Ions: sidebands and quantum gates

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Ingredients for QIPC

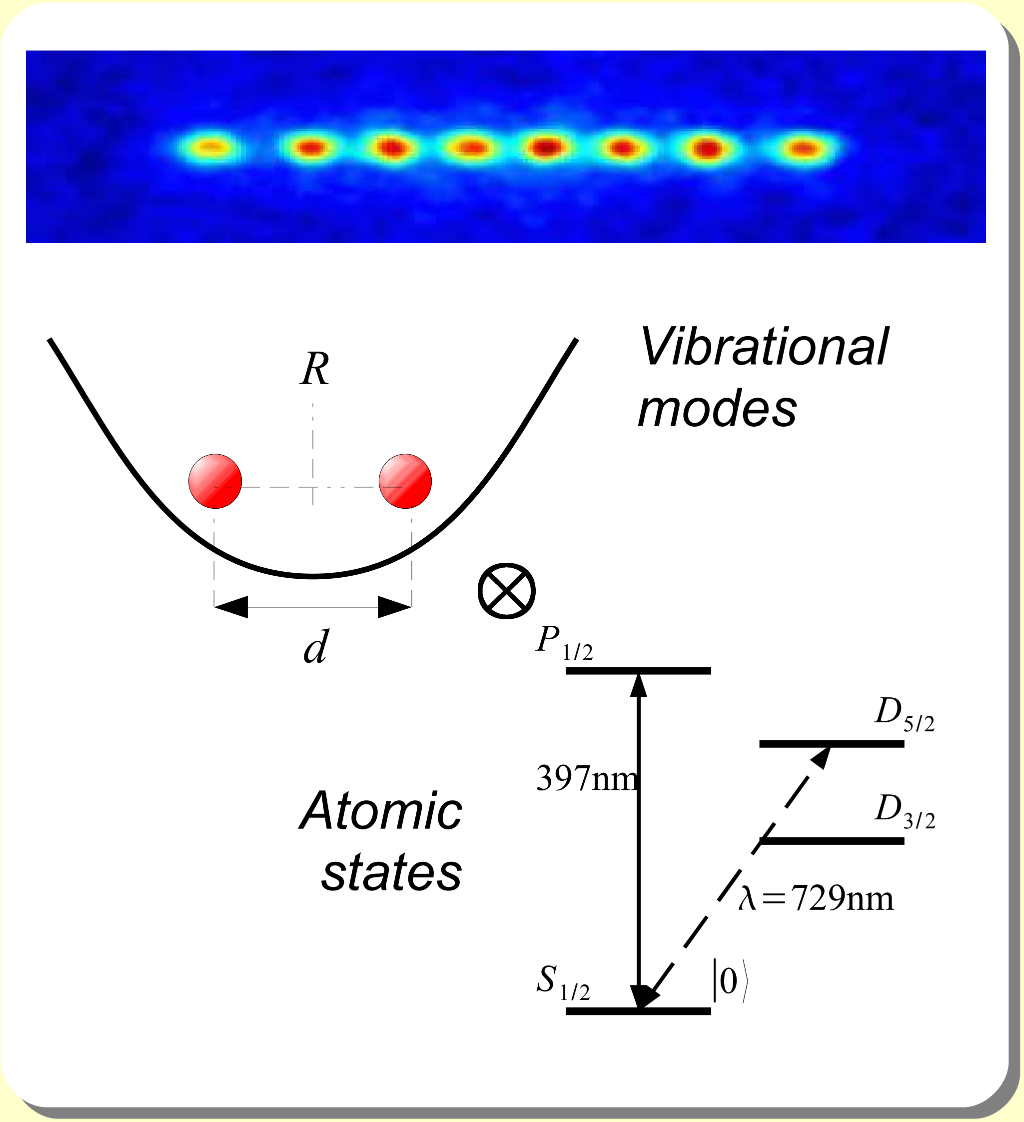
- ✓ Quantum degrees of freedom
- ✓ Local operations
- ✓ Measurements
- One of
  - Entangled state sources
  - **Universal 2qb unitaries**
- Error correcting schemes
- Large number of qubits

Basic quantum allgorithms

Q. Computation

Sidebands

Sidebands



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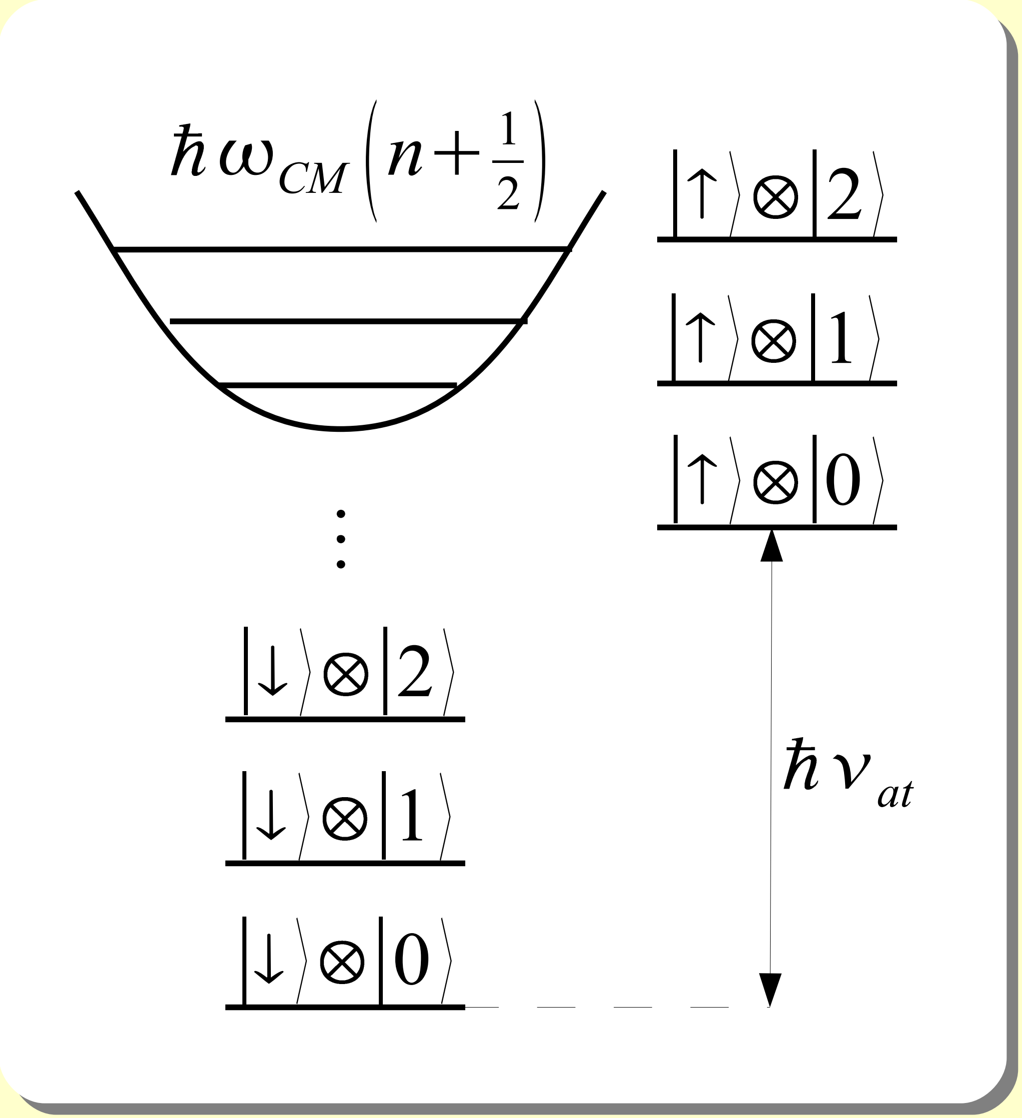

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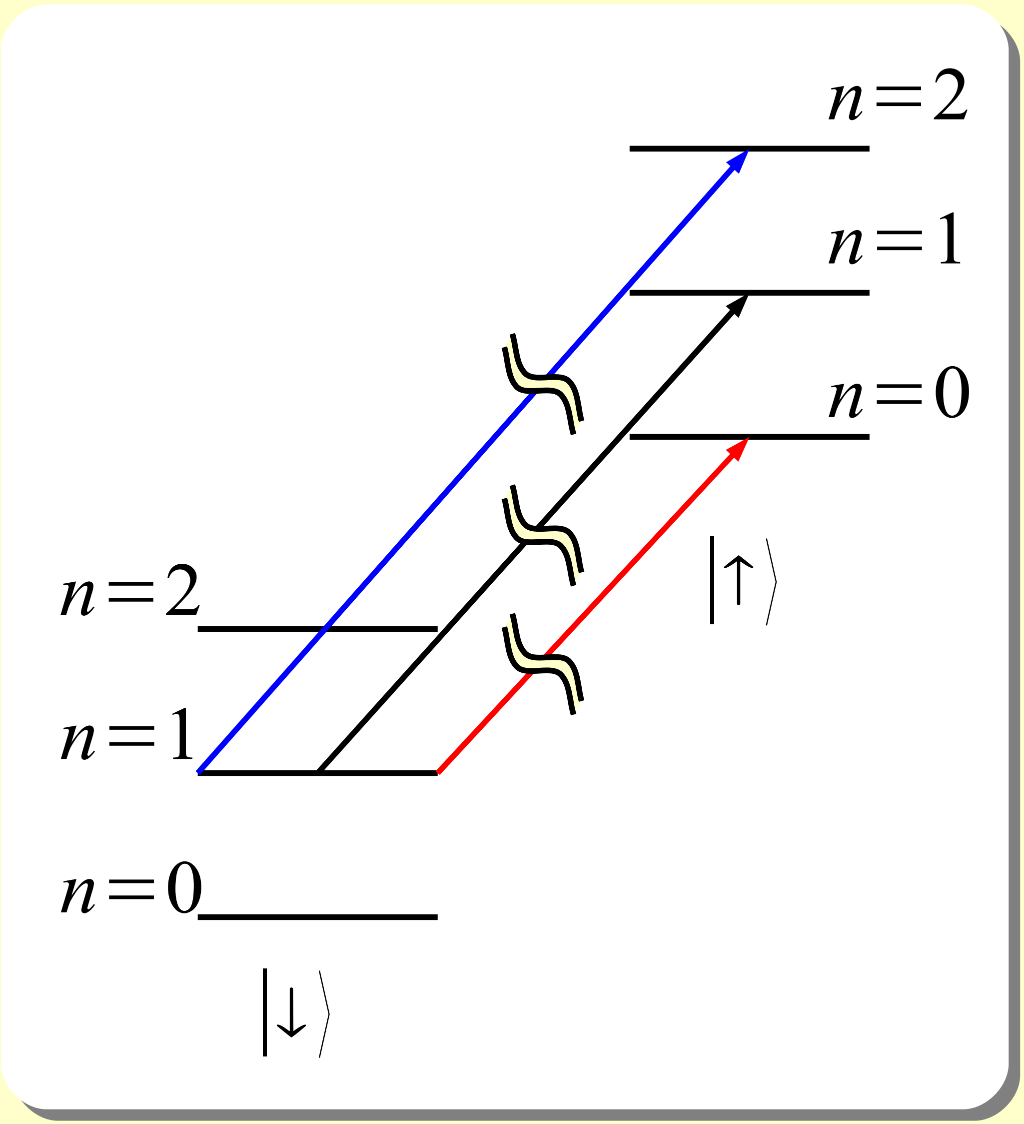

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  - Carrier: $\Delta n = 0$
  - Blue: $\Delta n > 0$
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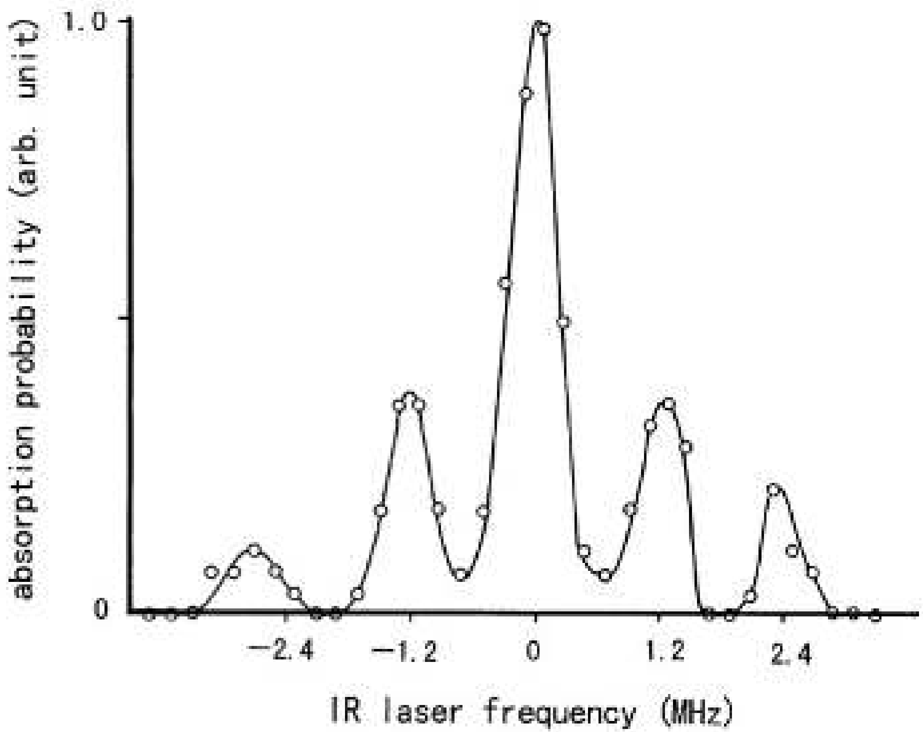

Fig. 5. The absorption spectrum of the $S_{1/2} - D_{5/2}$ electric-quadrupole-allowed transitions obtained from measuring quantum jumps in the 397-nm fluorescence

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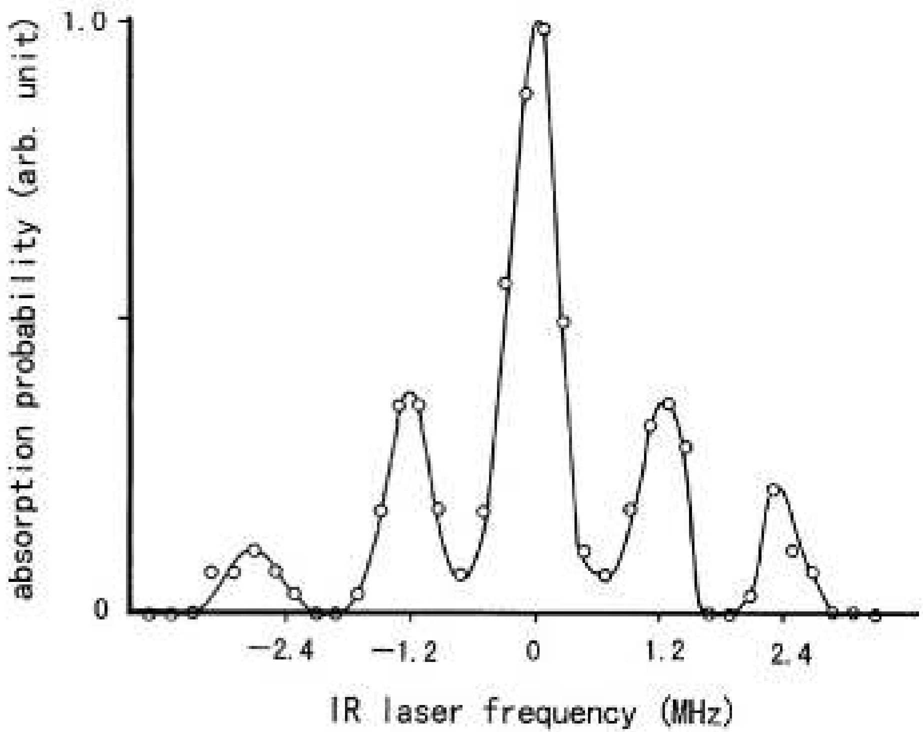
S. Urabe, Appl. Phys. B 67, 223 (1998)

Sidebands: why!



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- Why do sidebands happen?

$$H_{l-at} = \Omega \sigma^{+} a\, e^{ikx - i\omega t} + \ldots$$

- Position of the atom appears in the atom-light interaction.
- We have to replace it with the normal modes.
- After RWA matching the appropriate transition,

$$H_{l-at} = \Omega \eta \sigma^{+} a \times \begin{Bmatrix} a_{CM} \\ 1 \\ a_{CM}^{+} \end{Bmatrix}$$

Sideband cooling

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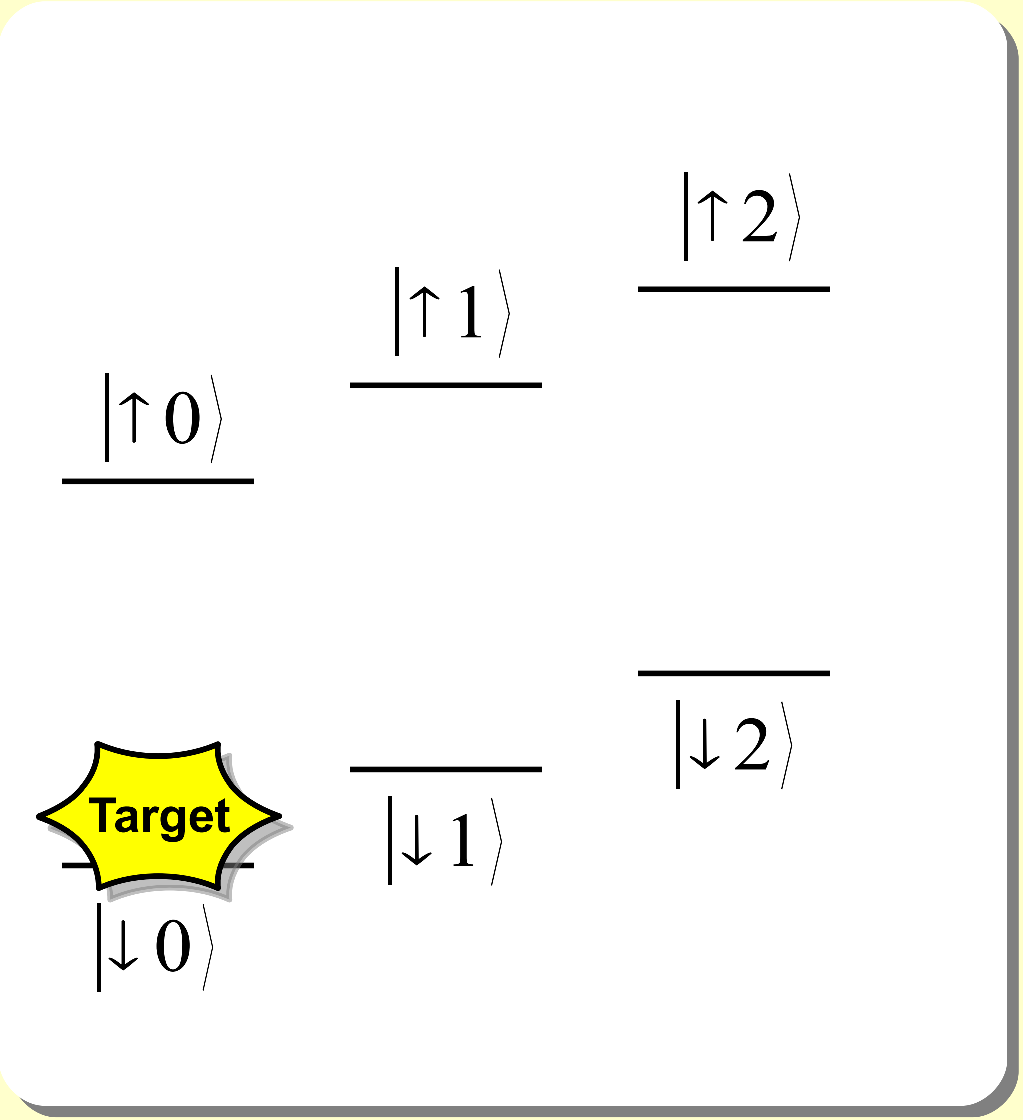

- For some quantum protocols we will need to get rid of all phonons
  - Cool to T=0
- Pump with red sideband
  - Too little energy
  - Remaining taken from vibrational modes
  - Population flow to absolute zero.
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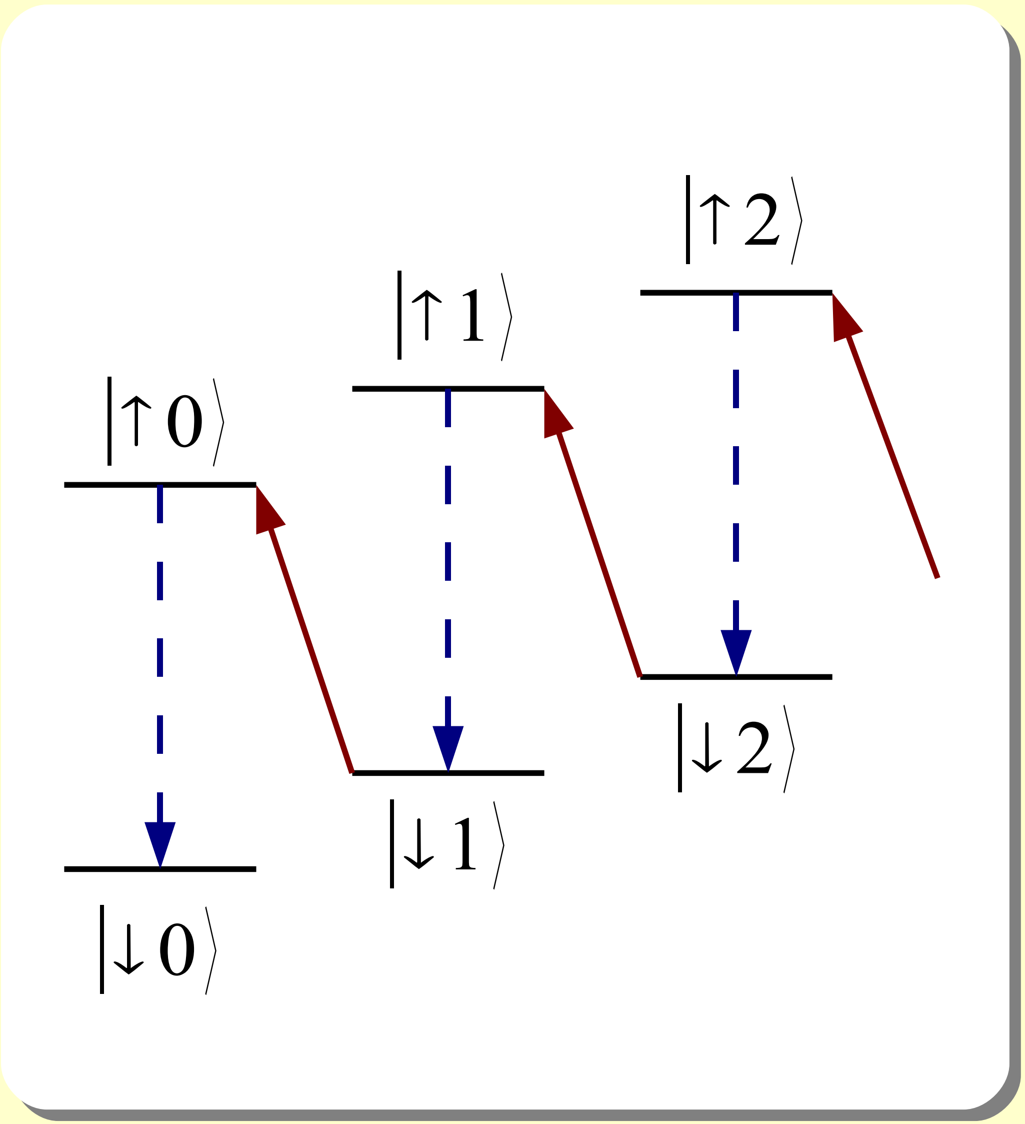

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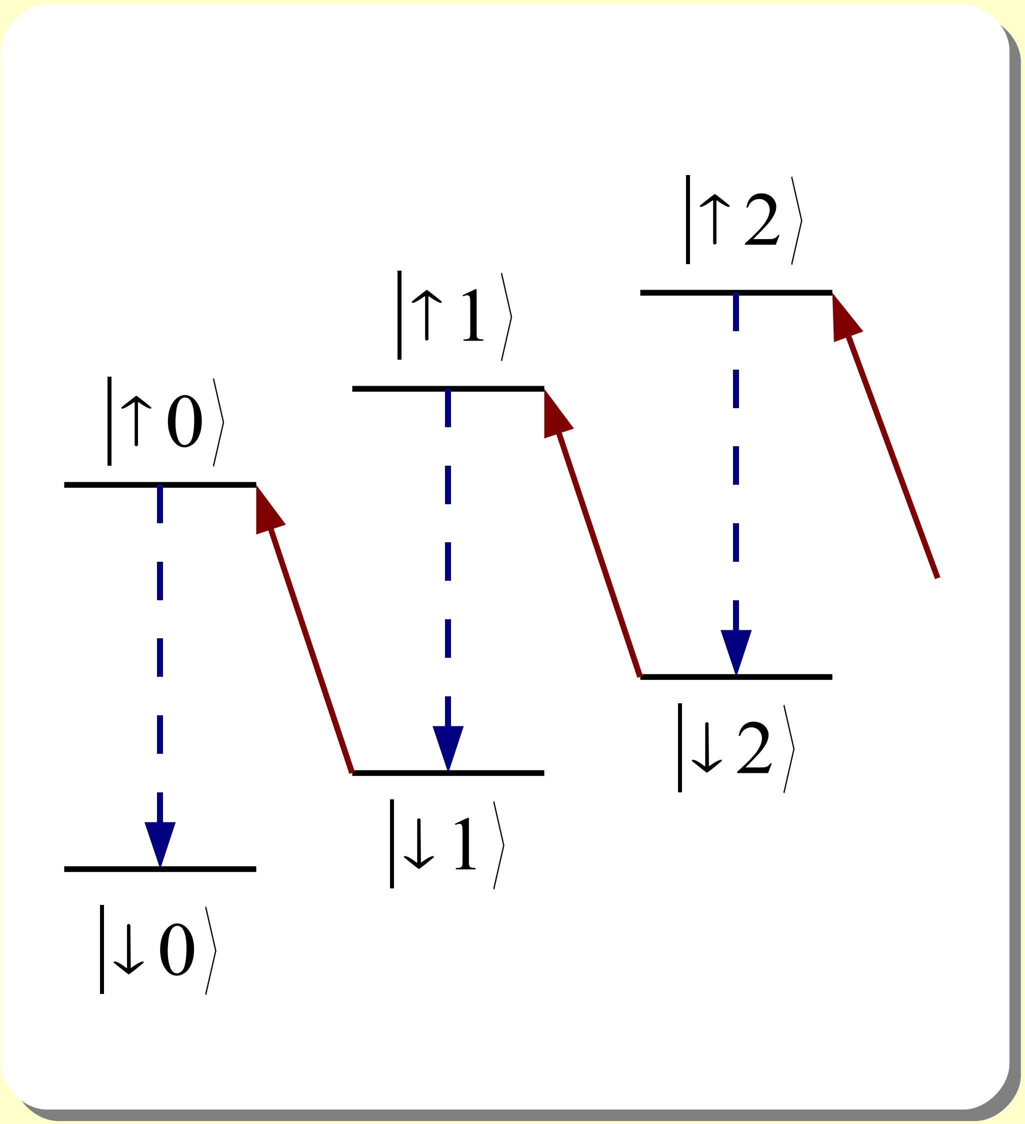
Important questions



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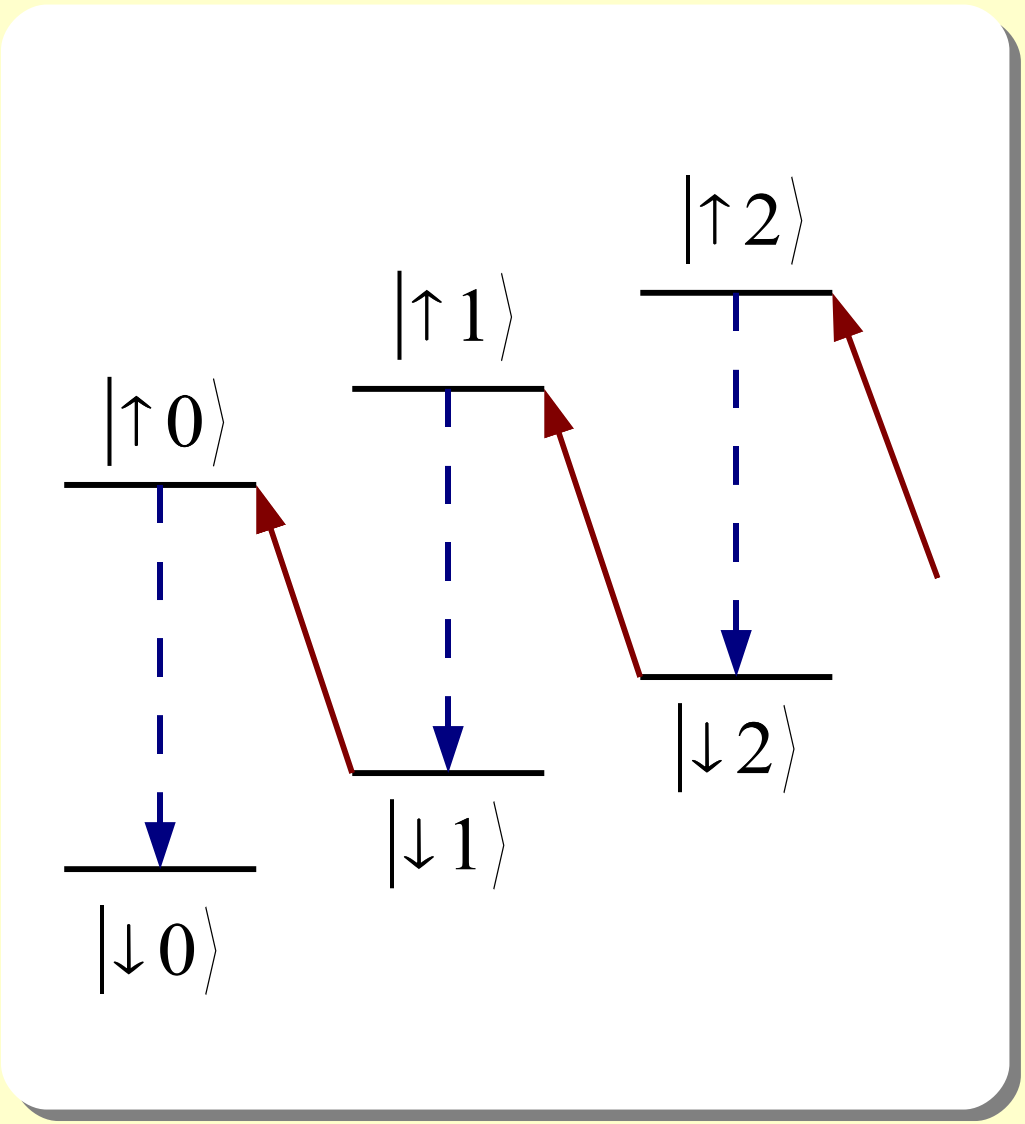
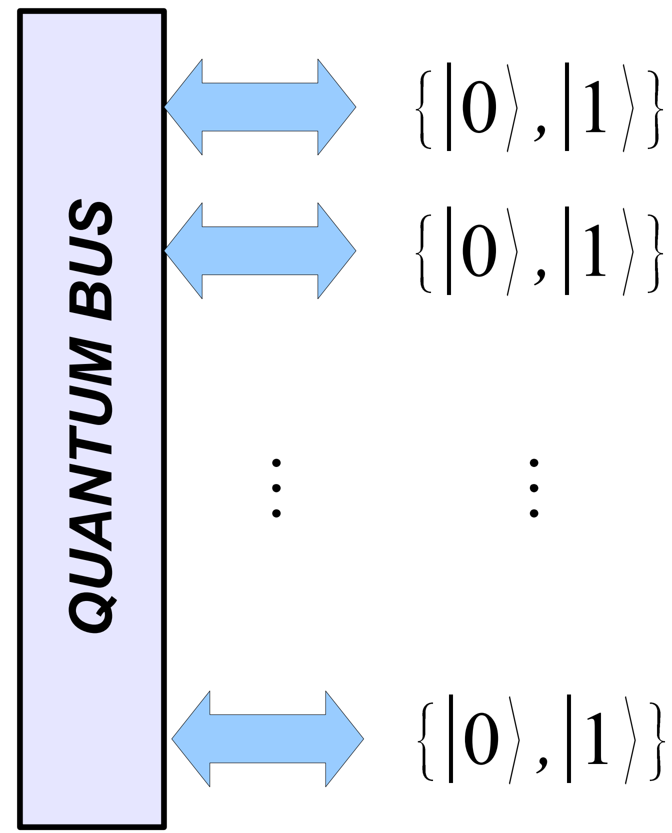


- How do we address side bands (1MHz / 1 Thz!!!)
  - AOM modulate the laser intensity at a frequency Δω
  - Pulse frequency gets incremented and decremented
- Really T=0? All modes?
  - No! Heating happens **while** we are cooling
  - Time limits: while cooling one mode, others heat.

Cirac-Zoller quantum gate

Cirac-Zoller gate

- We have seen that motional and internal states can be coupled.
- **Idea**: use motional states to mediate a quantum gate between qubits.
- Quantum information will be swapped between atoms and vibrational modes
  - Perfect control (cooling) of vibrations.



1) Swap atom – vib. mode

- For simplicity, we use letters (e,e',g) for atoms, numbers for phonons.
- We use a red sideband to map an excited state of the atom to a phonon

$$
\begin{array}{ccc}
 & U_m^{1,0} & \\
|g\rangle|g\rangle|0\rangle & \rightarrow & |g\rangle|g\rangle|0\rangle \\
|g\rangle|e\rangle|0\rangle & \rightarrow & |g\rangle|e\rangle|0\rangle \\
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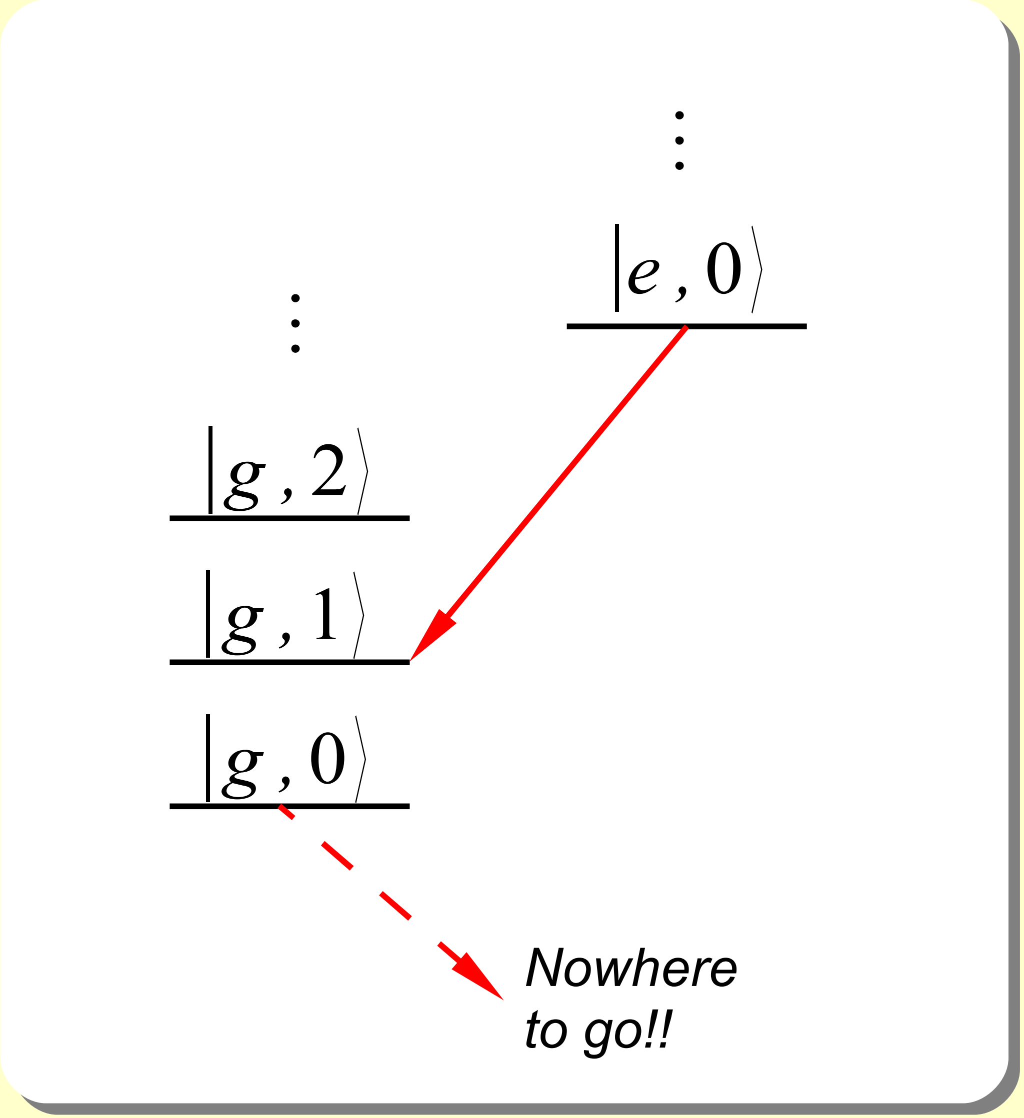

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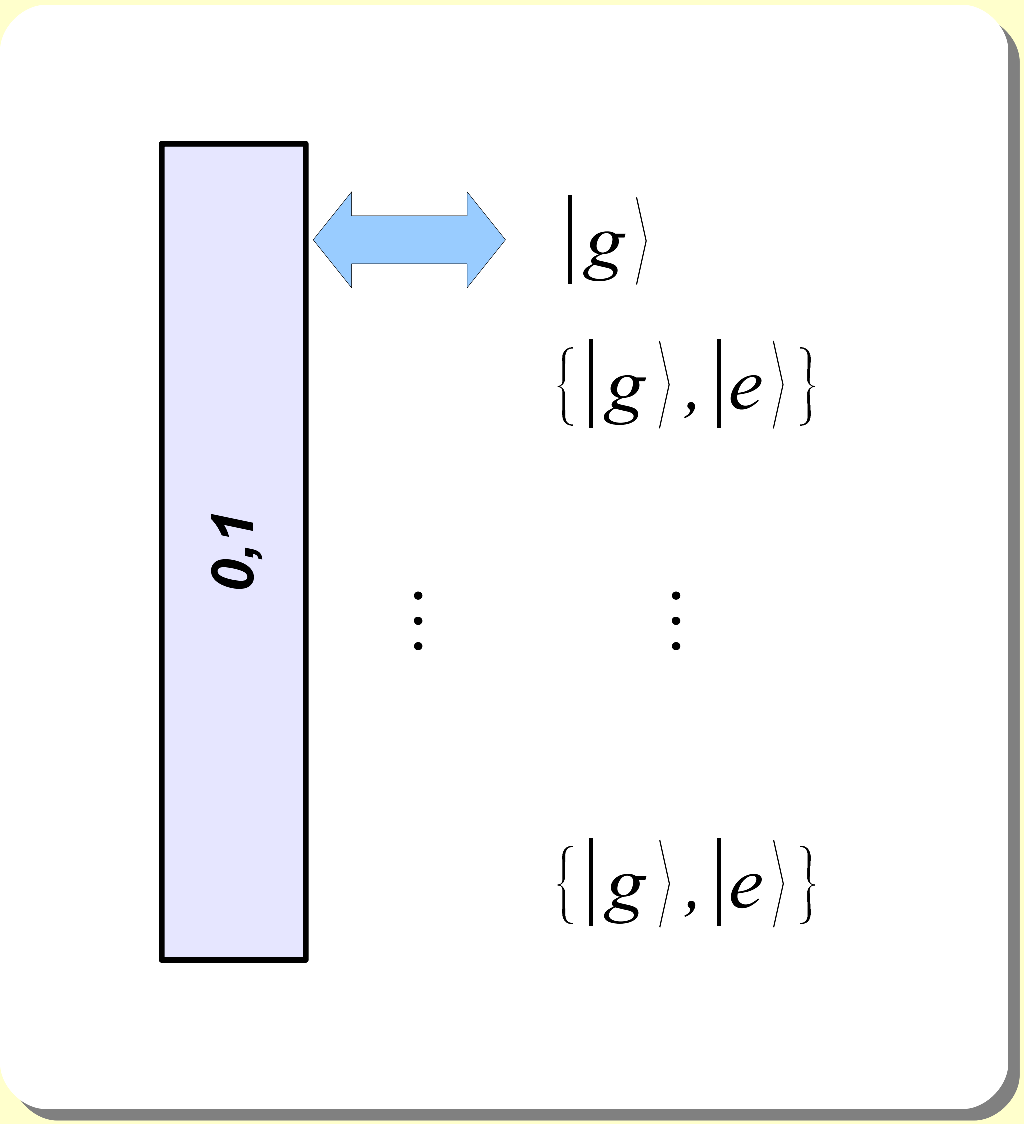

2) Perform a phase gate

- We do a 2π rotation on the other atom.
- The qubit states remain unchanged, but acquire a phase.

$$\begin{array}{ccc} |g\rangle|1\rangle & \rightarrow & -|g\rangle|1\rangle \\ |e'\rangle|0\rangle & \rightarrow & |e'\rangle|0\rangle \end{array}$$

- All together

$$U_n^{2,1}$$

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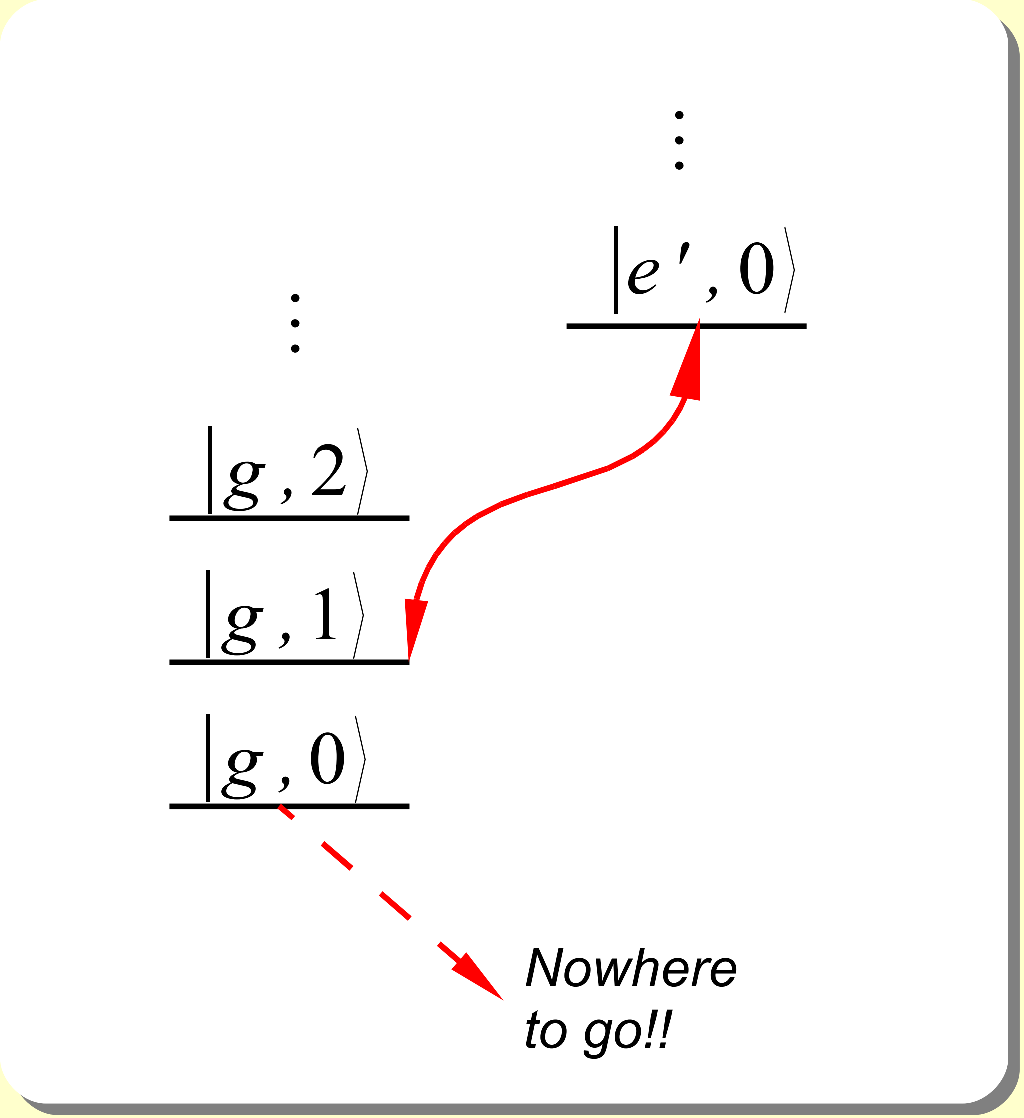

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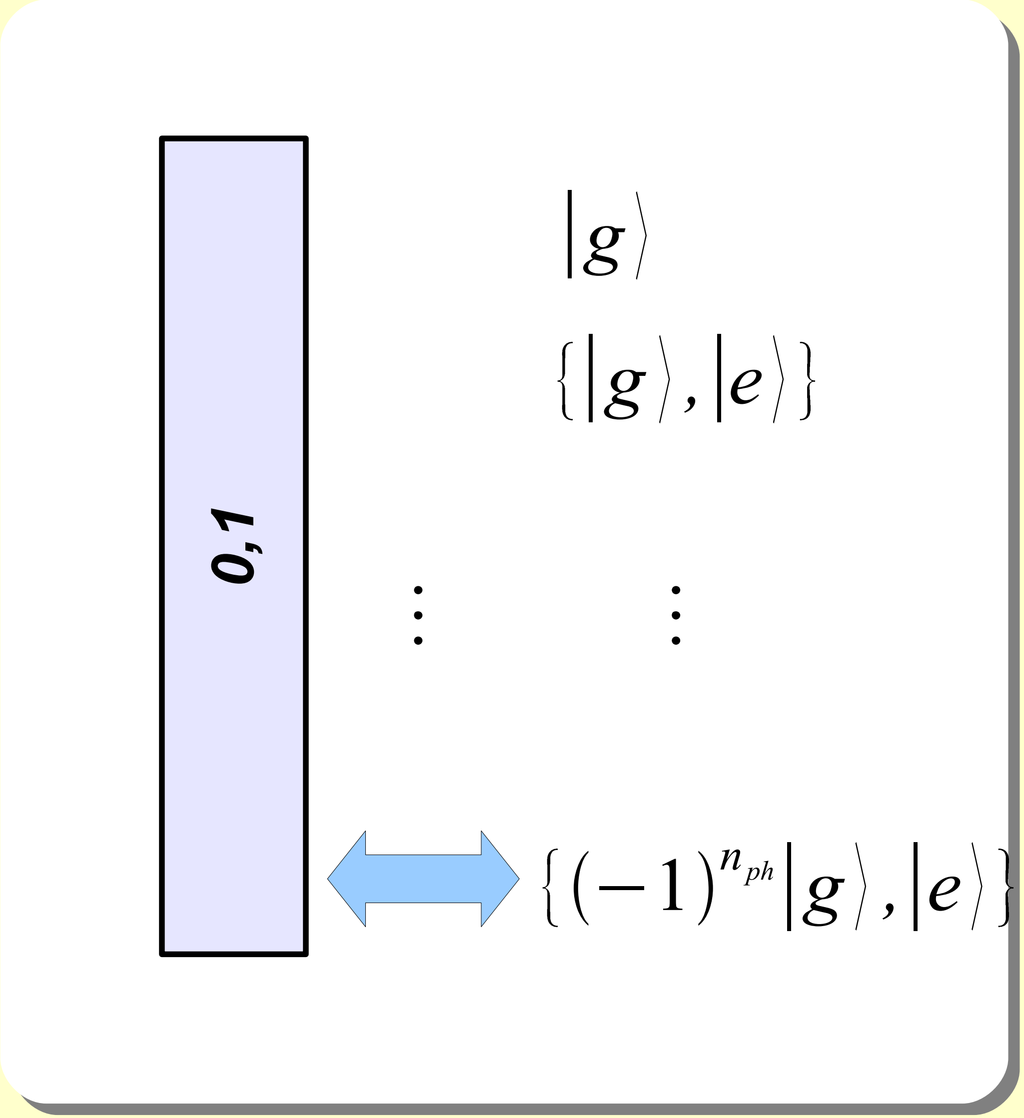


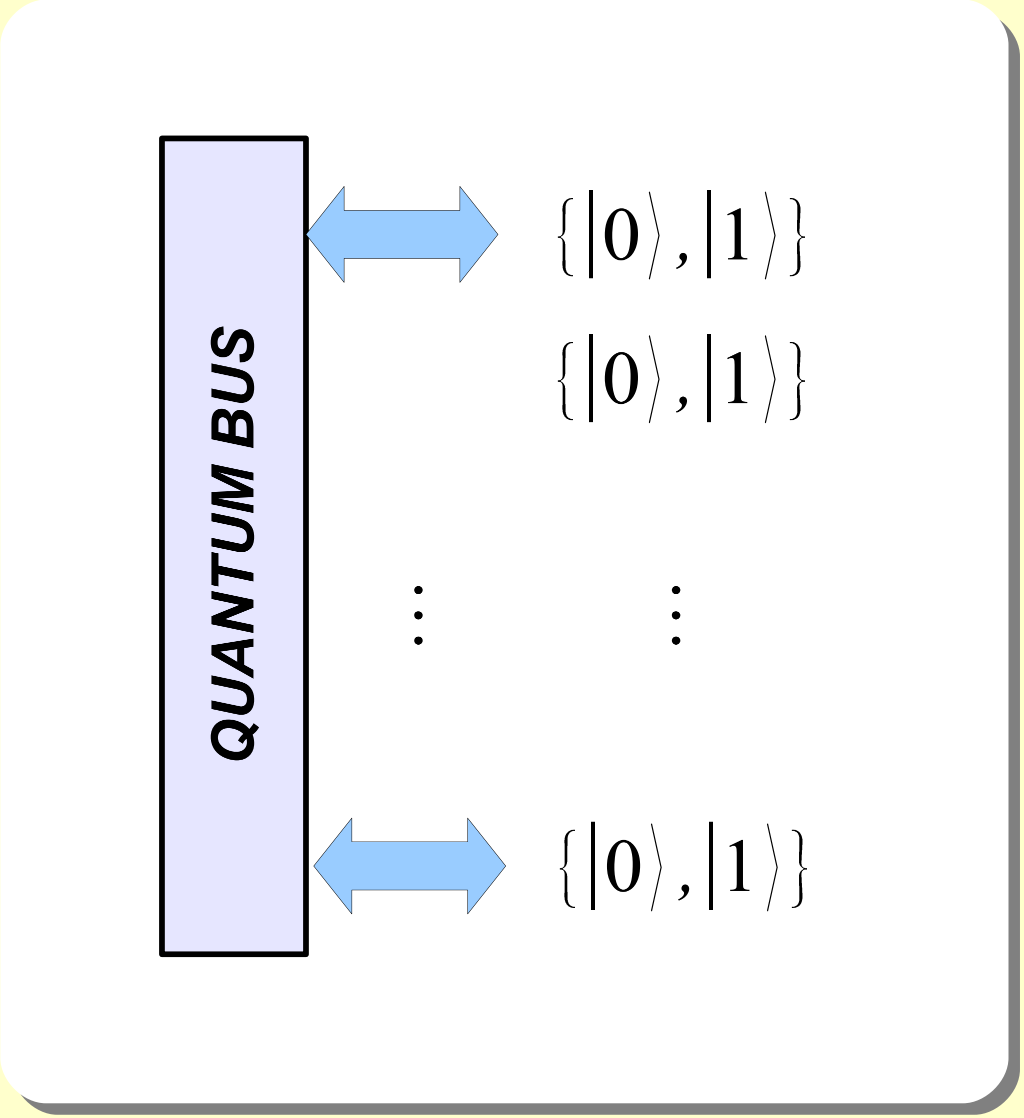
3) Swap back and combine

- We repeat the first gate, moving the QI from the bus to the first ion.

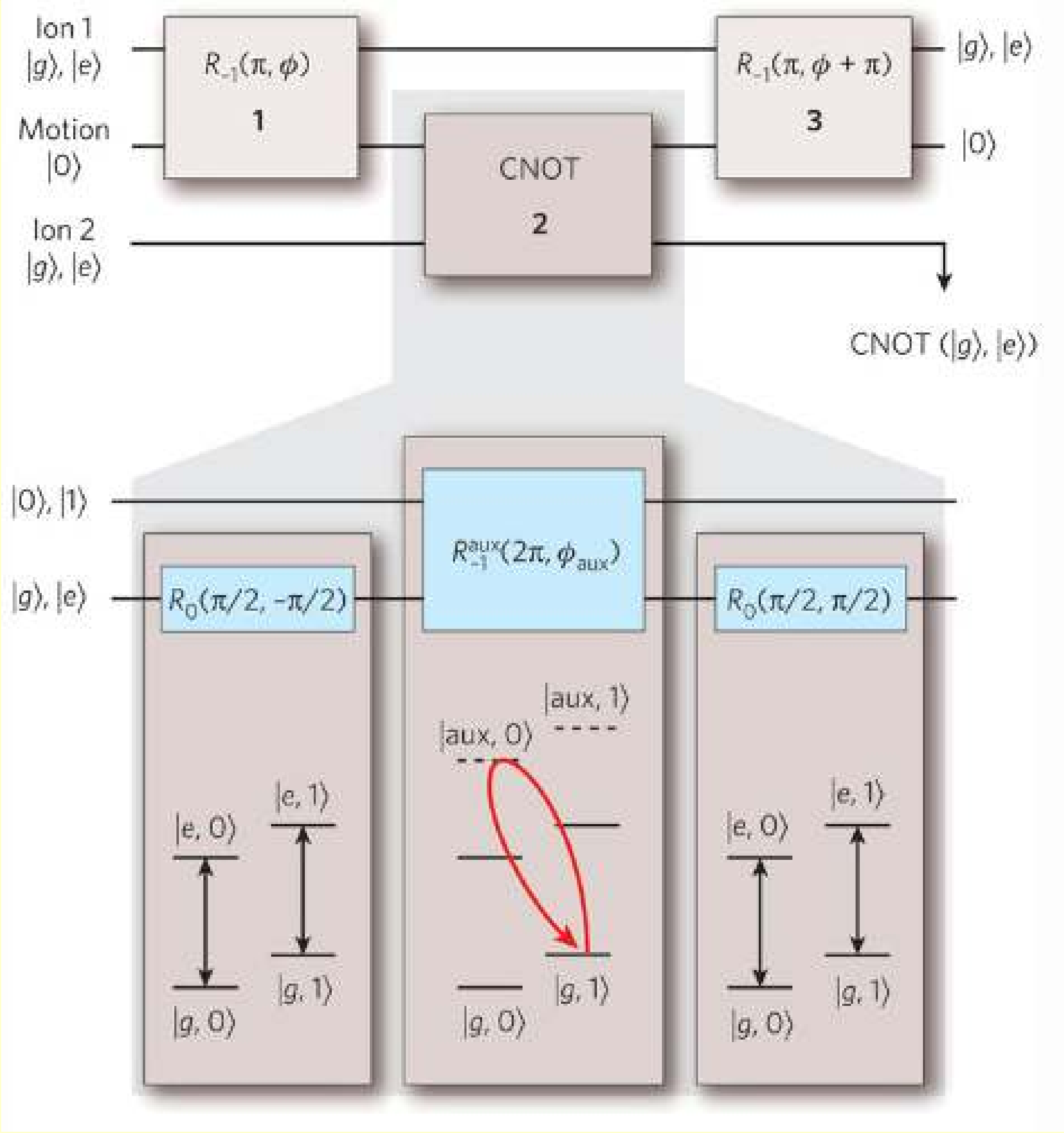
$$U_{total} = U_m^{1,0} U_n^{2,1} U_m^{1,0}$$

$$\begin{array}{lcl} |g\rangle|g\rangle|0\rangle & \rightarrow & |g\rangle|g\rangle|0\rangle \\ |g\rangle|e\rangle|0\rangle & \rightarrow & |g\rangle|e\rangle|0\rangle \\ |e\rangle|g\rangle|0\rangle & \rightarrow & |e\rangle|g\rangle|0\rangle \\ |e\rangle|e\rangle|0\rangle & \rightarrow & -|e\rangle|e\rangle|0\rangle \end{array}$$

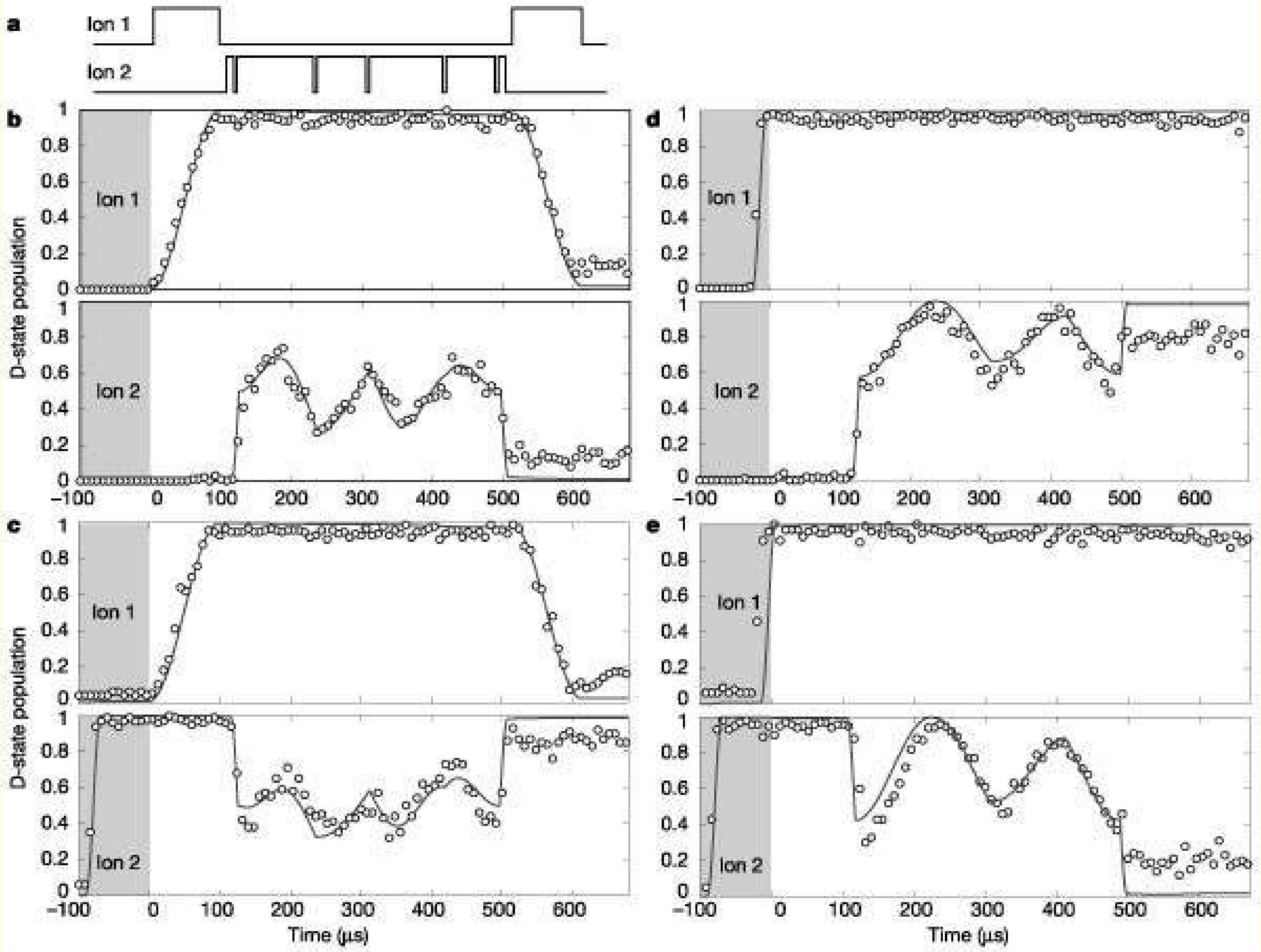
- A controlled-phase (CZ) gate.
- It is universal.
- Any pair of ions.



Cirac-Zoller gate ('95)



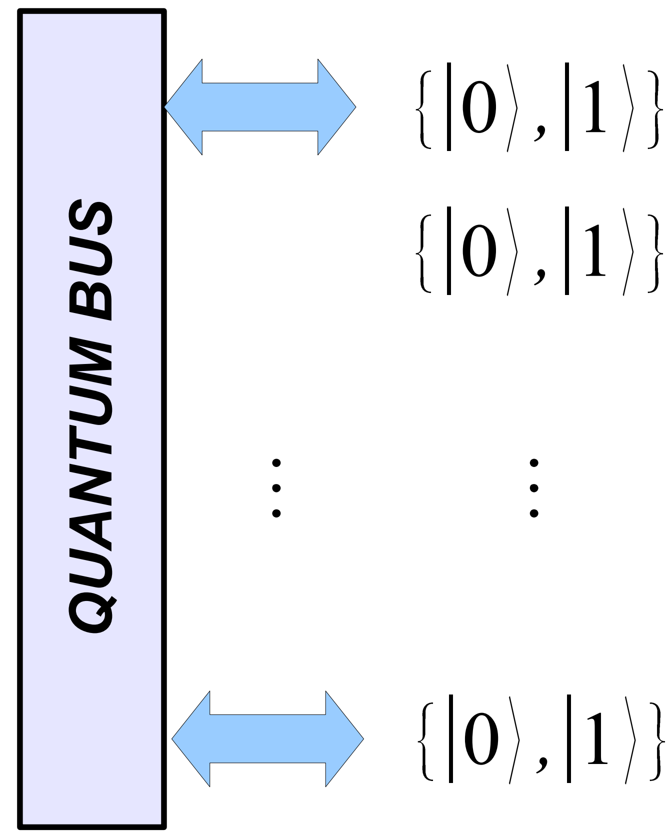
Implementation of CZ ('03)



Realization of the Cirac–Zoller controlled-NOT quantum gate
F. Schmidt-Kaler et al, Nature **422**, 408-411 (2003)

Drawbacks

- A very slow gate
  - Need to resolve sidebands.
  - Pulse duration >> 1/ω
  - Lamb-Dicke limit
- Vibrational bus cooled at T=0, no phonons.



Application: Q Teleportation

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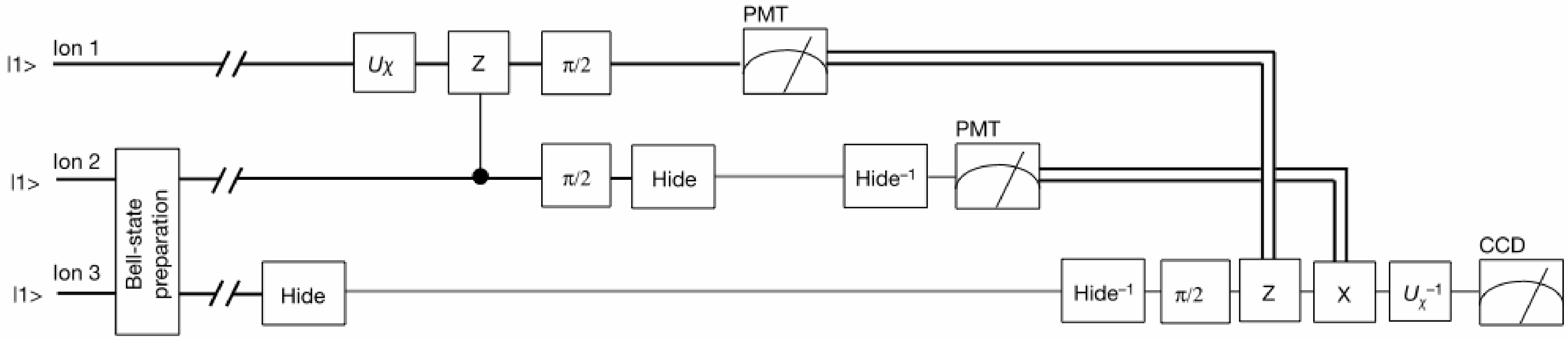

- We need a two qubit gate for entangling ions 2 and 3.
- We then apply the teleportation protocol, between ions 1 and 2
  - A two-qubit gate is needed for the Bell measurement
- A final measurement determines the local unitary to apply on the last qubit.

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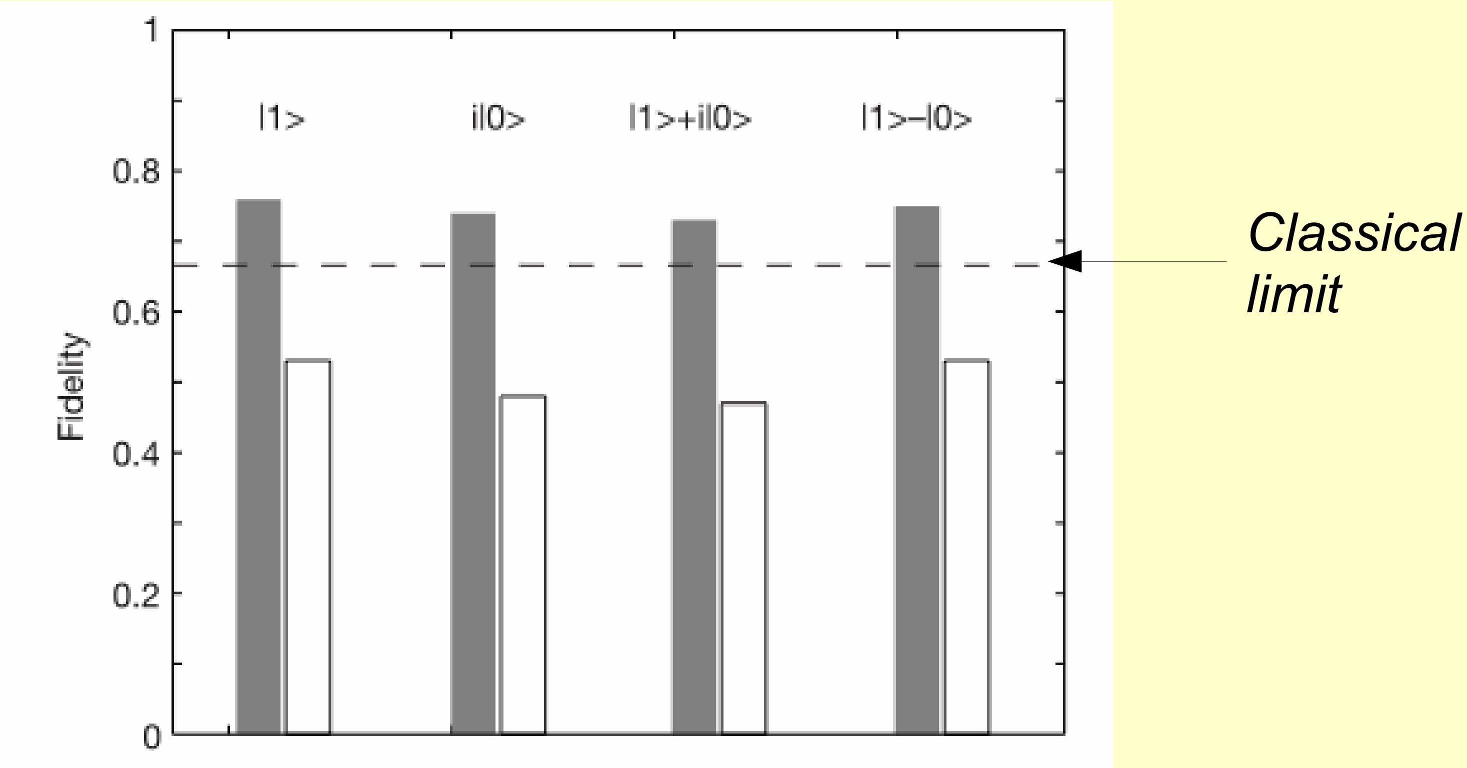

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