# Optical lattices

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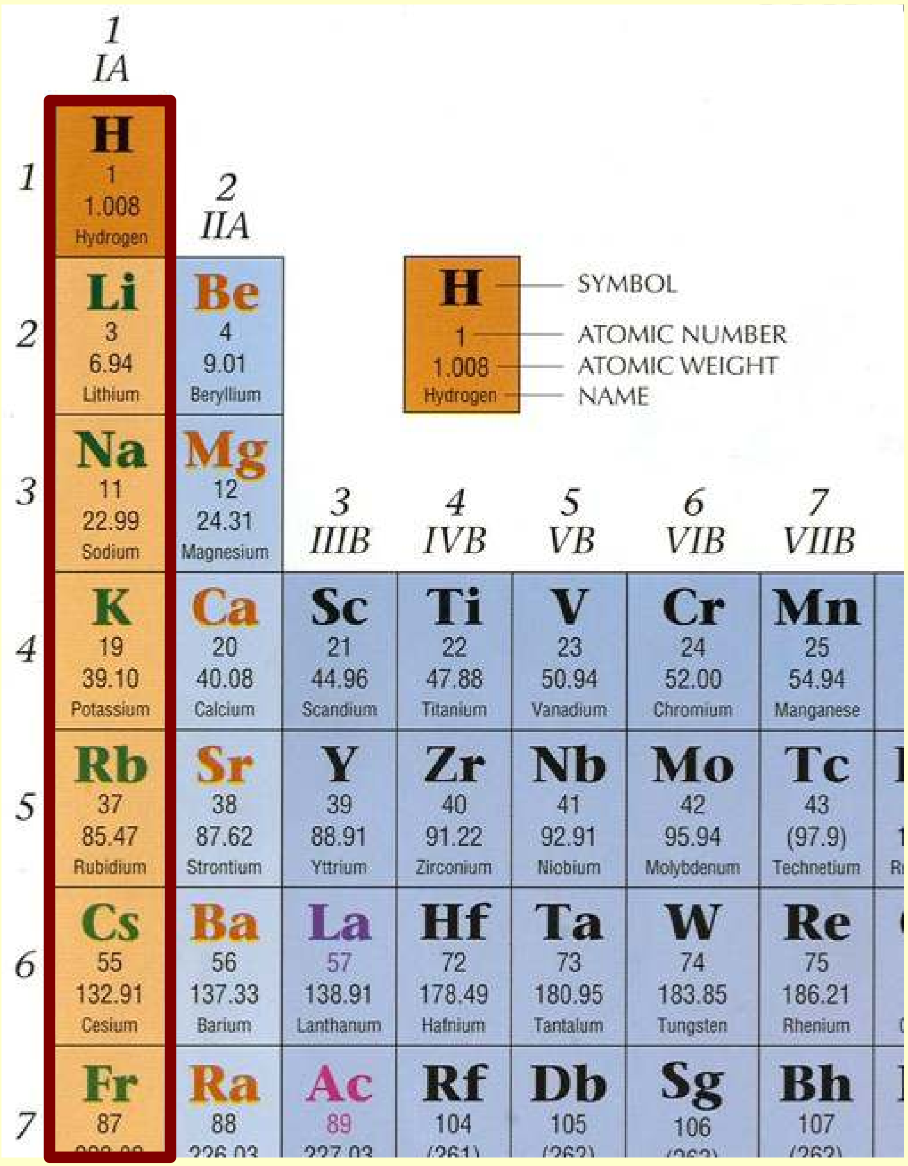
# Change of room

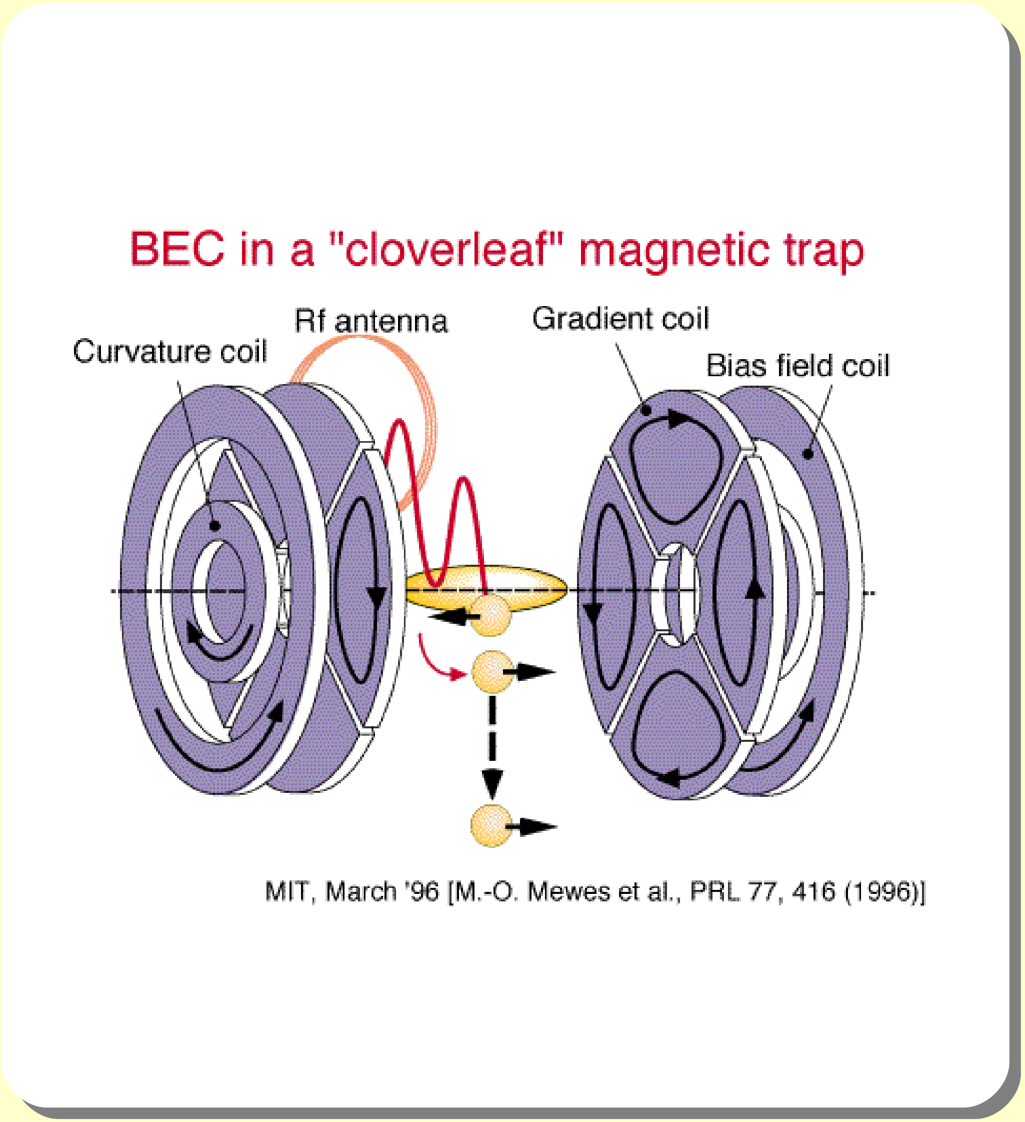
C5 – 1068
Sala de graus II
Monday – Tuesday

# Atoms



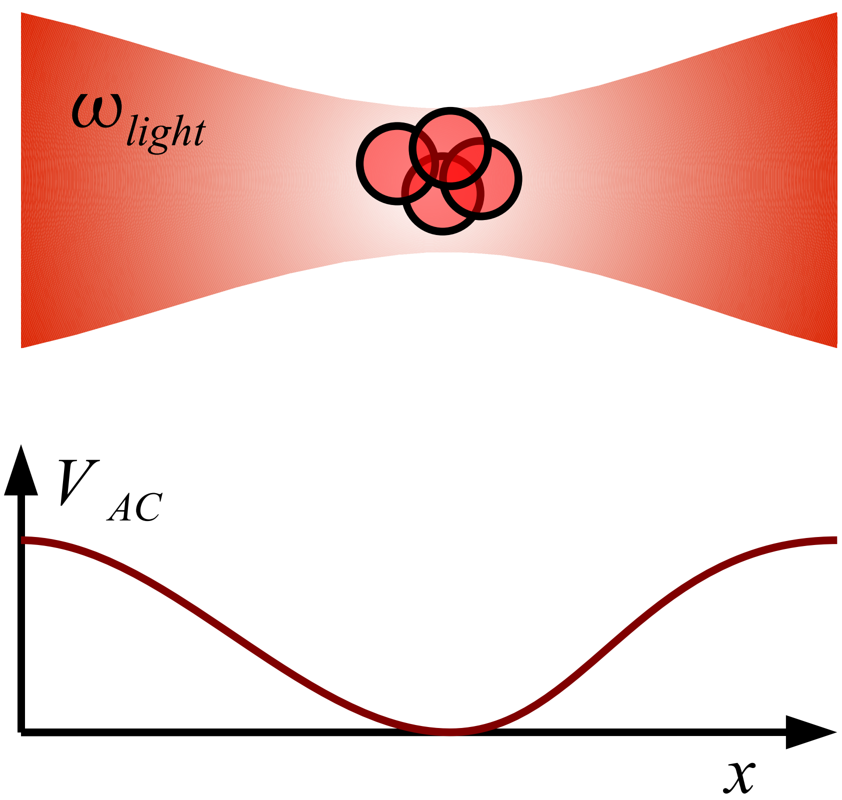

- Instead of ions, which couple strongly to the environment, use **neutral** atoms.
- Good:
  - Long lived coherences
  - Weaker interactions
  - Less energy to trap many
    - Large, scalable
- Bad:
  - Weak interactions
  - Trapping?
  - Cooling?

# Magnetic trapping




- We can trap the low-field seekers
  - Atoms that tend towards small value of |B|
- Majorana flips: when B=0, the atom magnetic moment can flip at no cost, becoming untrapped.
- Ioffe-Pritchard and cloverleaf traps combine quadrupole potentials with coils that shift the minimum to nonzero B.

# Light force

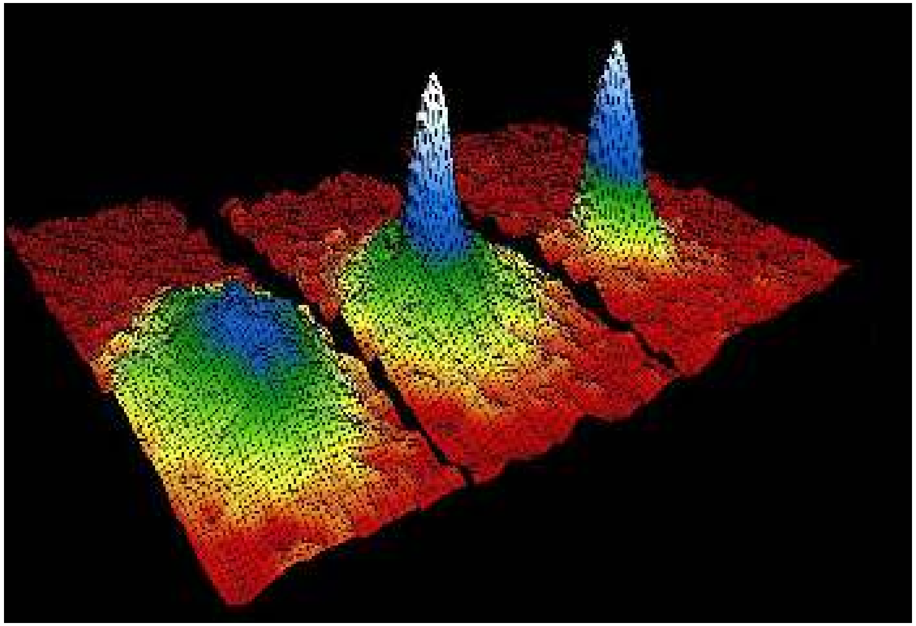



- When off-resonant light acts on an atom, it induces an energy shift
  - AC Stark shift

$$V_{AC} \sim -\frac{\Omega(x)^2}{\Delta}$$

- The potential depends on the light intensity.
- Different detunings
  - **Red:** attracted to maxima
  - **Blue:** attracted to minima

# Bose-Einstein condesate



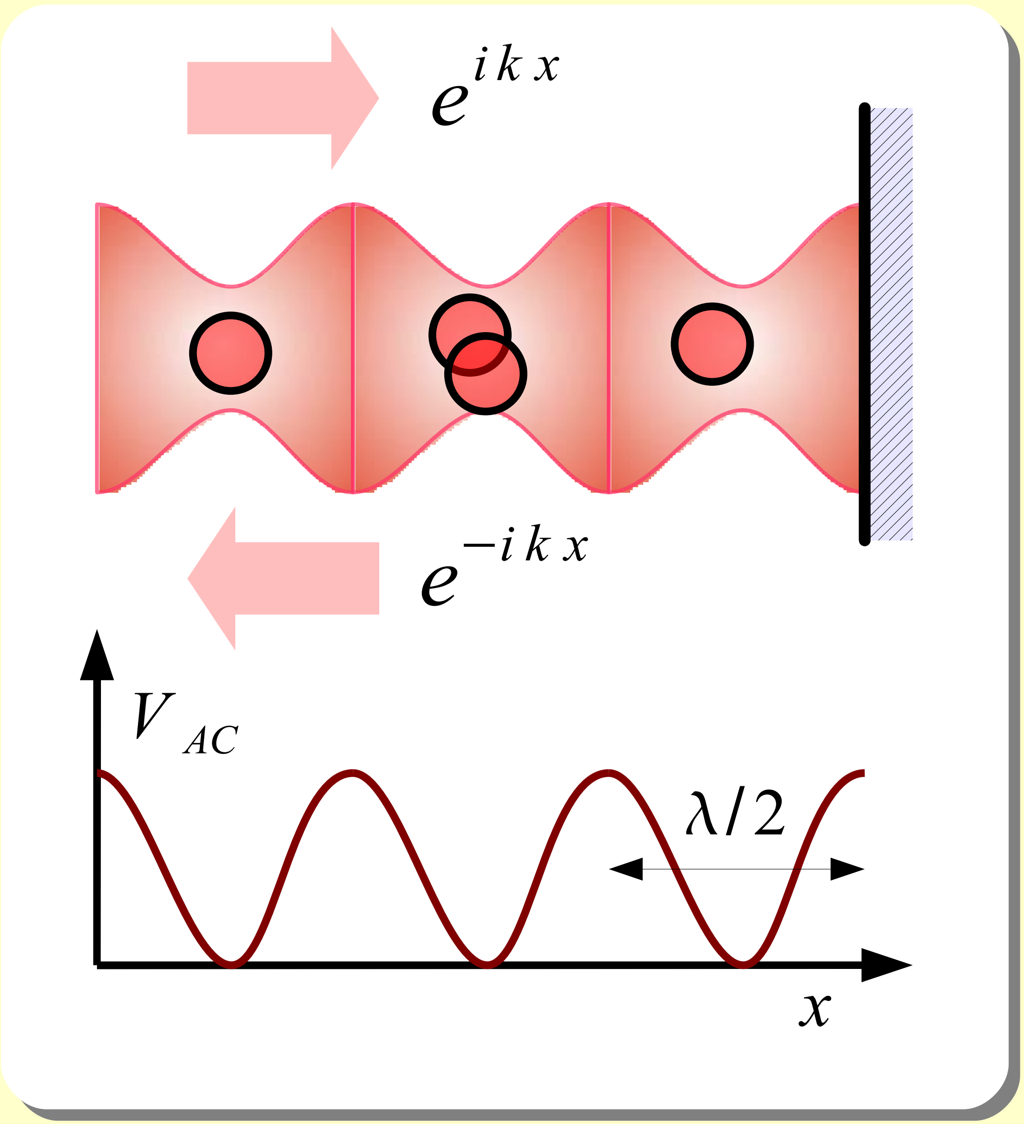
$$\psi(x_{1,}x_{2,}\ldots)=\phi(x_1)\phi(x_2)\cdots$$

$=$

Source of ultracold atoms, with the **same motional state**. We only care about internal state now!

# Isolating qubits

# Optical lattice




- Our goal is to trap an array of atoms that work as **qubits**
- We will create a standing wave of light

$$e^{ikx}+e^{-ikx}\sim\cos(kx)$$

- The AC Stark shift is created by this "cosine" mode.

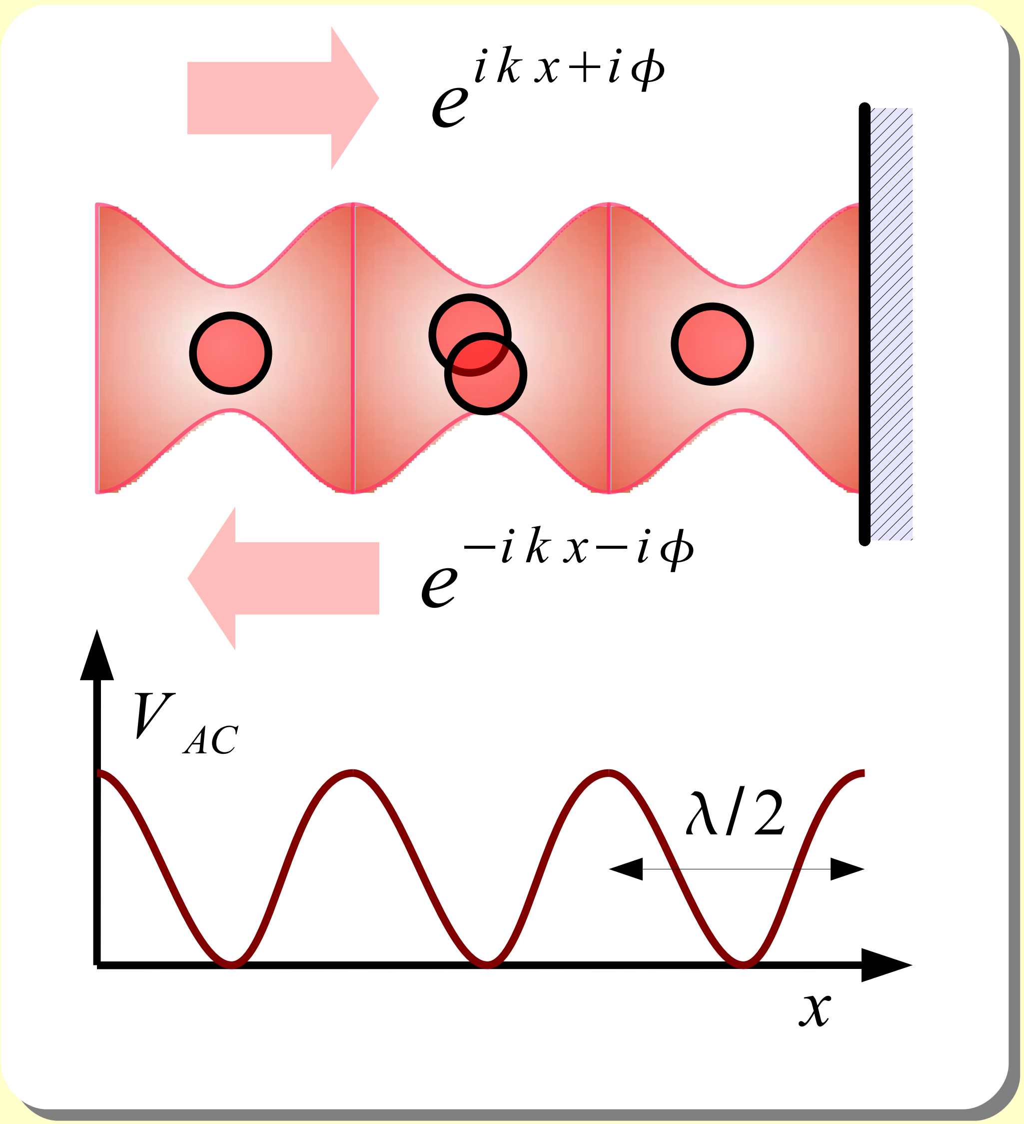
$$V\sim\cos(2\pi x/\lambda)^2$$

- Extremely tight confinement

$$\lambda_{Rb}\sim 850\,nm$$

D. Jaksch et al, Phys. Rev. Lett. 1998

# Optical lattice




D. Jaksch et al, Phys. Rev. Lett. 1998

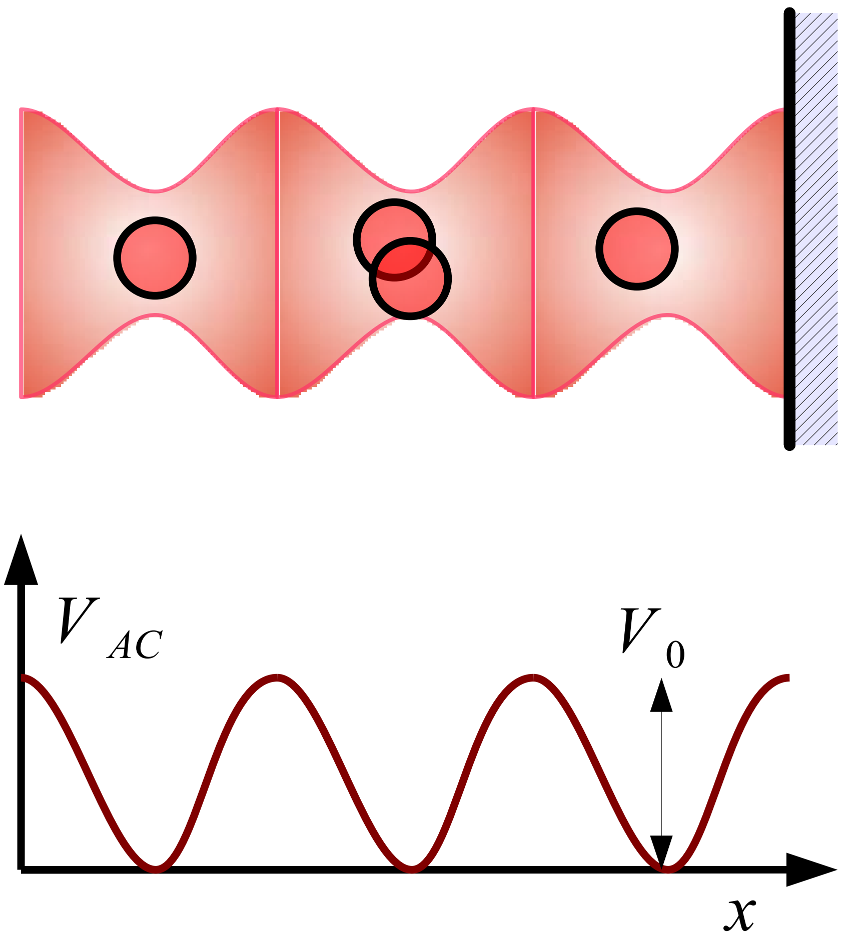
- Note the extreme sensitivity to the phase of the laser beams

$$V \sim \cos(2\pi(x+x_0)/\lambda)^2$$

$$x_0 = \lambda\phi/2\pi$$

- Any of these will break the experiment:
  - Light from different sources.
  - Mirror vibrates.
  - Phase drifts in laser
  - ...

# Optical lattice




D. Jaksch et al, Phys. Rev. Lett. 1998

- How strong is the confinement?
- Recoil energy: energy given by the photon to the atom

$$E_R = \frac{(\hbar k)^2}{2m}$$

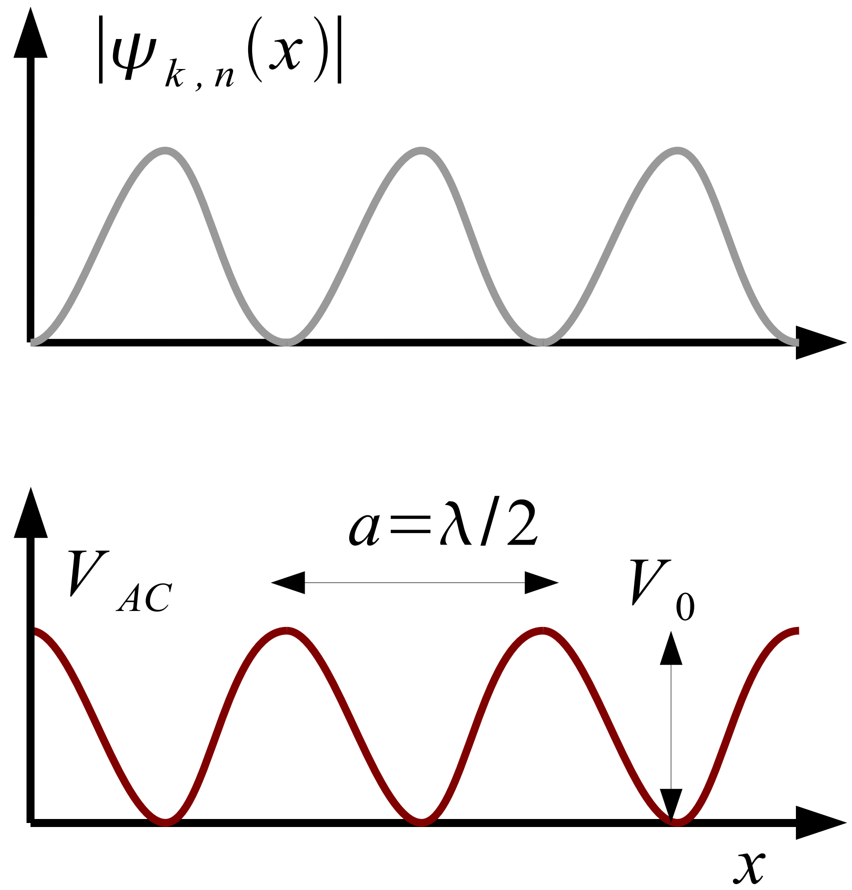
- Confinement can be strong

$$V_0 \sim (1-100) E_R$$

- What do the eigenstates look like?
- Can we trap isolated atoms?

# Single-particle wavefunctions

# Bloch wavefunctions




- For simplicity, we will impose periodic boundary conditions (the open b.c. can be derived from this case).
- Let us introduce translation operator, *T*

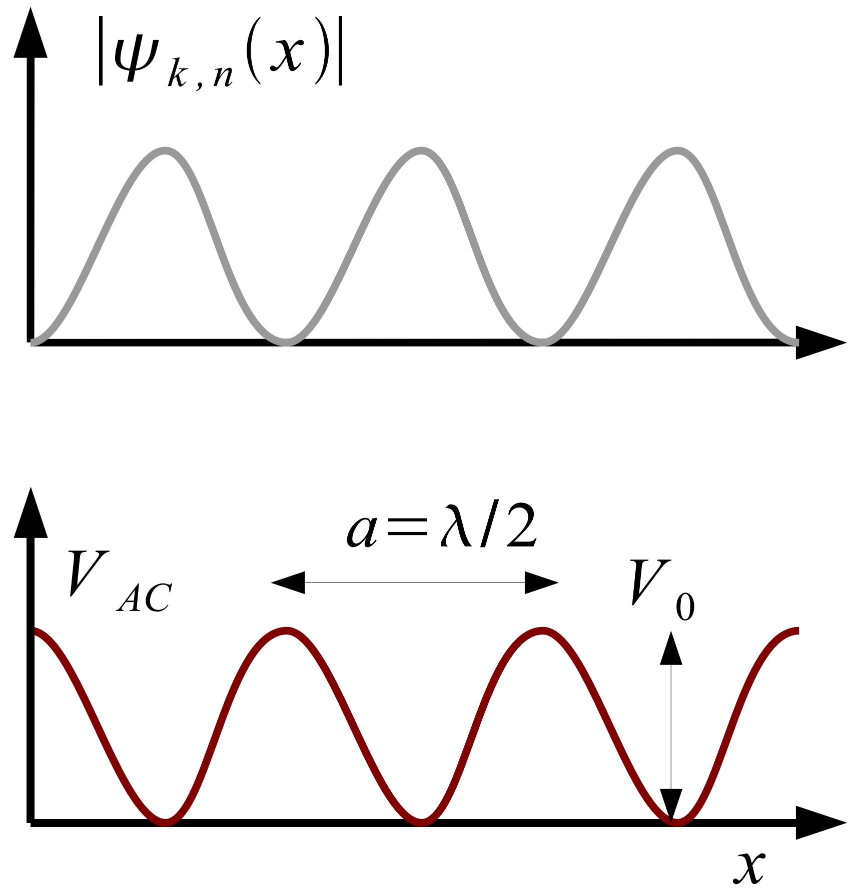
$$(T\psi_{kn})(x)=\psi_{kn}(x+a)$$

- If there are "N" sites, PBC

$$(T^N\psi_{kn})(x)=\psi_{kn}(x+aN)$$
$$=\psi_{kn}(x)$$

D. Jaksch, PhD thesis

# Bloch wavefunctions




D. Jaksch, PhD thesis

- Due to translational invariance, we can find a basis of common eigenstates of H and T

$$(T\psi_{kn})(x) = \psi_{kn}(x+a)$$
$$= e^{i\theta}\psi_{kn}(x)$$

- To match PBC

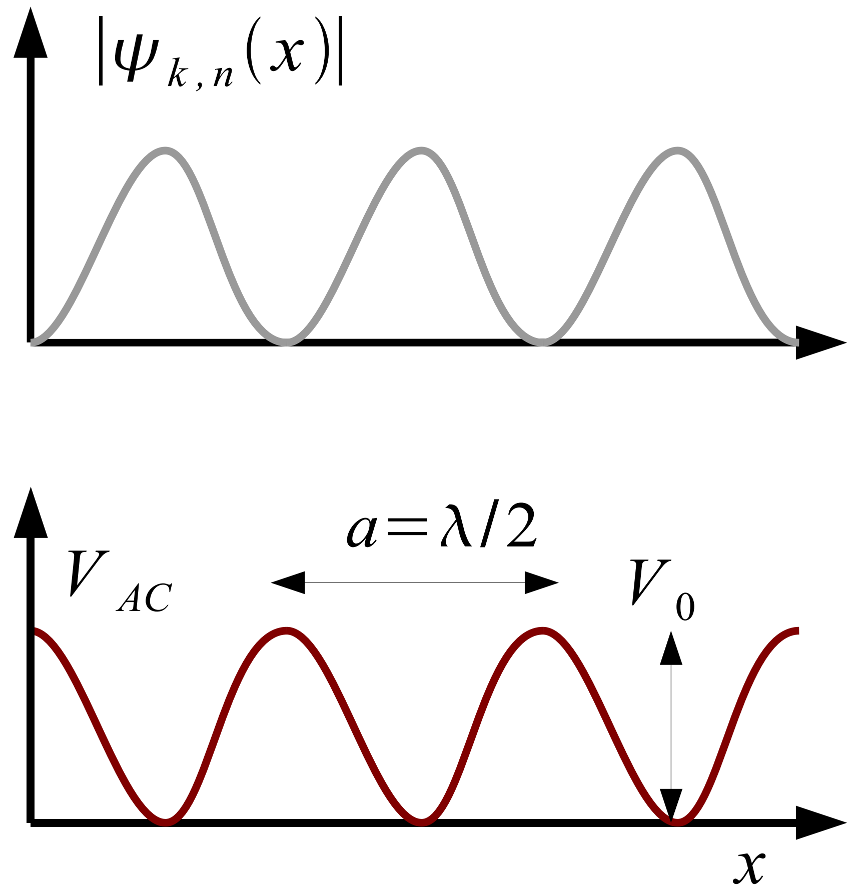
$$\theta = 2\pi m/N, \quad m \in \mathbb{Z}$$

We can write

$$k = 2\pi m/(Na)$$
$$\psi_{kn}(x) = e^{ikx} u_{kn}(x)$$

periodic Bloch wavef. “u”

# Bloch wavefunctions



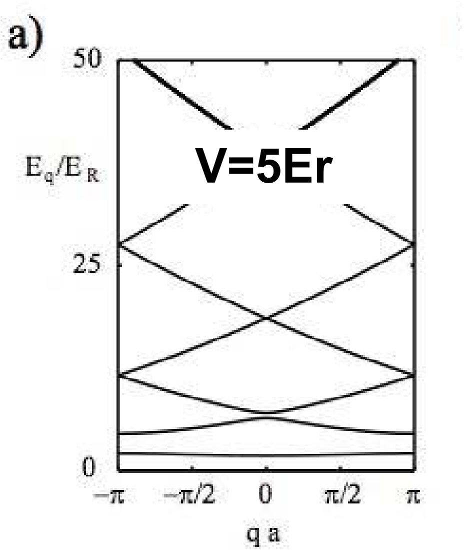

- The Bloch wavefunctions satisfy a Matthieu equation

$$\left[\frac{\hbar^2}{2\mathrm{m}}(-i\,\partial_x-k)^2+V_0\cos(kx)-E_{kn}\right]\times$$
$$\times u_{kn}(x)=0$$

- There may be additional quantum numbers, “n” which denote the **band** number.
- If the atom have internal states, these degrees of freedom should be added.

D. Jaksch, PhD thesis

# Band structure

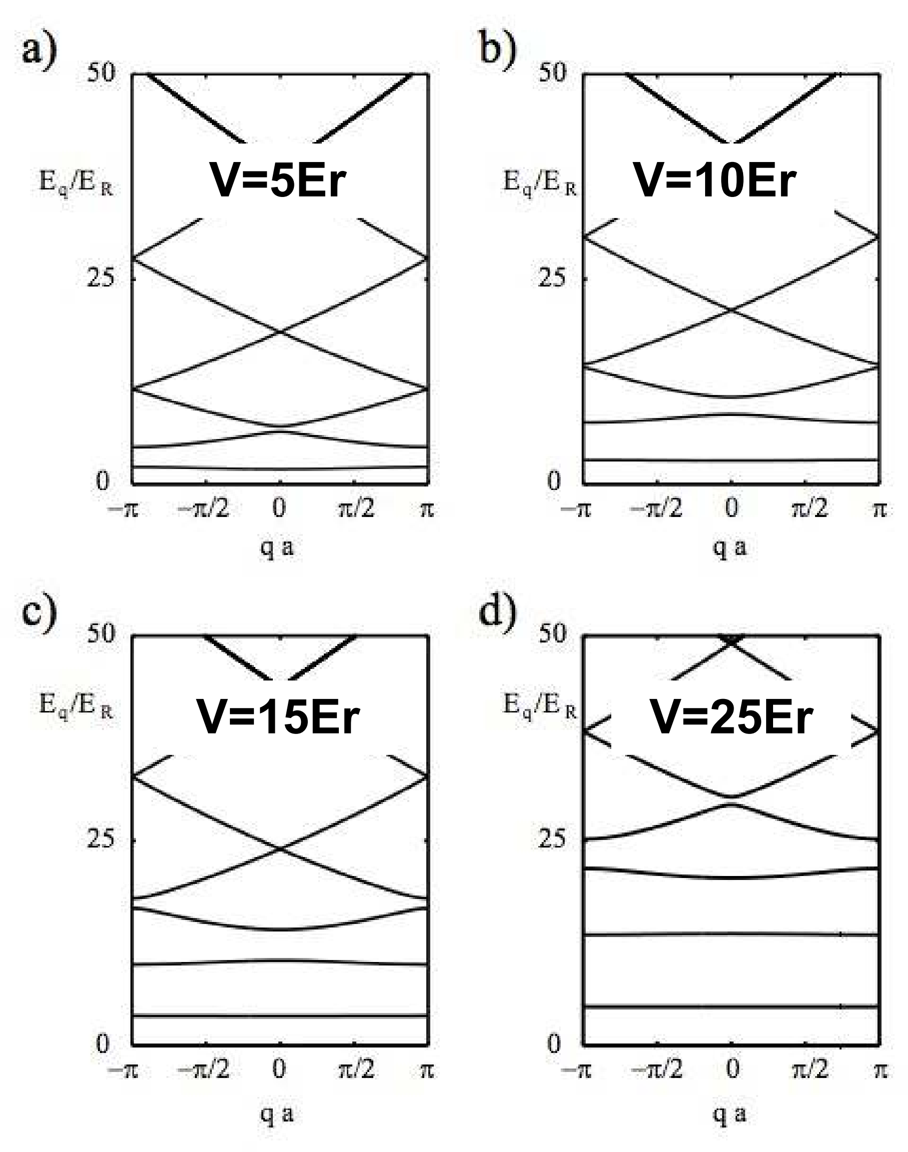



D. Jaksch, PhD thesis

- If the potential is weak, the energies are like free particles

$$E_k \sim (\hbar k)^2/2m$$

# Band structure




D. Jaksch, PhD thesis

- If the potential is weak, the energies are like free particles

$$E_k \sim (\hbar k)^2 / 2\mathrm{m}$$

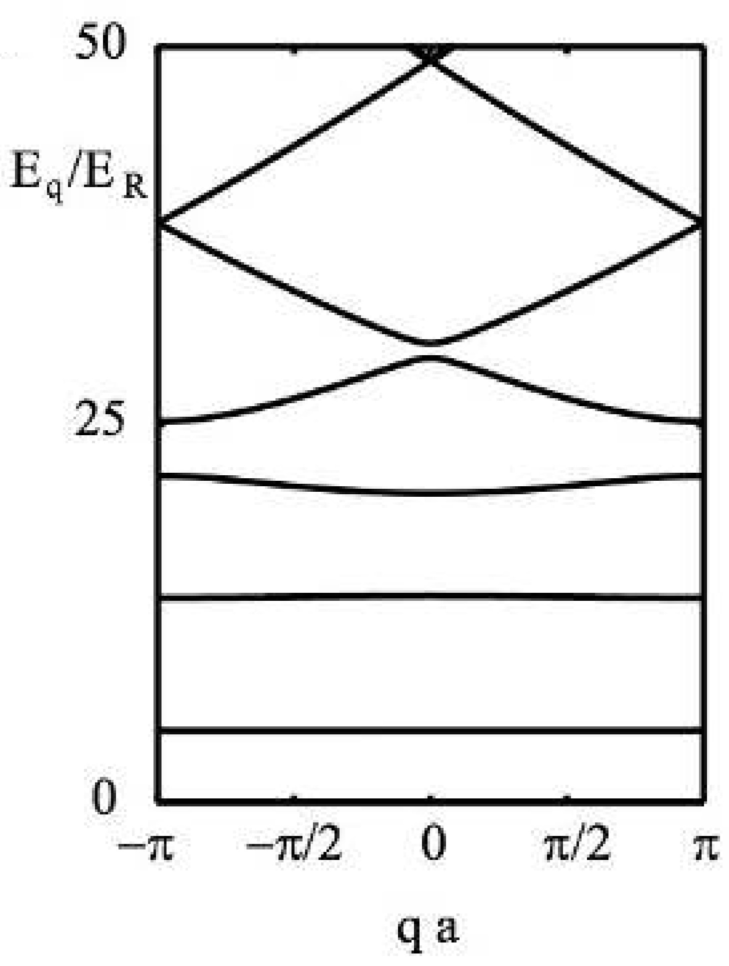
- As we make the potential stronger

$$V_0 \gg E_R$$

there appears a gap between bands and the lowest bands become flat

$$E_{k0} \sim const.$$

# Band structure

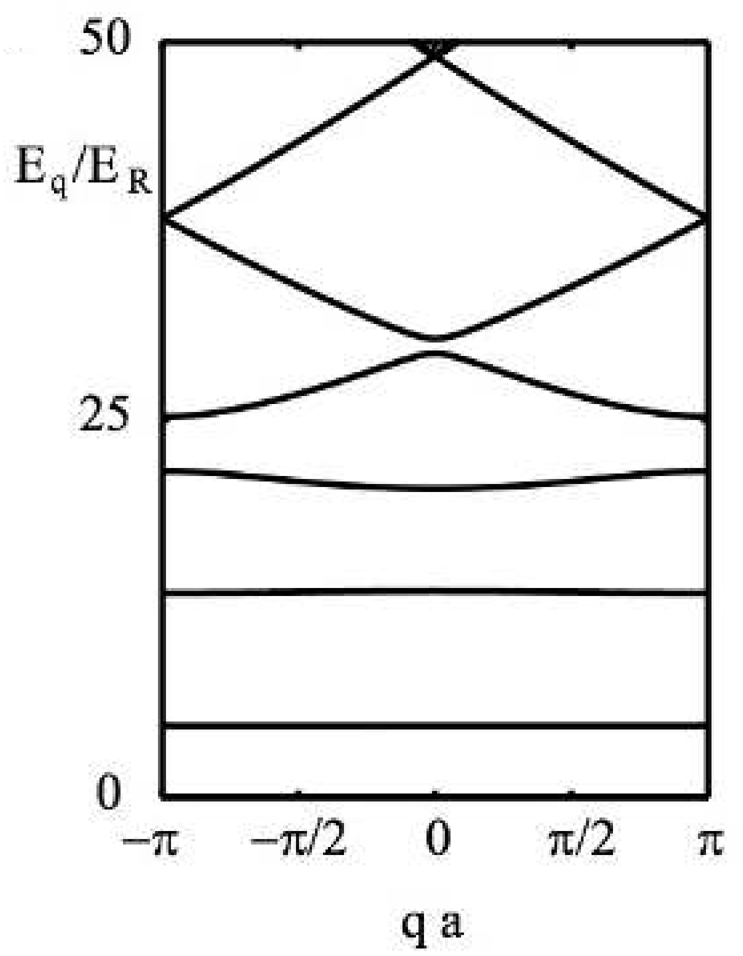



- Having a flat band means having N disconnected potential wells.

$$H_{1\text{part}} \sim E_0 \sum_k |k\rangle\langle k|$$

D. Jaksch, PhD thesis

# Band structure




- Having a flat band means having N disconnected potential wells.

$$H_{1\text{part}} \sim E_0 \sum_k |k\rangle\langle k|$$

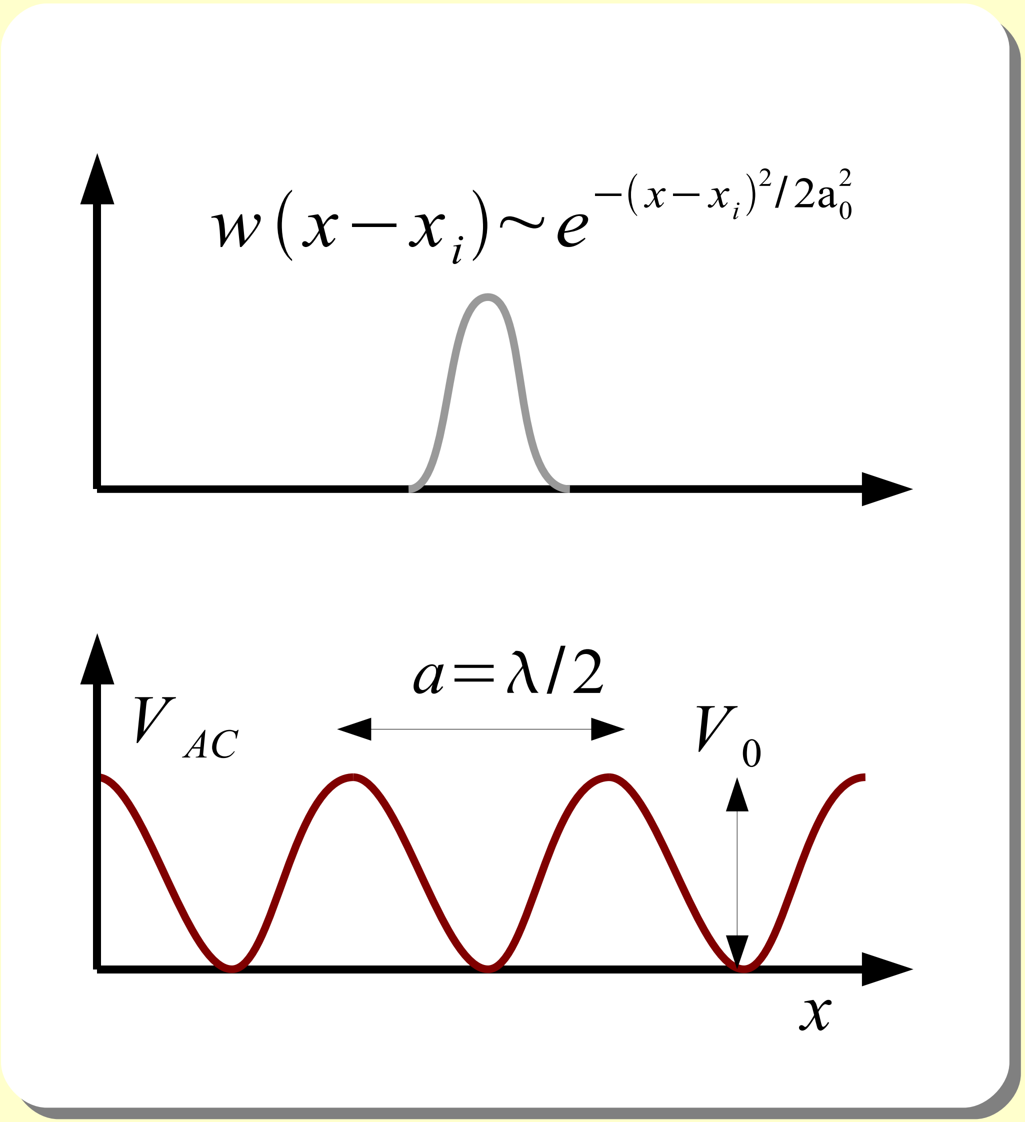
- It is obvious that for infinitely deep lattices we have isolated atoms

$$H_{1\text{part}} \sim E_0 \sum_{i=1}^{N} |i\rangle\langle i|$$

on the i-th lattice sites

D. Jaksch, PhD thesis

# Band structure



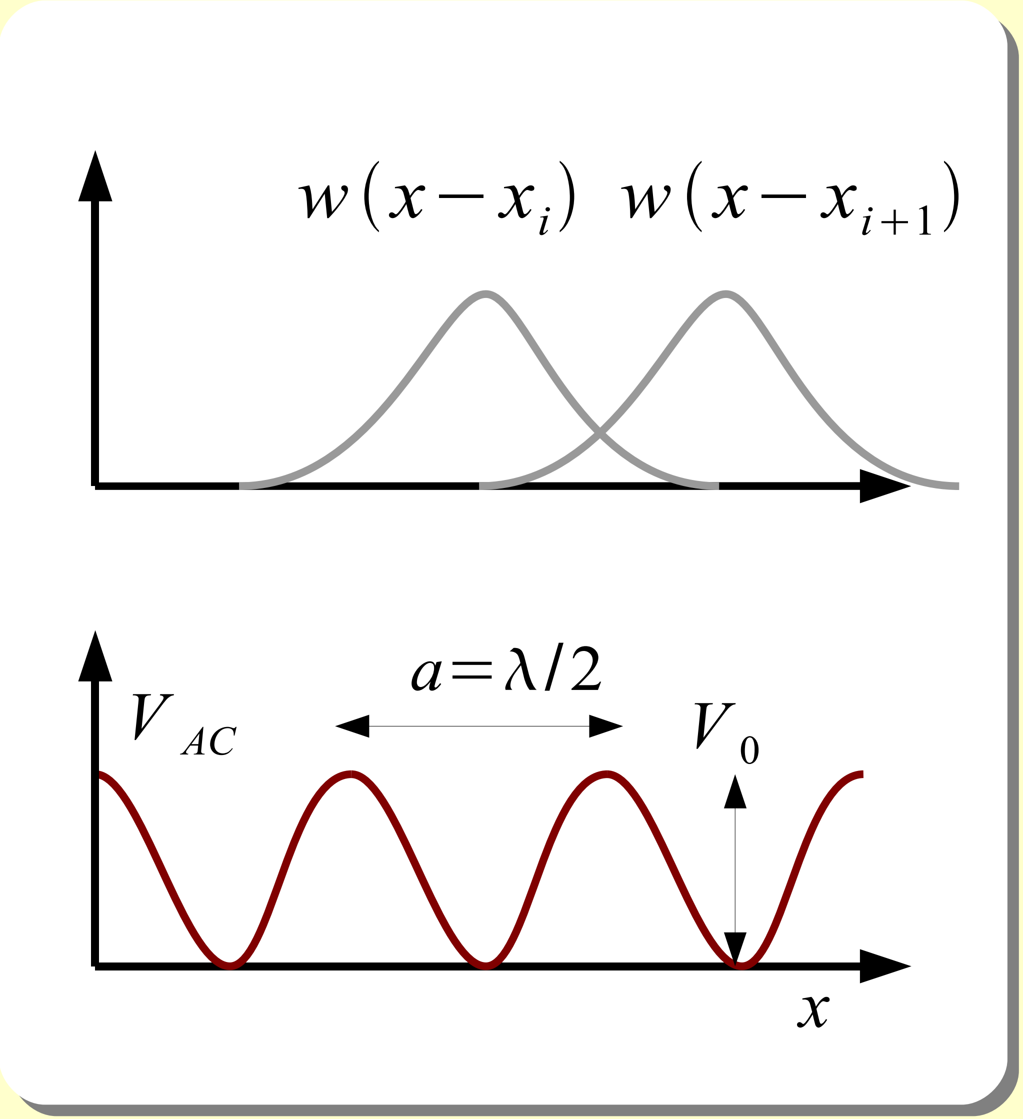

- The wavefunction is approx. the one of a harmonic potential around the minimum.
- Using a harmonic approximation we find

$$\hbar\omega \sim \sqrt{V_0 E_R}$$

$$a_0 \sim \lambda\sqrt{\frac{E_R}{V_0}} \ll \lambda$$

D. Jaksch, PhD thesis

# Band structure




D. Jaksch, PhD thesis

- For other intermediate regimes

$$E_R \leqslant V_0$$

we can use Wannier wavepackets.

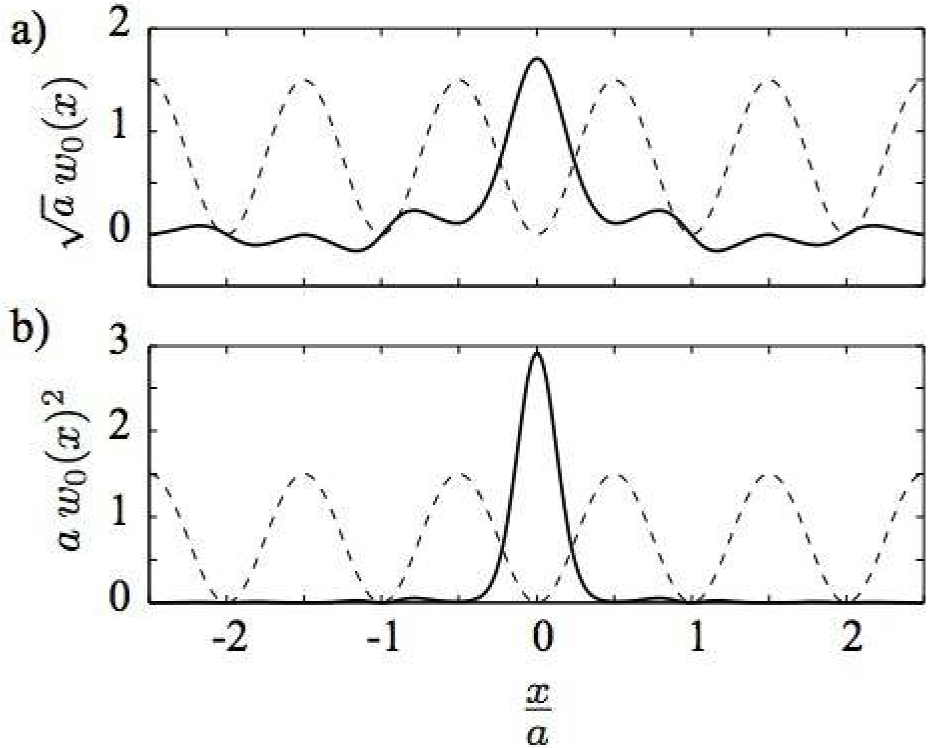
- They are localized wavefunctions

$$w(x-a\times j)=\sum_k e^{-ikaj} u_{k,n=0}(x)$$

that form an orthonormal set

$$\int_0^L w(x-aj)\,w(x-ak)\,dx=\delta_{jk}$$

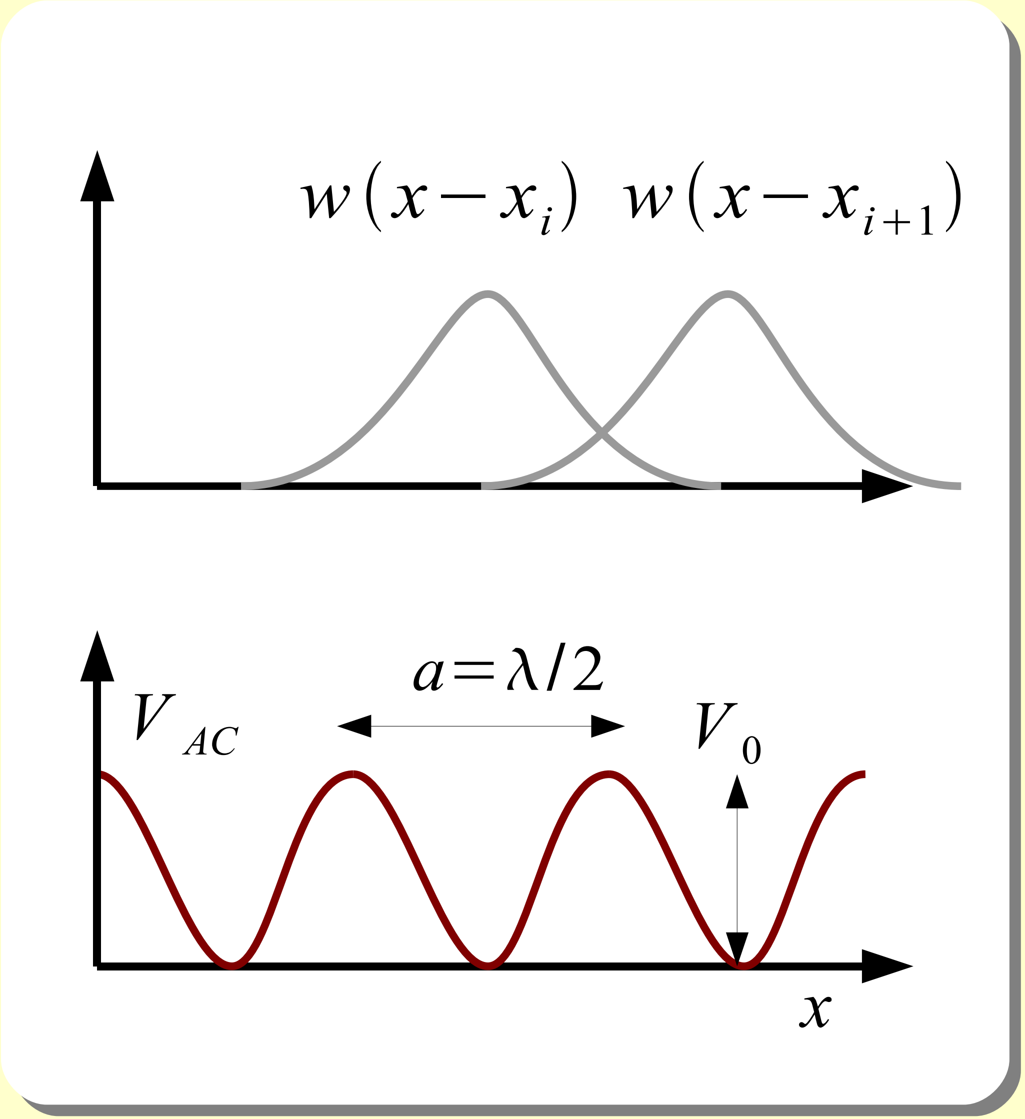
# Band structure




- The Wannier wavefunctions
  - Actually decay more slowly than an exponential.
  - They are not positive
  - The overlap between two is not zero.
  - But the scalar product is zero.
  - They form a complete basis.

D. Jaksch, PhD thesis

# Band structure




D. Jaksch, PhD thesis

- In this basis we can approximate the Hamiltonian.
- The model now includes an effective hopping

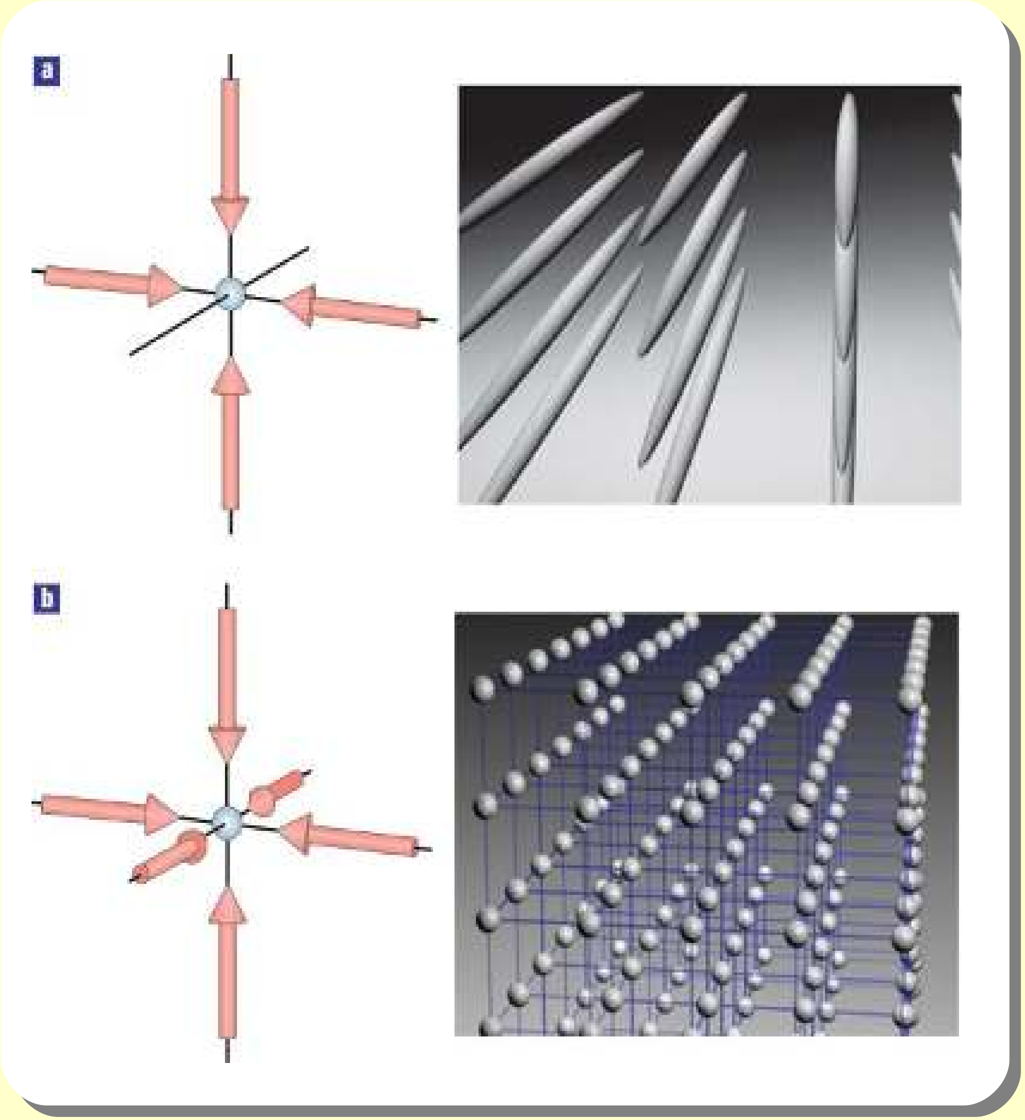
$$J \propto \int w(x)\, H\, w(x-x_1)\, dx$$

$$H_{1\mathrm{part}} \sim J \sum_i \left[ |i+1\rangle\langle i| + |i\rangle\langle i+1| \right]$$

- Note that J is exponentially suppressed as $V_0$ grows

# Physical consequences

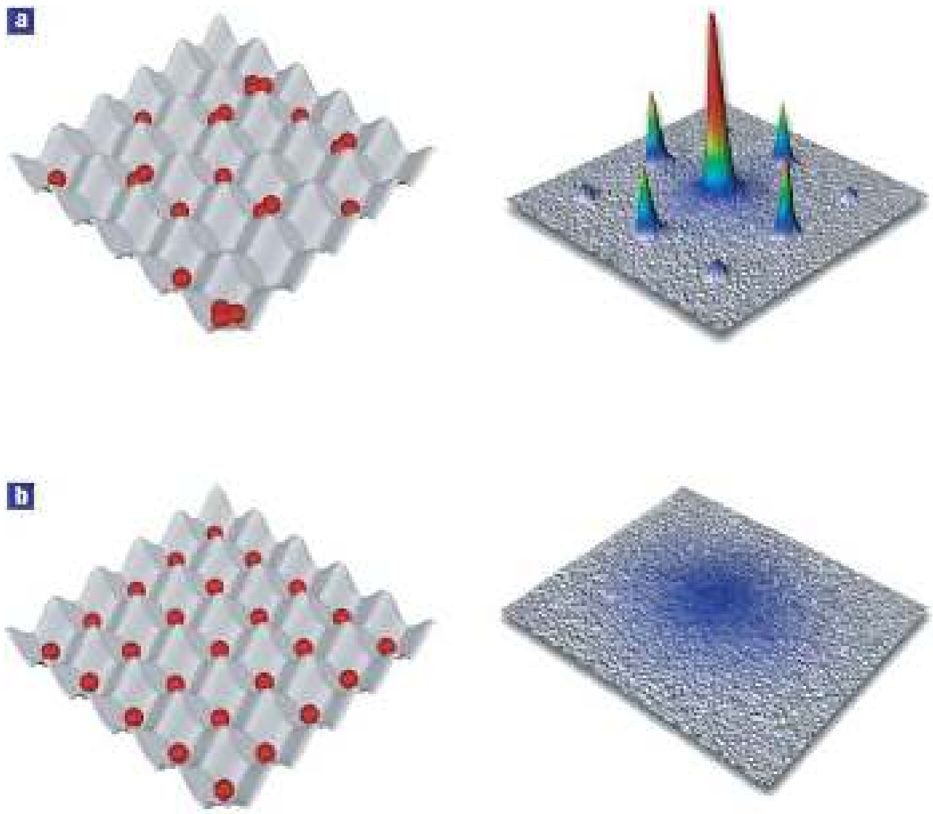
# Variable geometry




- One-, two- and thre-dimensional structures.
- Very stable in space and time.
- Optical wells do not vibrate: no heating.
- Spontaneous emission still limits lifetime to seconds or minutes
  - Can be reduced with blue detuned lattices

Immanuel Bloch, Nature Physics **1**, 23 - 30 (2005)

# Two trapping regimes



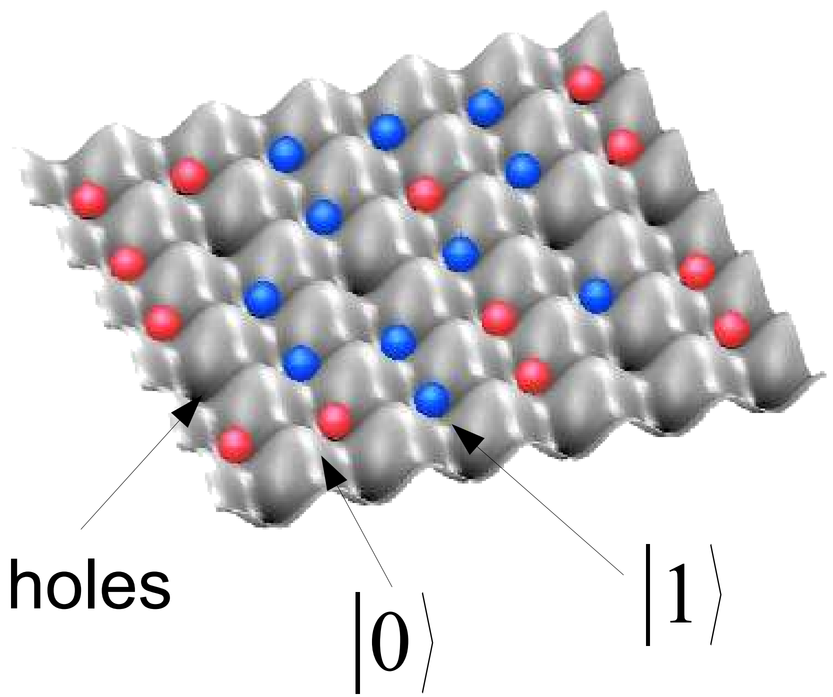

$$J, U \sim 1 - 10\, kHz$$

$$T \sim 50\, nK$$

- When lattice is weak, atoms may move
  - lattice superfluid
- When lattice is deep, atoms localize
  - lattice insulator
  - an array of perfectly isolated qubits.

Immanuel Bloch, Nature Physics **1**, 23 - 30 (2005)

# Quantum register




$$N_{1D} \sim 10 - 100$$

$$N_{2D} \sim 100 - 10^4$$

$$N_{3D} \sim 10^6$$

- One atom per lattice site, acting as a possible qubit.
- Information stored in the state of the atom.
- A macroscopic number of qubits...
- ...or a huge number of copies of the same experiment.
- May introduce **defects** when loading the lattice
  - holes, double loading, etc.