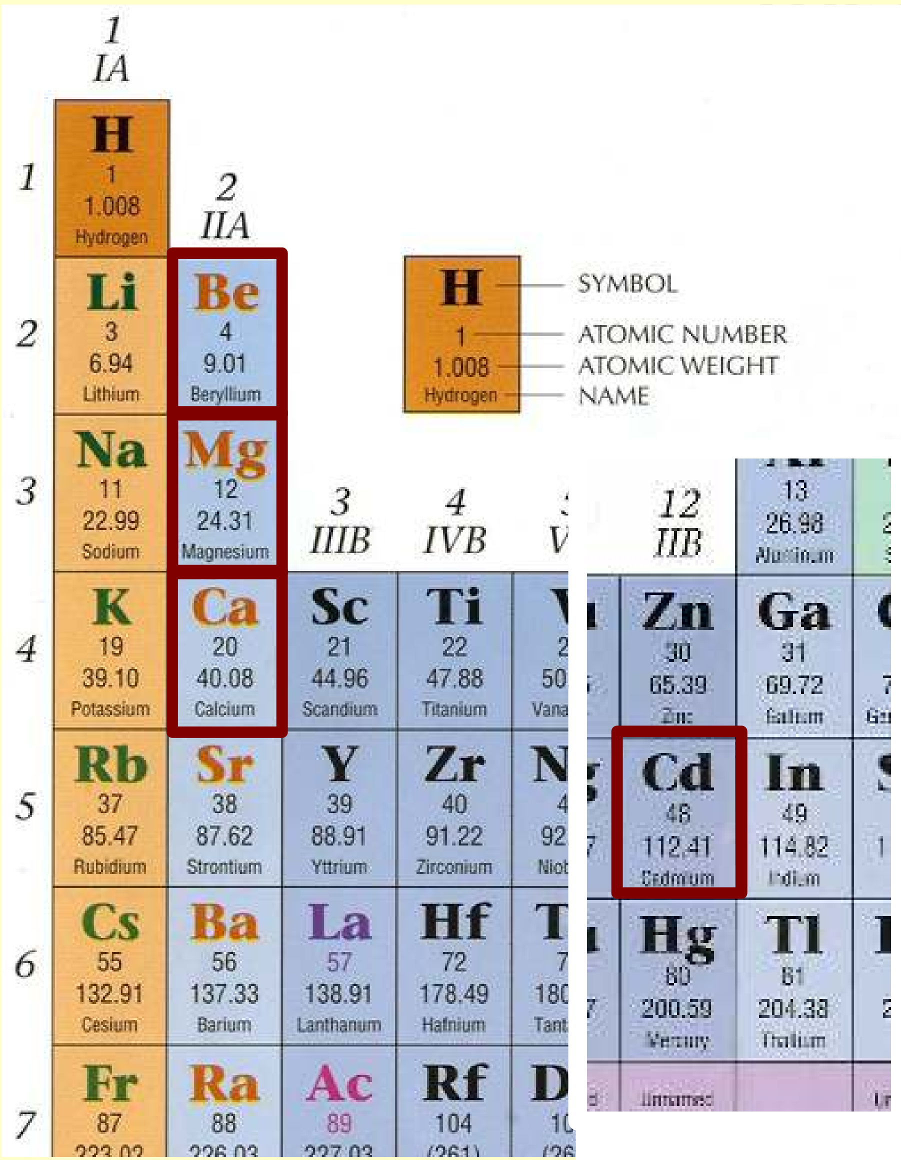
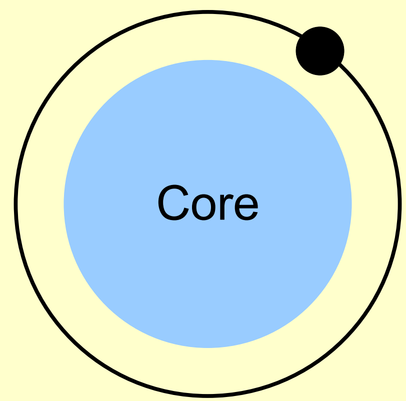


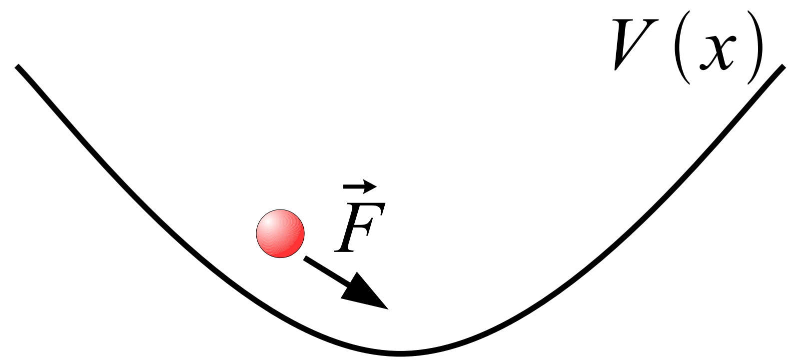
- Atoms that have been subtracted one of the outer electrons.
- Normally restricted to hydrogen-like final states



- Charged, react to electric and magnetic fields.

Trapping

- To trap we need a potential and a local minimum of energy.
- This is impossible with static electric fields
  - No divergence means derivative cannot change sign.
  - Maxima and minima are located at the charges.
- We will need time dependence.



Desired	***Maxwell Eq***
$\vec{F} = \frac{dV}{dx} = 0$	$\vec{F} = e\vec{E}$
$\frac{d^2V}{dx^2} > 0$	$\nabla \cdot \vec{E} = 0$

Micromotion

- Assume that a particle is subject to a rapidly oscillating potential

$$\ddot{x} = g(x)\cos(\omega t)$$

- The change is so fast, that the motion can be split into a slow drift and rapid oscillations

$$x(t) = x_0(t) + x_1(t), \quad |\dot{x}_0| \ll |\dot{x}_1|$$

- Given that the fast displacements are small on average

$$\ddot{x} \simeq \ddot{x}_1 \simeq [g(x_0) + g'(x_0)x_1]\cos(\omega t) \simeq g(x_0)\cos(\omega t)$$

they can be integrated giving a particle that follows the force

$$x_1(t) = -\frac{1}{\omega^2} g(x_0)\cos(\omega t)$$

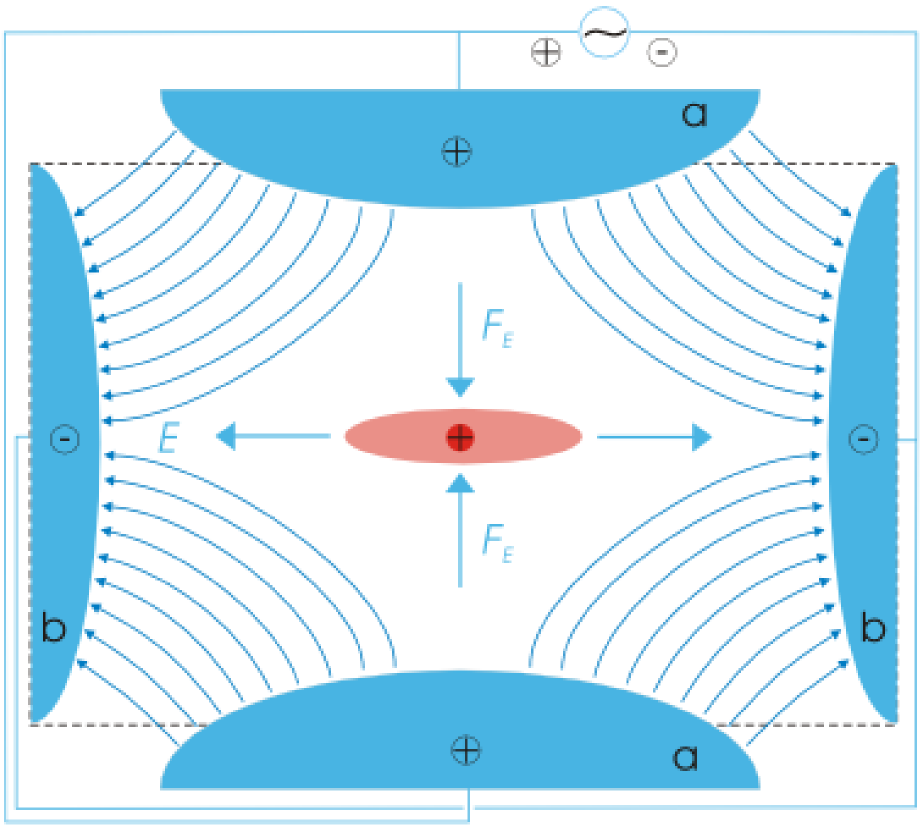
http://www.nature.com/physics/looking-back/boot/index.html

Ponderomotive force

- So now we have to combine
$$\begin{cases} \ddot{x}_0 \simeq g'(x_0)\, x_1 \cos(\omega t) \\ x_1 \simeq -\dfrac{1}{2\omega^2} \cos(\omega t)^2 \dfrac{d}{dx} g^2 \end{cases}$$
- Substituting the micromotion, we obtain an average force
$$x_1(t) = -\frac{1}{\omega^2} g(x_0) \cos(\omega t)$$
- On average over a period, we obtain the ponderomotive force
$$\ddot{x}_0 \simeq -\frac{1}{4\omega^2} \frac{d}{dx} g^2$$
- For a charged particle $F_{eff} = -\dfrac{e^2}{4\omega^2} \nabla |\bar{E}|^2 \rightarrow V_{eff} = \dfrac{e^2}{4\omega^2} |\bar{E}|^2$

http://www.nature.com/physics/looking-back/boot/index.html

Paul traps



- Our oscillating potential will be hyperbolic

$$V(\vec{r}, t) \sim (x^2 - y^2)\cos(\omega t)$$

- The resulting force is harmonic

$$V_{eff} \sim \frac{e^2}{4\omega^2}(x^2 + y^2)$$

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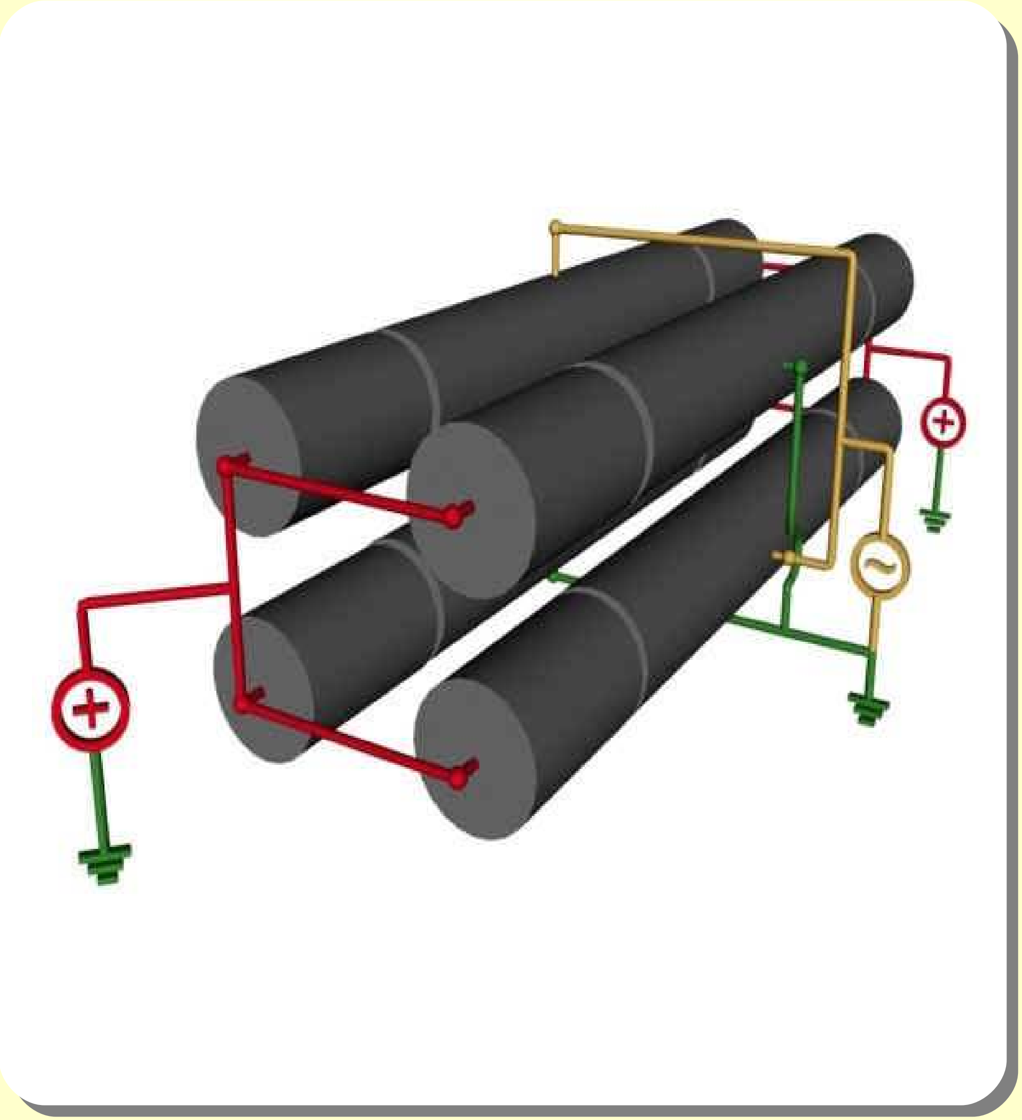

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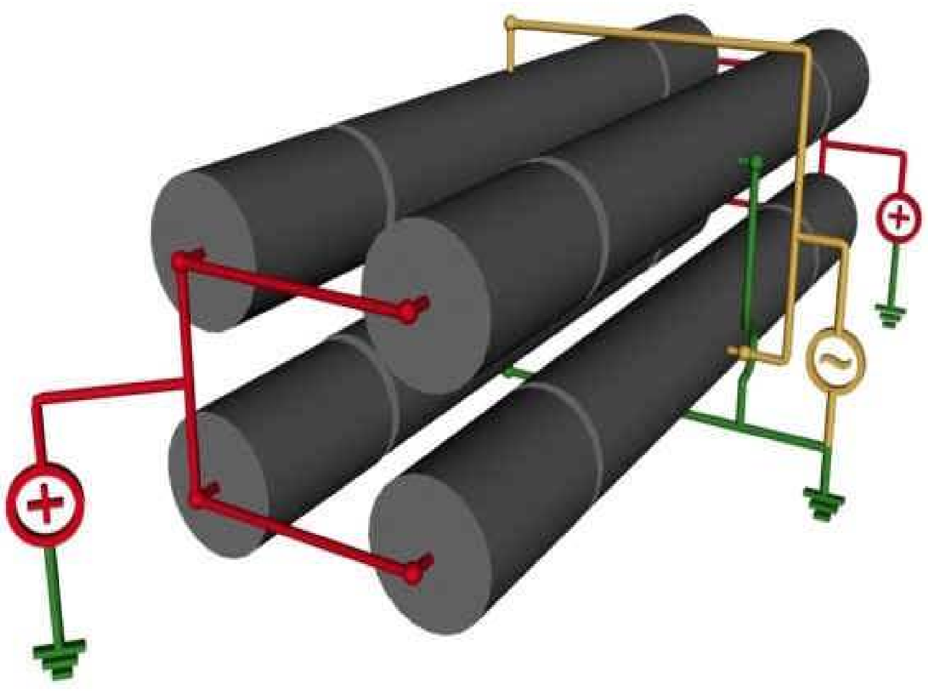
- The potential is created by four rods in alternating potentials

Paul traps



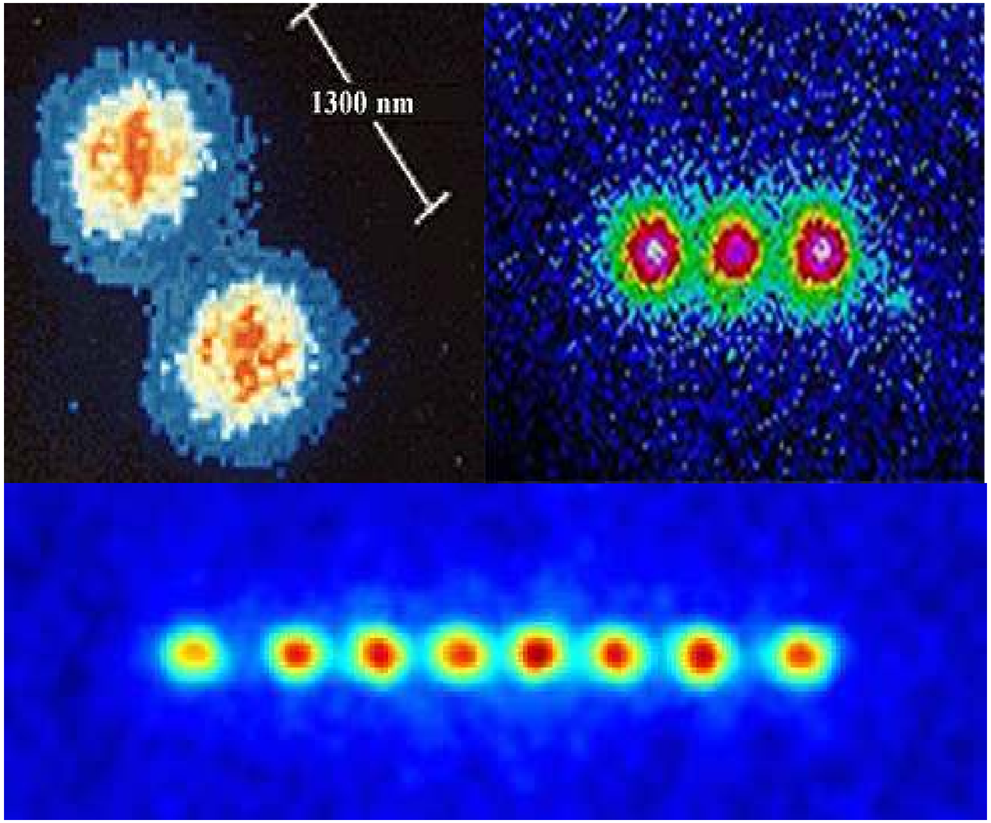
- The outer rods confine particles longitudinally

$$V_{eff} \sim \frac{1}{2} m \omega_{perp}^2 (x^2 + y^2) + \\ + \frac{1}{2} m \omega_z^2 z^2$$

- Tight transverse confinement

$$\omega_z \ll \omega_{perp}$$

Paul traps



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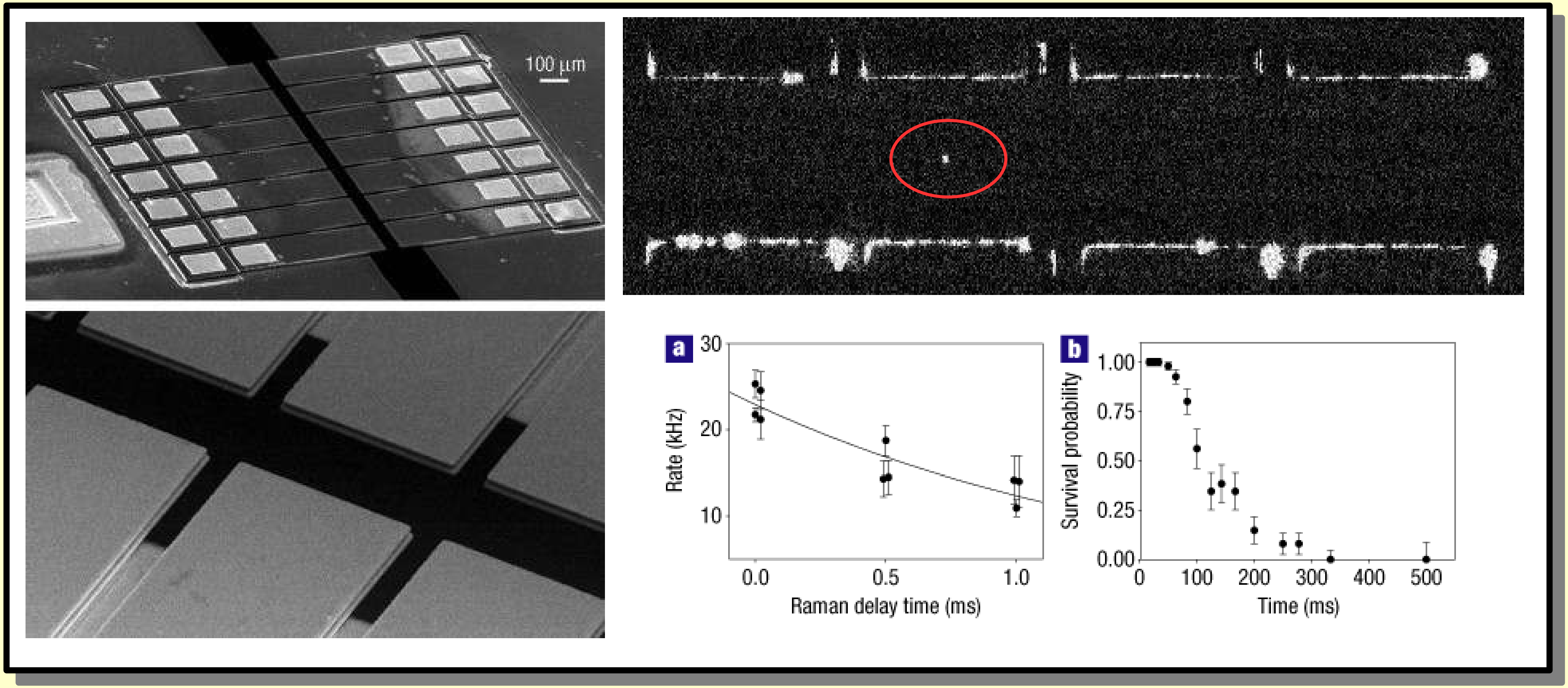
- Tight transverse confinement

$$\omega_z \ll \omega_{perp}$$

$$\omega \sim 2\pi \times (2-4)\, MHz$$

- Metastable Coulomb crystals.

Microtraps



- Repeat the same design using microfabricate circuits.
- More electrodes: finer control over ions positions and potentials.
- Nasty effect due to interaction with surfaces – decoherence.

Quantization of motion

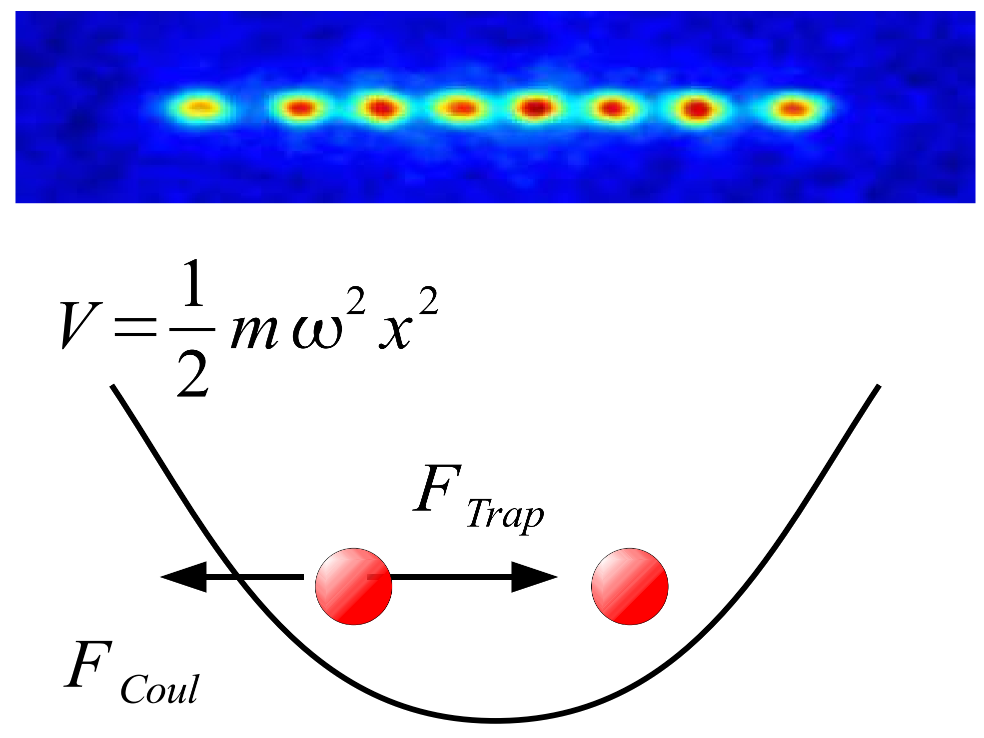
Normal modes

- Hamiltonian combines trapping and interaction

$$V_{tot} \sim \sum_{i<j} \frac{e^2}{4\pi\epsilon_0} \frac{1}{x_i - x_j} + \sum_i \frac{1}{2} m \omega_z^2 x_i^2$$

- Equilibrium configuration at energy minimum

$$\left(\frac{\partial V_{tot}}{\partial x_i} \right)_{x_i^{(0)}} = 0$$



Normal modes

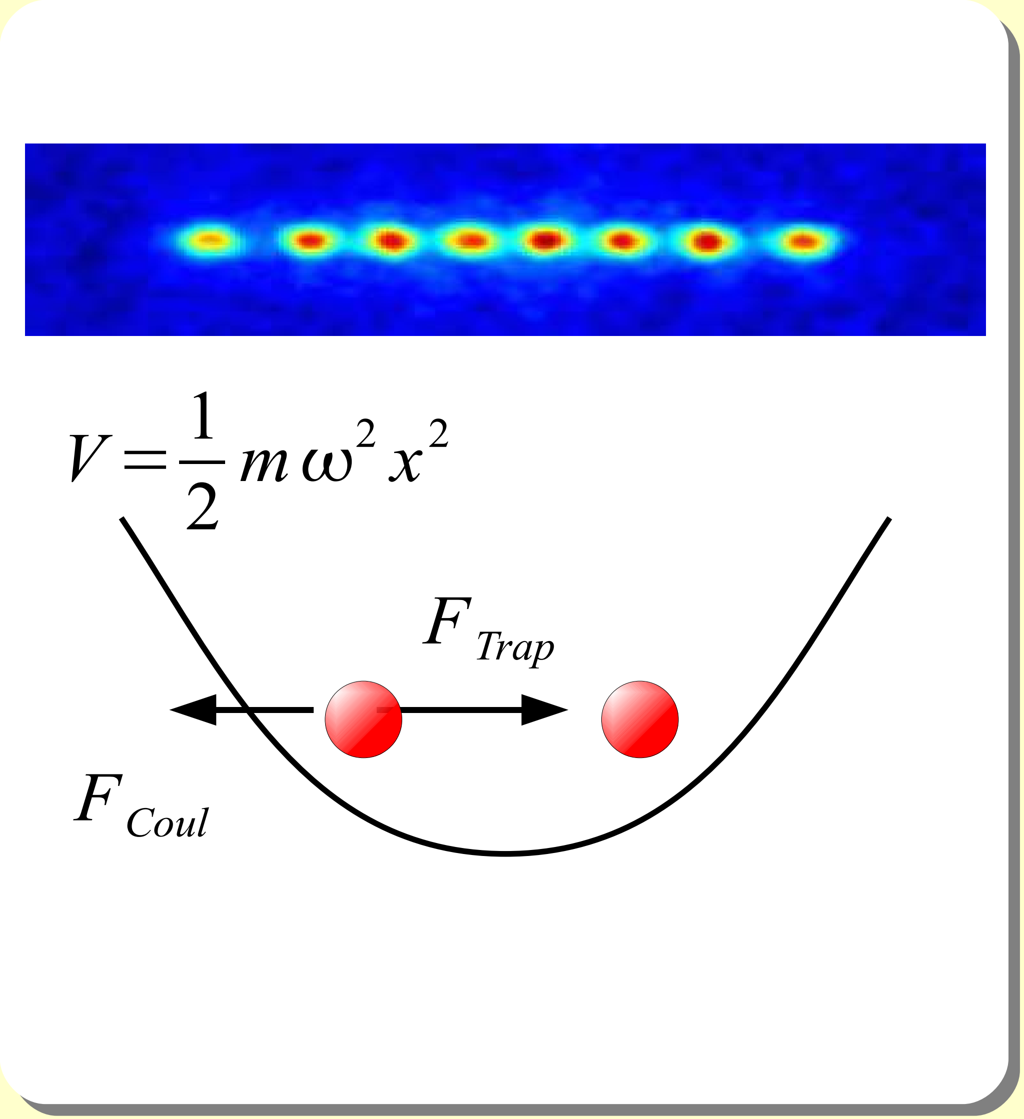
- Around the minimum we can expand energy

$$H \sim \sum_{i,j} \left(\frac{\partial^2 V}{\partial x_i \partial x_j} \right)_{x_i^{(0)}} \Delta x_i \Delta x_j$$

$$+ \sum_i \frac{p_i^2}{2\mathrm{m}}$$

- This Hamiltonian can be diagonalized using normal modes

$$H \sim \sum_K \left[\frac{1}{2m} P_K^2 + \frac{1}{2} m \omega_k^2 Q_K^2 \right]$$



A real-life example

Two ions

- Use CM and relative coord.

$$x_1 = R + \frac{1}{2} d$$

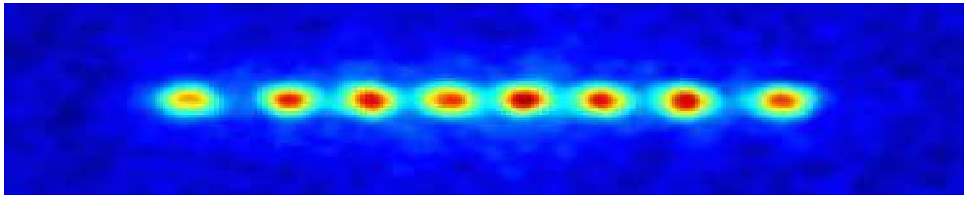
$$x_2 = R - \frac{1}{2} d$$

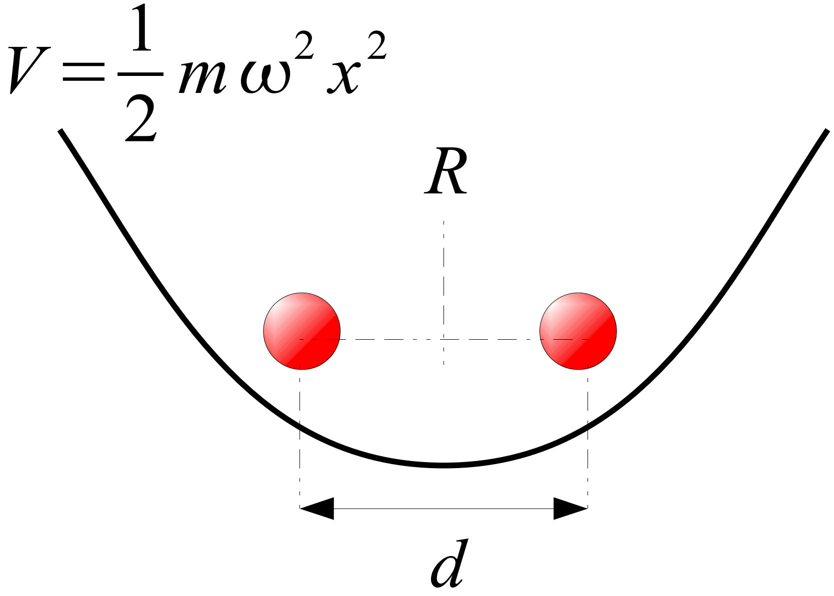
- The harmonic potential becomes

$$x_1^2 + x_2^2 = 2R^2 + \frac{1}{2} d^2$$

- Potential splits

$$V_{tot} = V_{CM}(R) + V_{st}(d)$$





Two ions

- The CM oscillates with frequency ω

$$H_{CM}=\frac{1}{m}P^2+m\omega^2 R^2$$

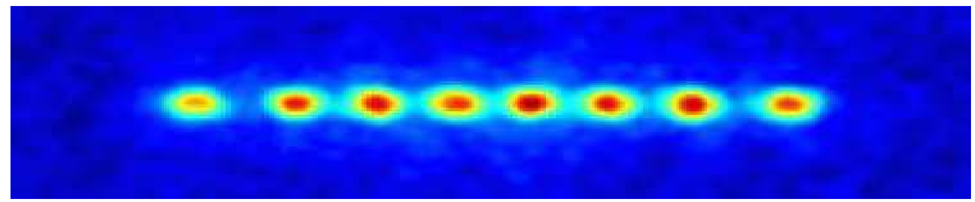
- The stretch mode has

$$V_{tot}\sim\frac{e^2}{4\pi\epsilon_0}\frac{1}{d}+\frac{1}{4}m\omega_z^2 d^2$$

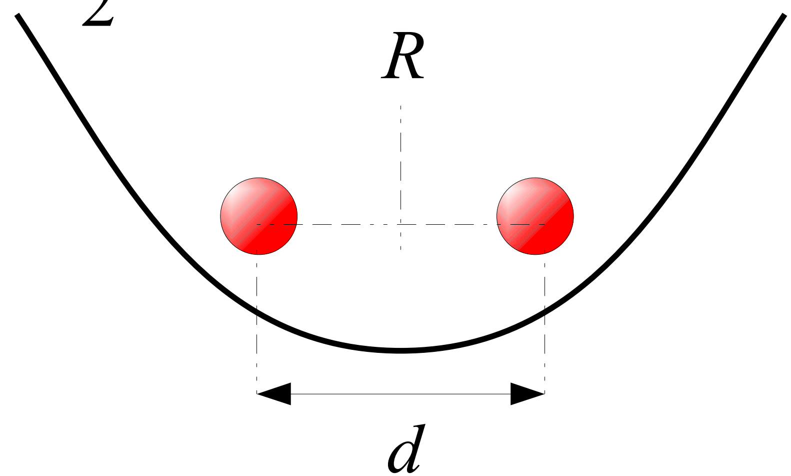
with equilibrium at

$$0=-\frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0^2}+\frac{1}{2}m\omega_z^2 d_0$$

$$d_0=\left(\frac{e^2}{2\pi\epsilon_0 m\omega^2}\right)^{1/3}$$



$$V=\frac{1}{2}m\omega^2 x^2$$



Two ions

- The second order terms become

$$V_{st} \sim \left[\frac{e^2}{2\pi\epsilon_0} \frac{1}{d_0^3} + \frac{1}{2} m \omega_z^2 \right] \times$$

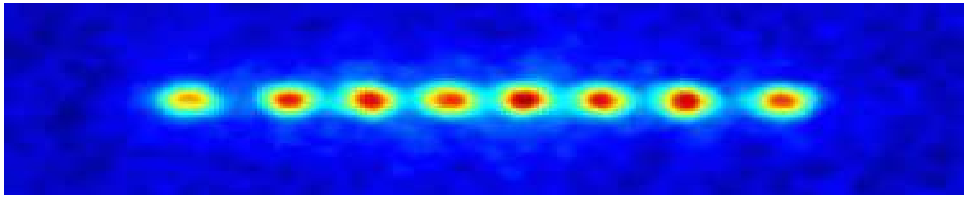
$$\times \frac{1}{2} q^2$$

- Using the value of d0

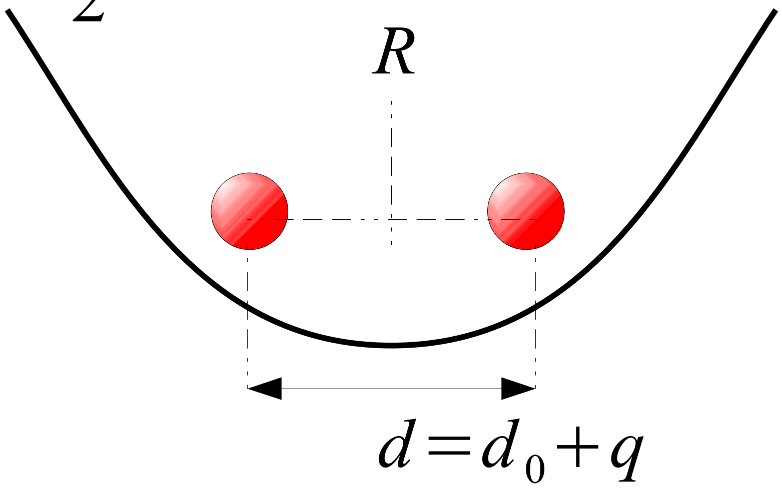
$$H_{st} = \frac{1}{4\mathrm{m}} p^2 + \frac{3}{4} m \omega_z^2 q^2$$

with normal frequency

$$\omega_{st} = \sqrt{3}\, \omega$$



$$V = \frac{1}{2} m \omega^2 x^2$$



Two ions

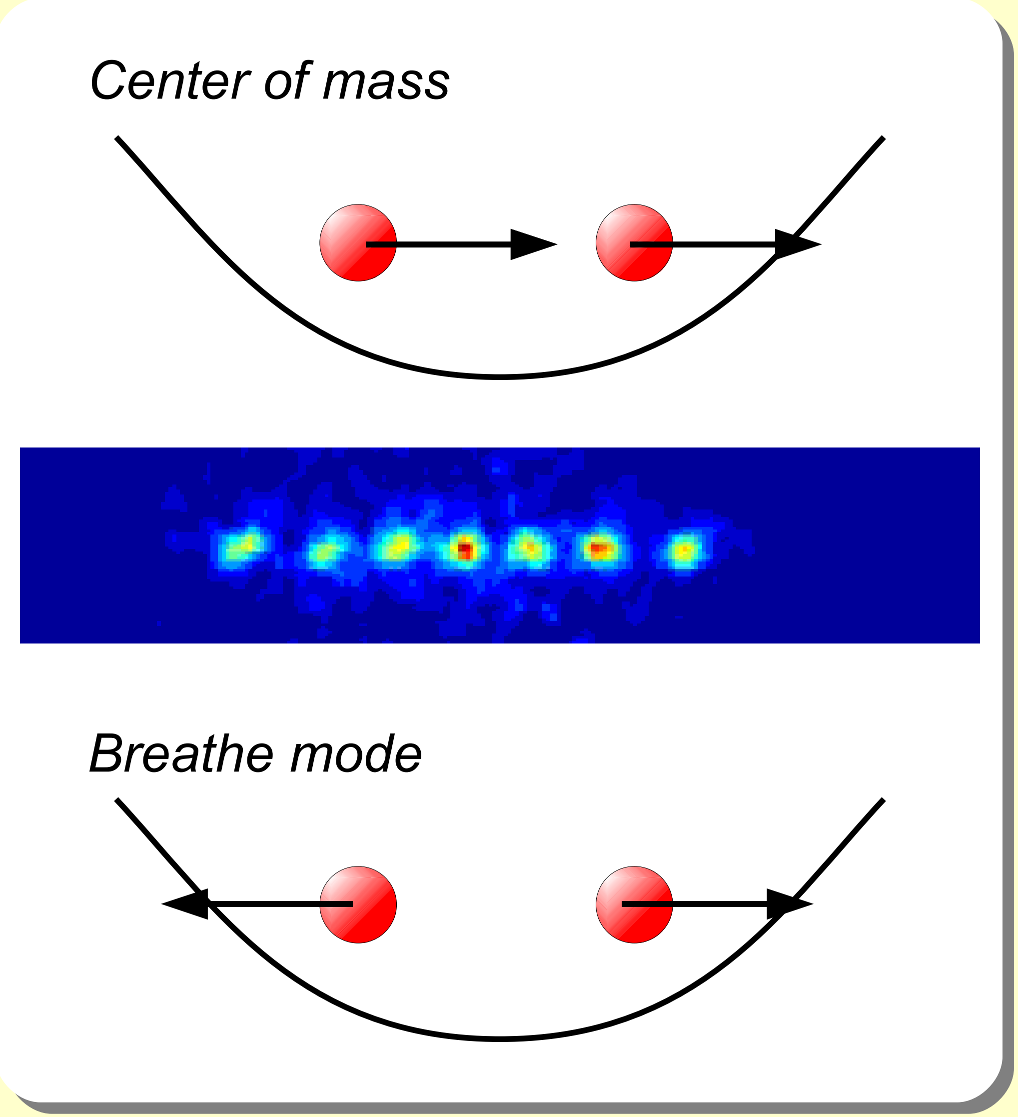
- Center of mass mode

$$\omega_{CM} = \omega$$

- Stretch or "breath" mode

$$\omega_{st} = \sqrt{3}\,\omega$$

- Very different frequencies
  - Will be able to address them spectroscopically.



Full quantum model

- We combine the vibrational and atomic states

$$H = \Delta \sigma_1^z + \Omega \sigma_1^x + \Delta \sigma_2^z + \Omega \sigma_1^x + \\ + \hbar \omega a_{CM}^+ a_{CM} + \hbar \sqrt{3} \omega a_{st}^+ a_{st}$$

- Complex energy ladders, mixture of atomic and vibrational states: **sidebands**
- But **no mechanism for interaction** between qubits.
  - Coulomb interaction does not distinguish spin
  - We need to **induce** some coupling bw motion and qubits.