Ions: geometric phase gates

J. J. García-Ripoll

IFF, CSIC Madrid

(16-4-2009)

Forced harmonic oscillator

Forced harmonic oscillator

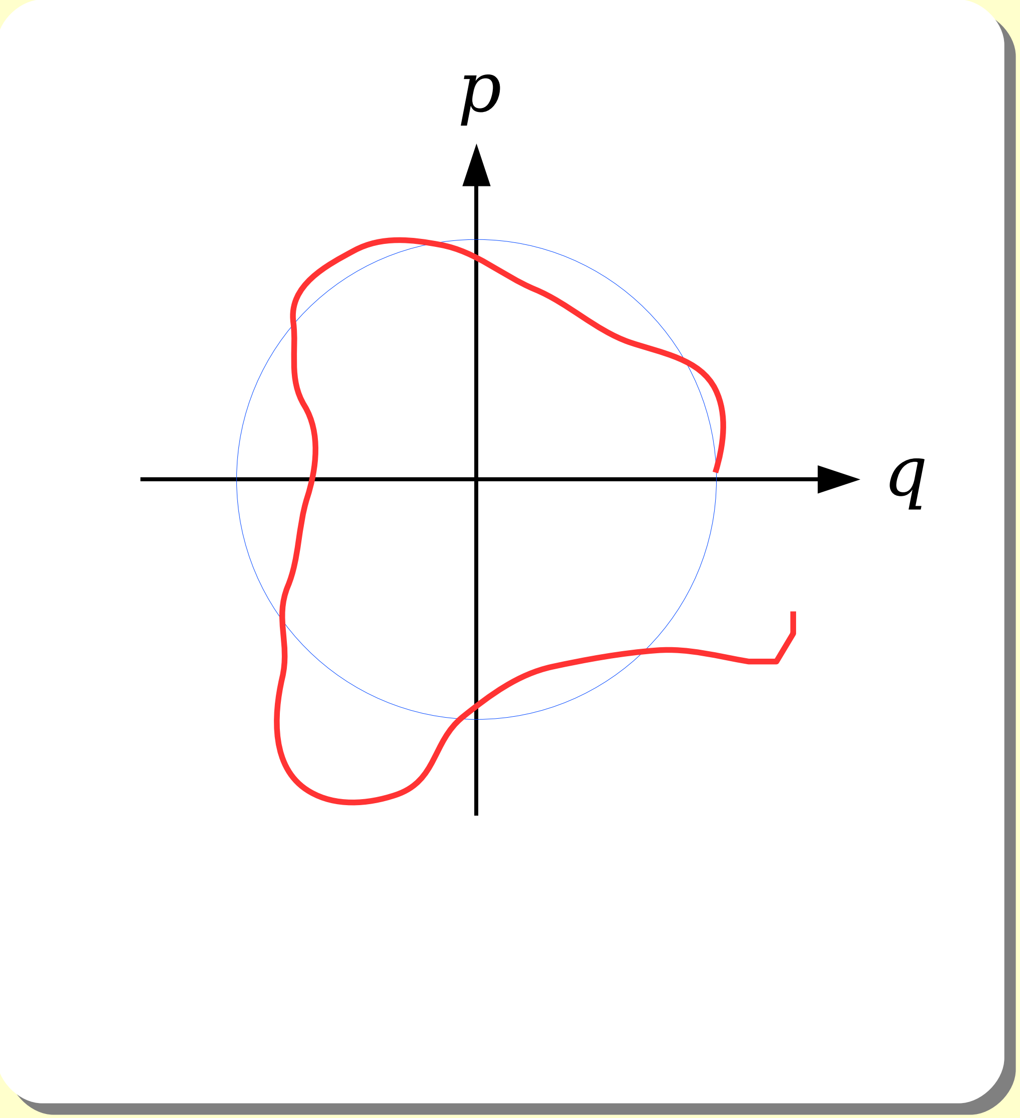
- A single oscillator mode subject to a time-dependent force

$$H = \frac{1}{2}\omega(p^2 + q^2) - F(t)x$$

- Simpler with Fock ops

$$H = \omega a^+ a - \frac{F}{\sqrt{2}}(a + a^+)$$

  - Free evolution means rotations in phase space
  - The force distorts the orbits.



Forced harmonic oscillator

- This problem can be integrated using coherent states

$$\left|\psi(t)\right\rangle = e^{i\phi(t)}\left|z(t)\right\rangle$$

- The evolution equations are

$$\frac{dz}{dt} = -i\omega z + i\frac{1}{\sqrt{2}}F(t)$$

$$\frac{d\phi}{dt} = \frac{1}{2\sqrt{2}}F(t)(\bar{z}+z)$$

- The displacement becomes

$$z(t) = e^{-i\omega t}z(0) - \frac{i}{\sqrt{2}}\int_0^t F(\tau)e^{-i\omega(t-\tau)}d\tau$$

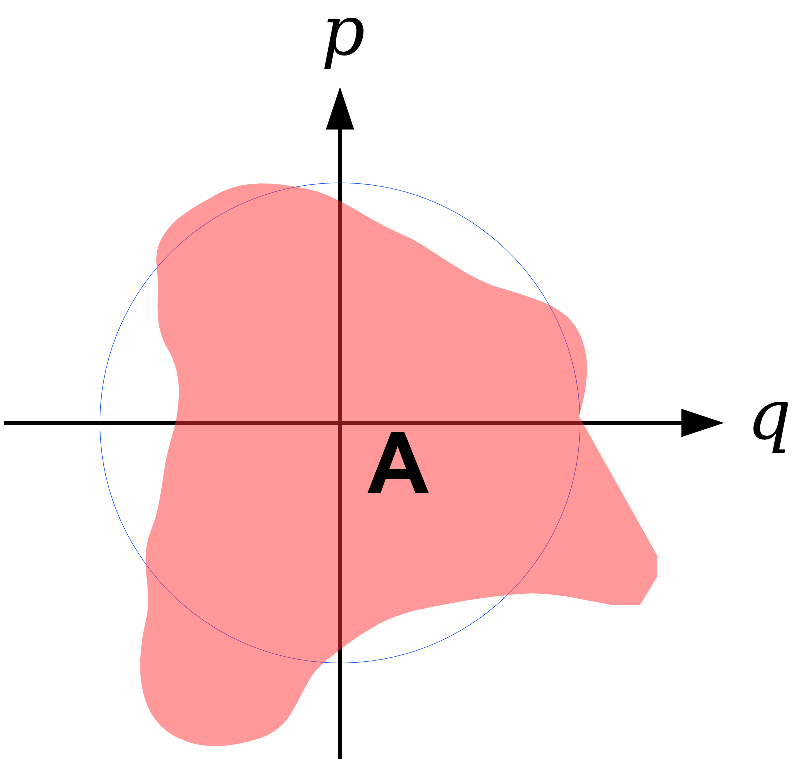
Geometric phase

- The total phase can be expressed in terms of the coherent state path

$$\frac{d\phi}{dt} = Im \frac{dz}{dt}\bar{z}$$

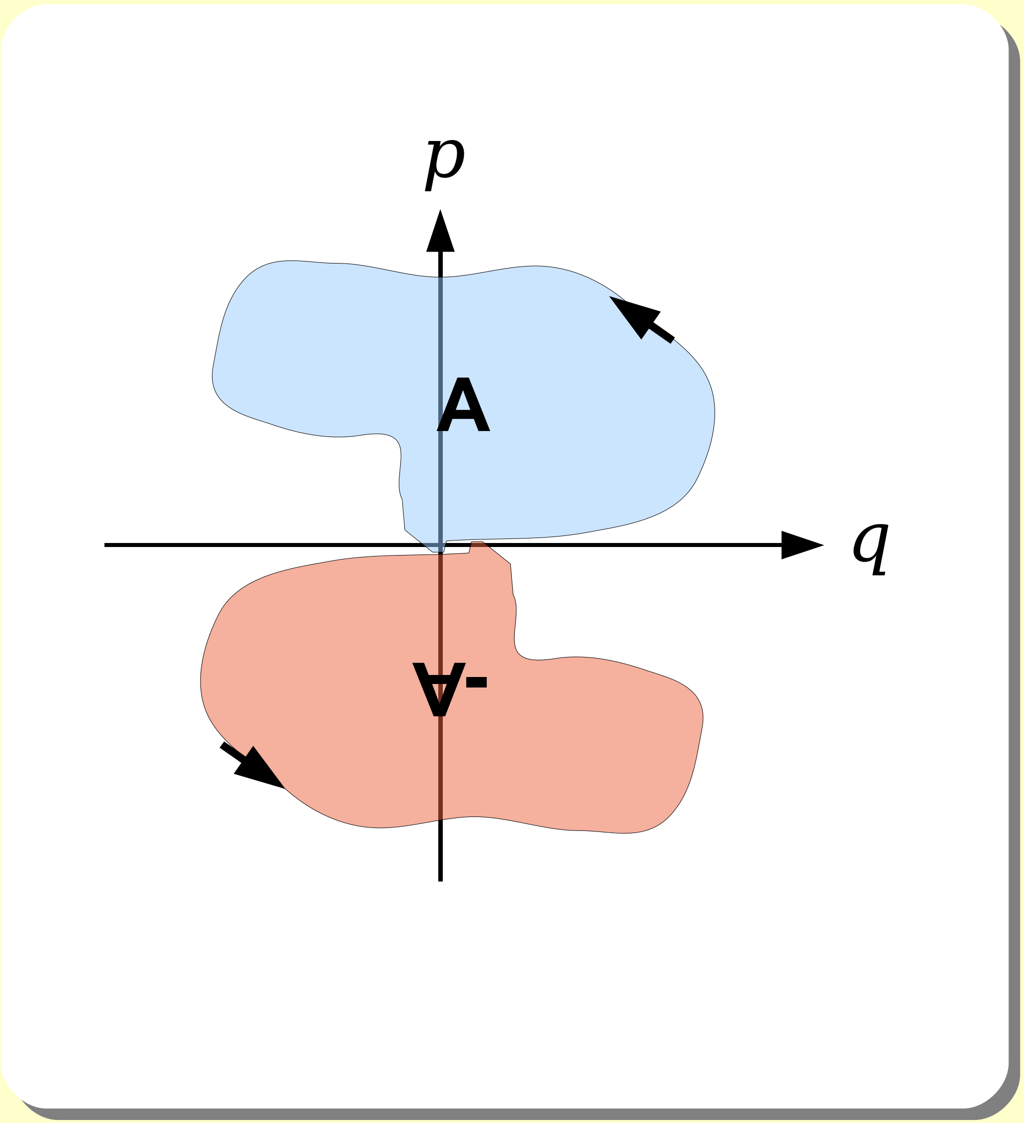
$$= \frac{dp}{dt} q - p \frac{dp}{dt}$$

$$= 2\frac{dA}{dt}$$



Geometric quantum gate

Geometric quantum gate



- We will now use state-dependent forces

$$H = \omega a^{+} a - \frac{F}{\sqrt{2}} \sigma_z (a + a^{+})$$

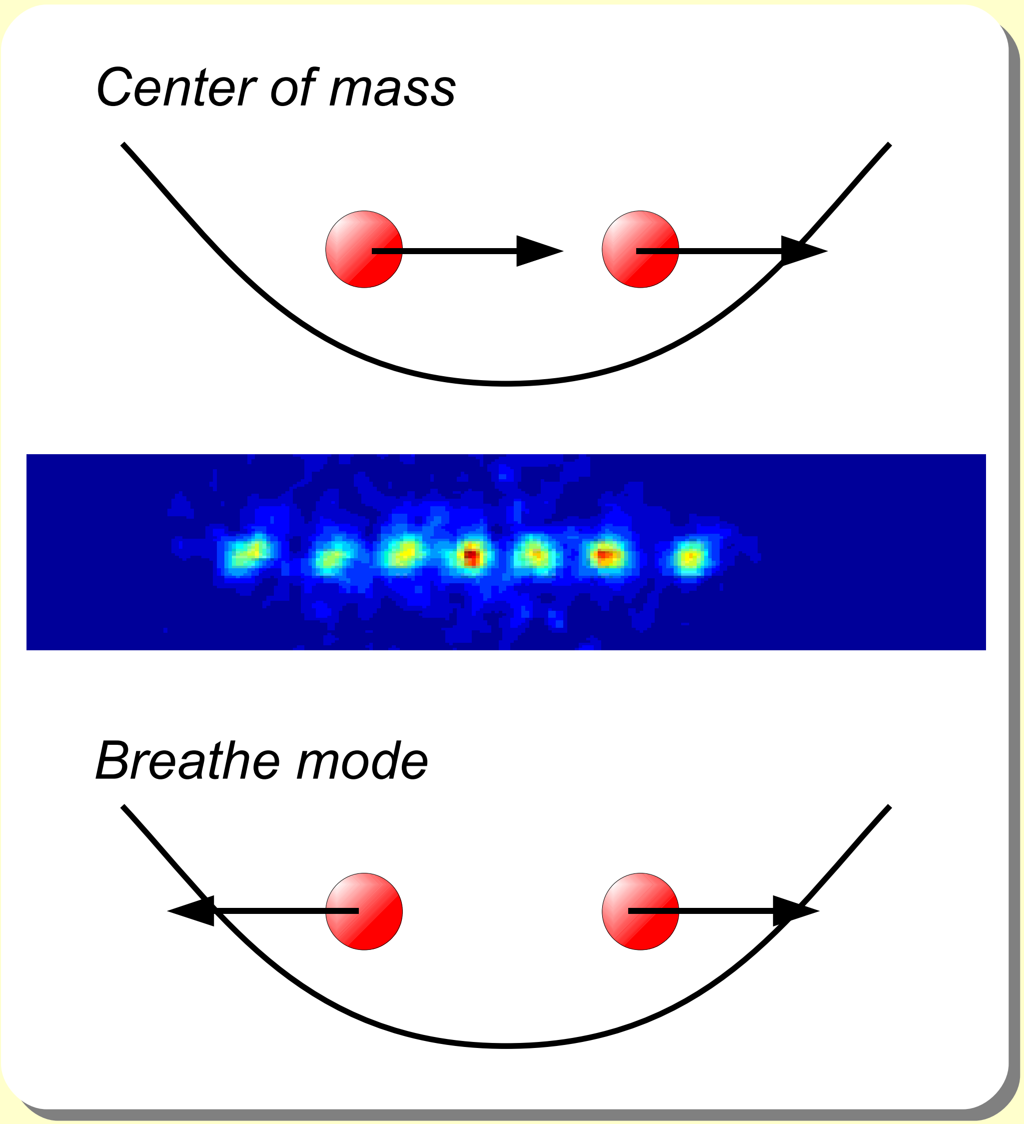
- The phase depends on the operator

$$\phi = \int_0^t \int_0^{\tau_1} e^{i\omega(\tau_1 - \tau_2)} F(\tau_1) F(\tau_2) \times$$

$$\times \sigma_z^2 \, d\tau_1 \, d\tau_2$$

- Not very useful :-)

Geometric quantum gate



- In an experiment with more than one ion, we have more vibrational modes

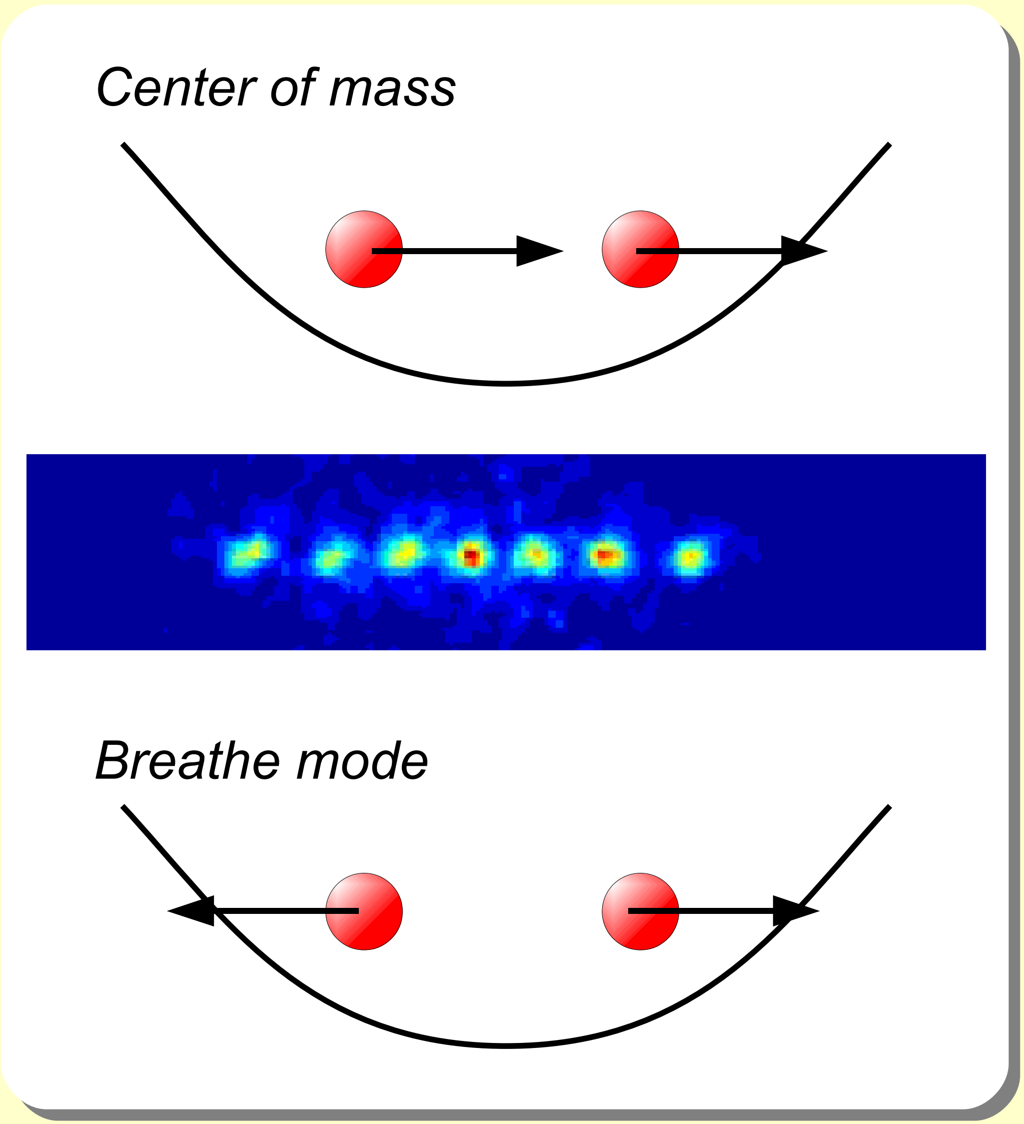
$$H = \omega_{CM} a_{CM}^{+} a_{CM} + \omega_R a_R^{+} a_R - F(t)\sigma_1^z x_1 - F(t)\sigma_2^z x_2$$

- Note that x1 and x2 do no longer make sense

$$x_1 = R + \frac{1}{2} d$$

$$x_2 = R - \frac{1}{2} d$$

Geometric quantum gate



- In an experiment with more than one ion, we have more vibrational modes

$$H=\omega_{CM}a_{CM}^{+}a_{CM}+\omega_{R}a_{R}^{+}a_{R}-F(\sigma_1^z+\sigma_2^z)\frac{1}{\sqrt{2}}(a_{CM}+a_{CM}^{+})-F(\sigma_1^z-\sigma_2^z)\frac{1}{2\sqrt{2}}(a_R+a_R^{+})$$

- We obtain phases that depend on

$$A(\sigma_1^z+\sigma_2^z)^2+B(\sigma_1^z-\sigma_2^z)^2\sim\sim J\sigma_1^z\sigma_2^z$$

Geometric quantum gates

Complete solution

$$|\psi(t)\rangle = U_{rot} U_{tr} U_{ph} |\psi(0)\rangle$$

$$U_{rot} = \prod_k e^{-i\nu_k a_k^\dagger a_k t}$$

Free evolution

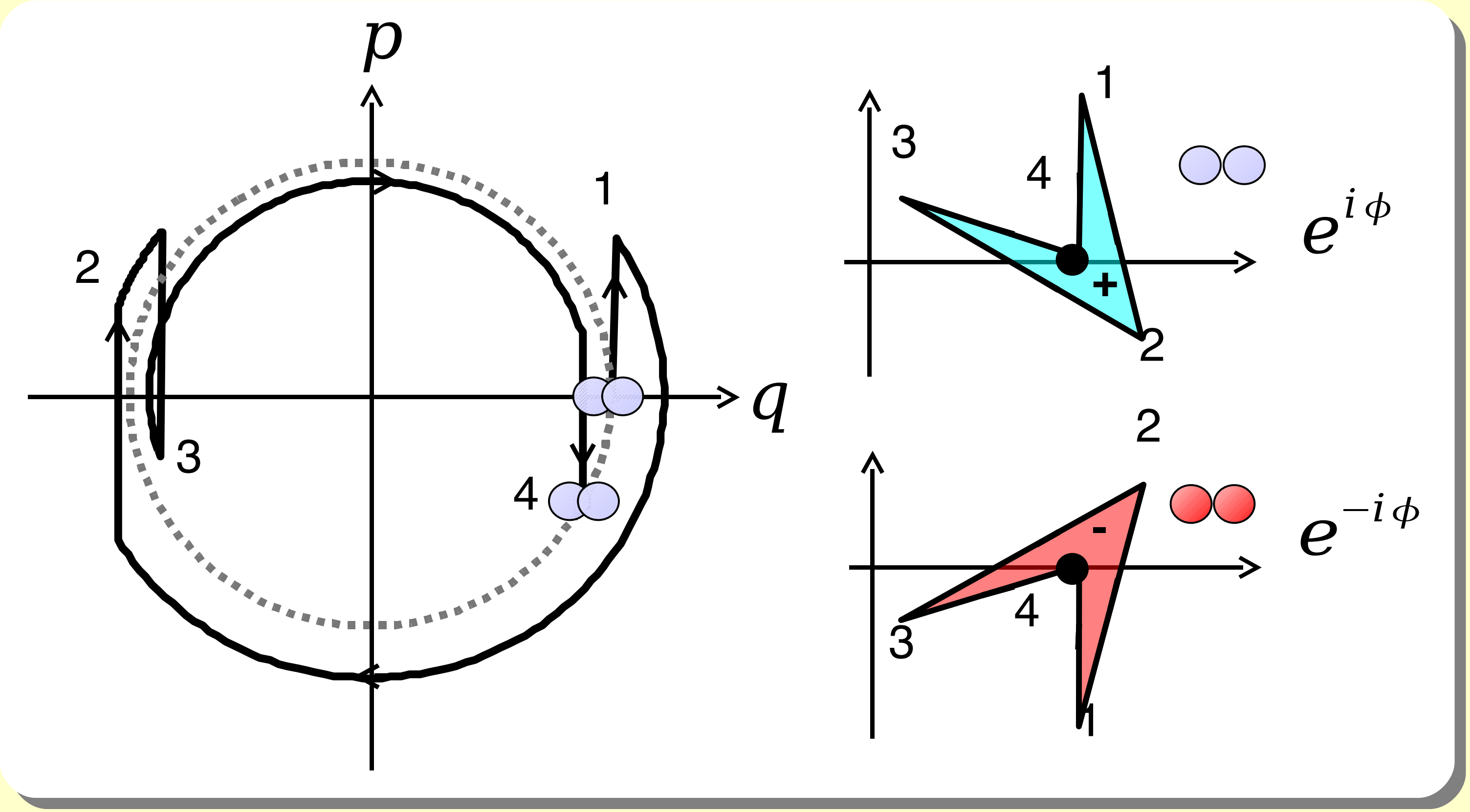
$$U_{tr}(\sigma) = \prod_k e^{z_k a_k^\dagger - \bar{z}_k a_k}$$

Final displacement

$$U_{ph} = \exp\left(i \sum_{ij} J_{ij} \sigma_z^i \sigma_z^j \right)$$

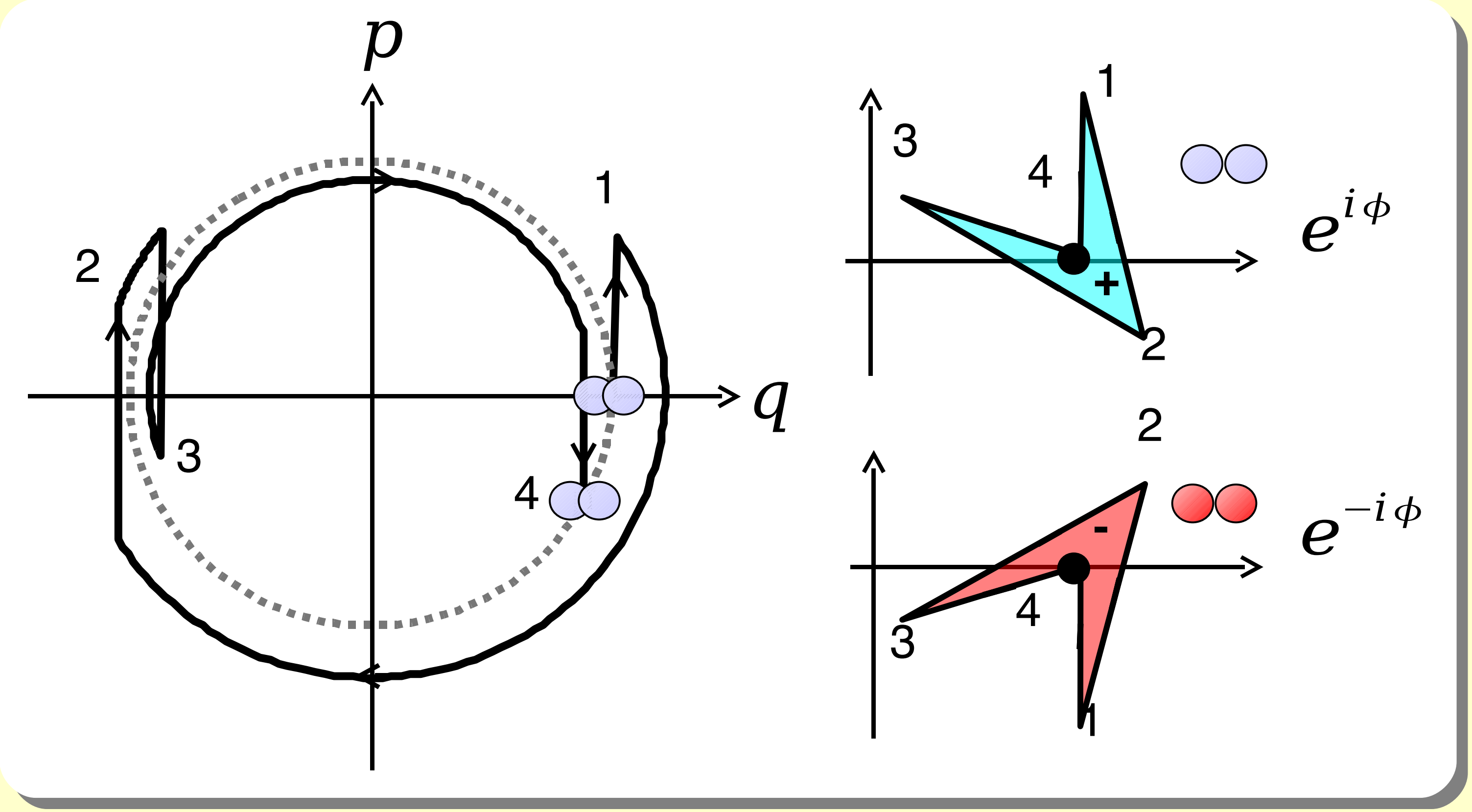
Geometrical phases

Example: instantaneous kicks



- Depending on the internal state of the ions, we get different oriented paths and phases.

Example: instantaneous kicks

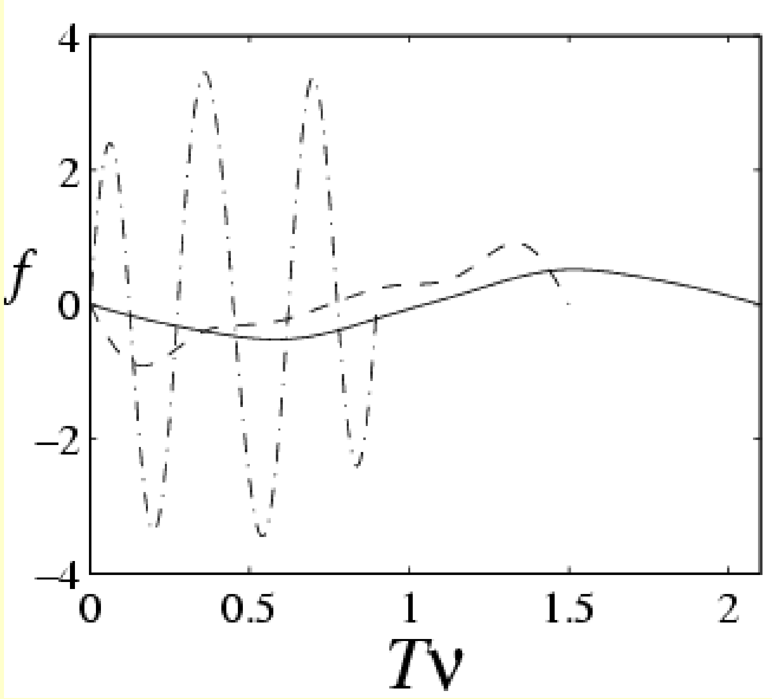


- In order to disentangle motion and internal state, we have to return to the original orbit.

$$\int_0^T F(\tau) e^{-i\omega_{CM}\tau} = \int_0^T F(\tau) e^{-i\omega_R \tau} = 0$$

Optimal controls

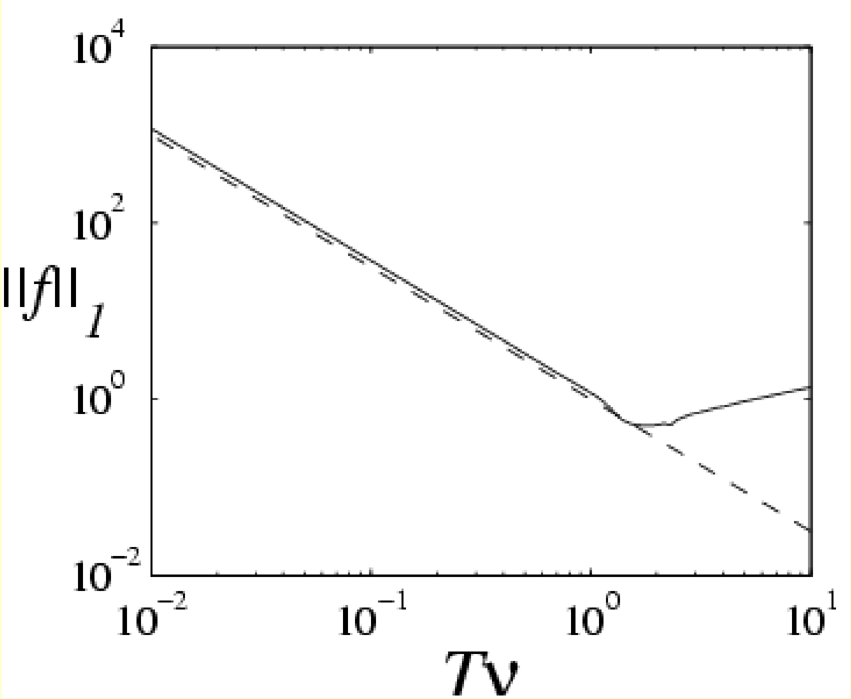
Sample forces



- Very good solutions already for 3 modes

$$F(t)=\sum_{i=-1}^{1} c_i e^{i\omega 2\pi n t/T}$$

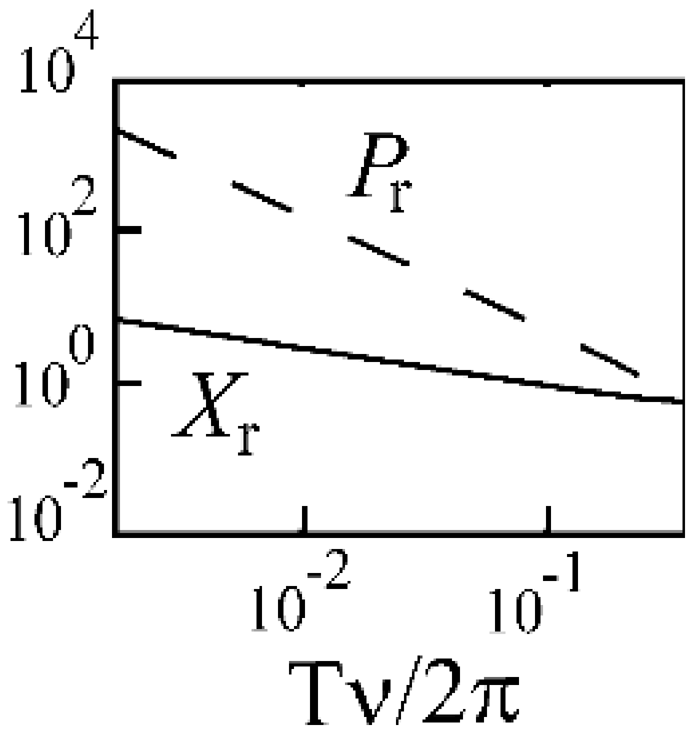
- **Intensity vs time**



- Faster gates need stronger forces

$$|F|^2 \sim T^{-3/2}$$

Optimal controls



- For very fast quantum gates, the ion makes large excursions in phase space.
- The anharmonic terms in the Hamiltonian may become relevant

$$H = \sum_k \omega_k a_k^+ a_k + O(V''' x^3)$$

- A simple estimate gives

$$T_{min} \simeq 10^{-3} / \omega_{trap}$$