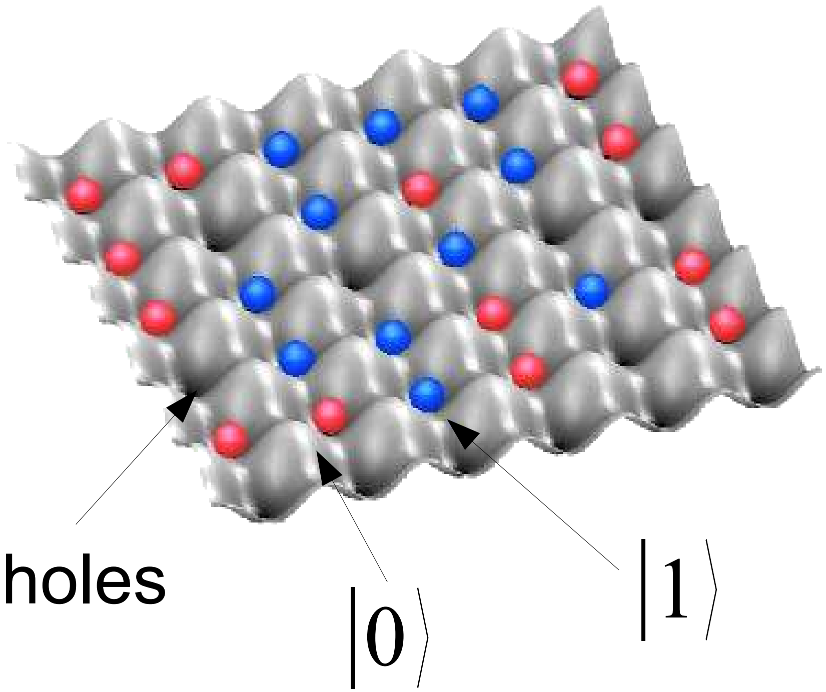
Optical lattice & Quantum computation

J. J. García-Ripoll
IFF, CSIC Madrid

(17-4-2009)

Quantum register



$$N_{1D} \sim 10 - 100$$

$$N_{2D} \sim 100 - 10^4$$

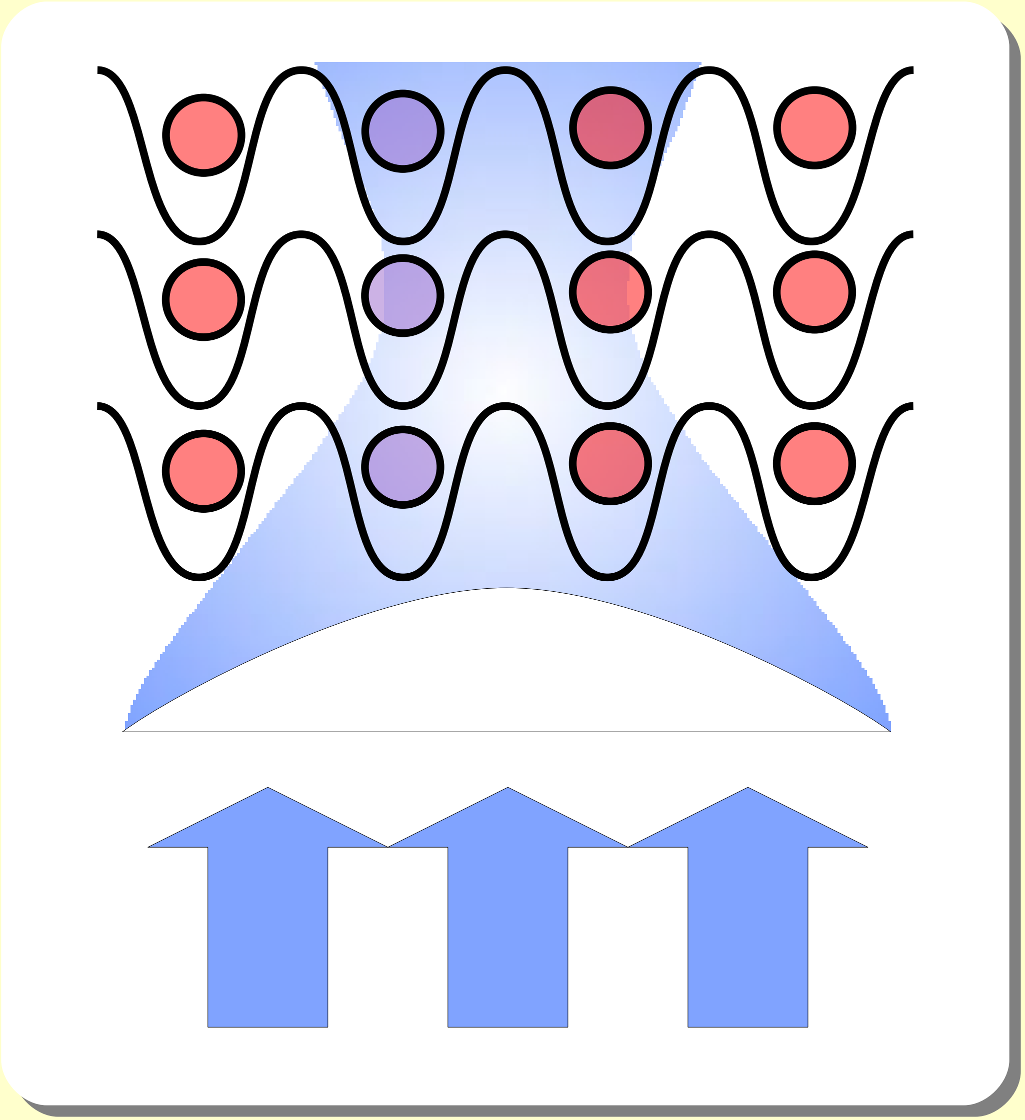
$$N_{3D} \sim 10^6$$

- One atom per lattice site, acting as a possible qubit.
- Information stored in the state of the atom.
- A macroscopic number of qubits...
- ...or a huge number of copies of the same experiment.
- May introduce **defects** when loading the lattice
  - holes, double loading, etc.

Single-qubit operations

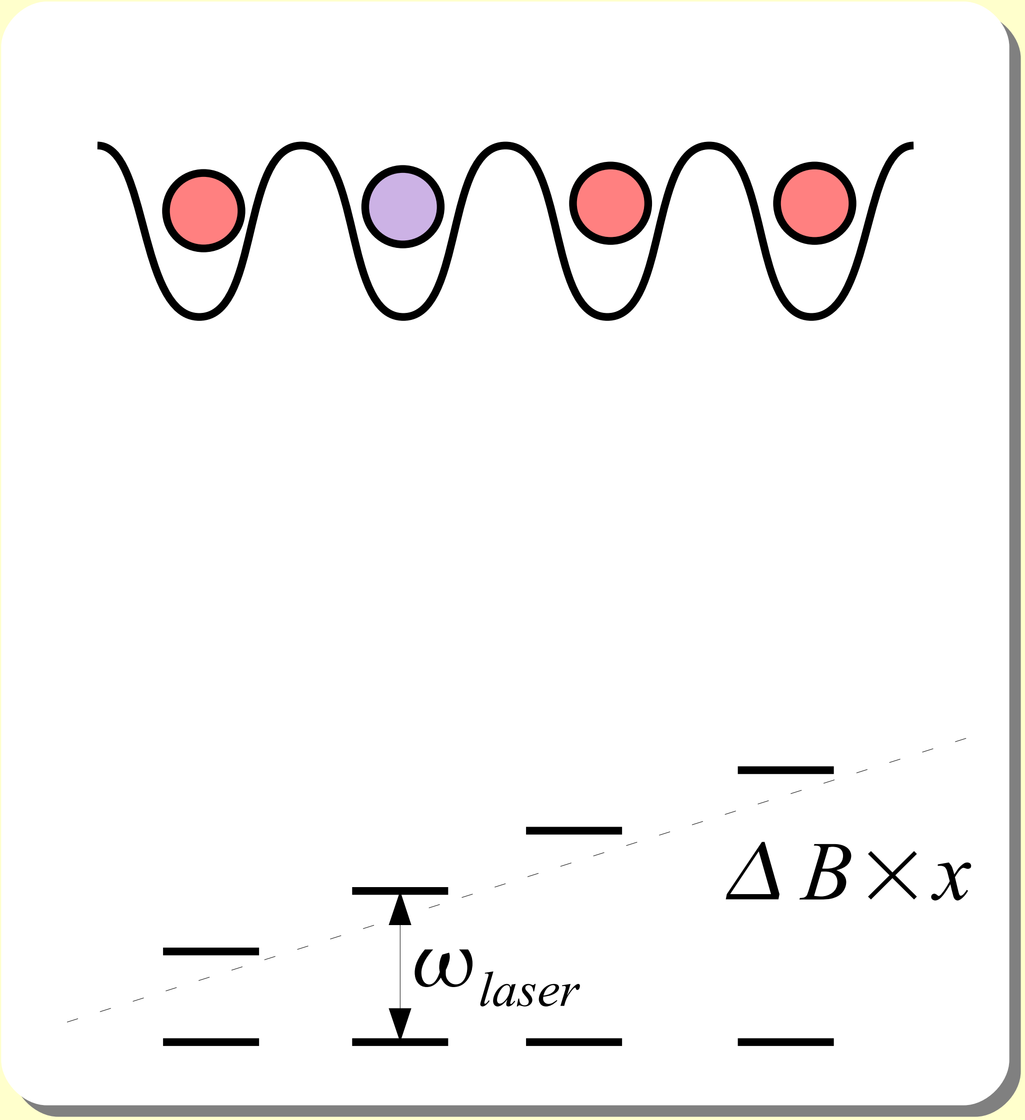
Single-qubit operations

- We have seen how to change the state of qubits.
- But in the lattice it is difficult to achieve individual addressing:
  - manipulate qubits separately
- One reason, diffraction limit:
  - no focusing below wavelength
- Other reason, many layers of atoms in between.



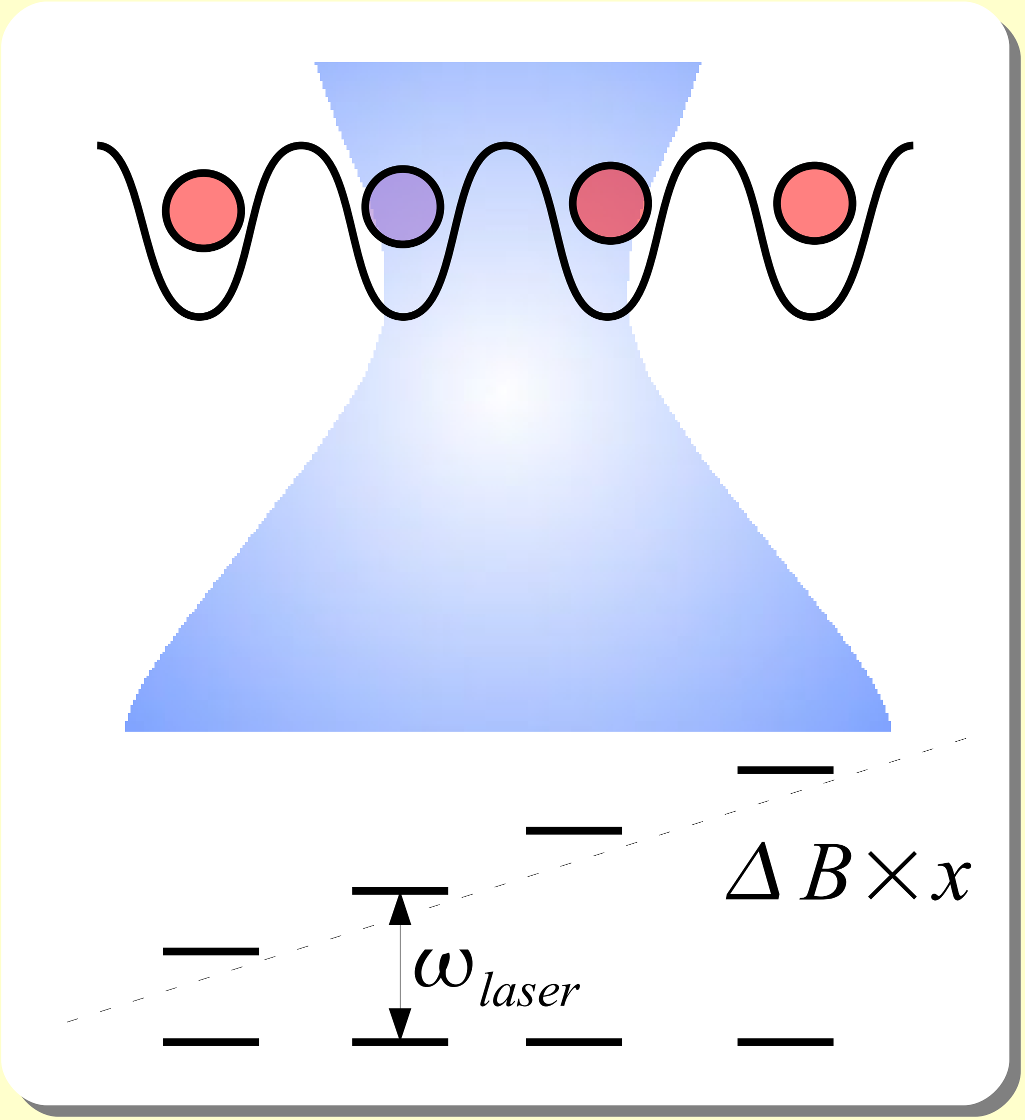
Single-qubit manipulations

- Applying EM field gradients, the energy levels of the atoms change.



Single-qubit manipulations

- Applying EM field gradients, the energy levels of the atoms change.
- Only some of them become sensitive to the lasers.

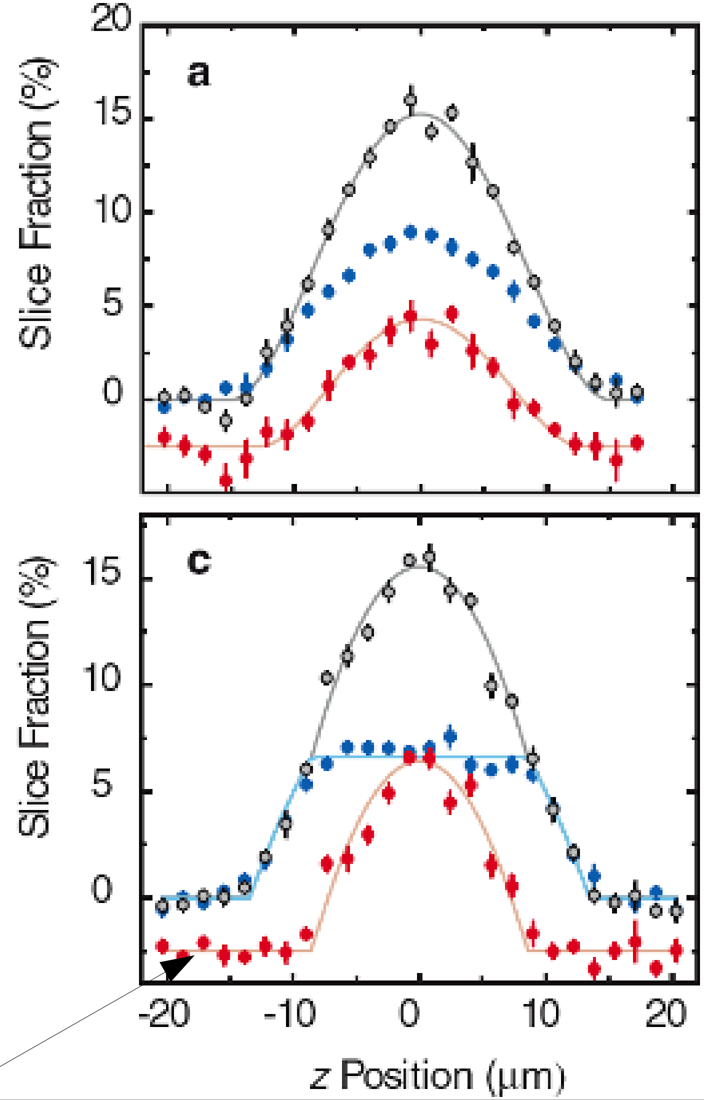


Single-qubit manipulations

- Applying EM field gradients, the energy levels of the atoms change.
- Only some of them become sensitive to the lasers.
- Similar in principle to NMR techniques:
  - spectrally resolved qubits.

Single-qubit manipulations

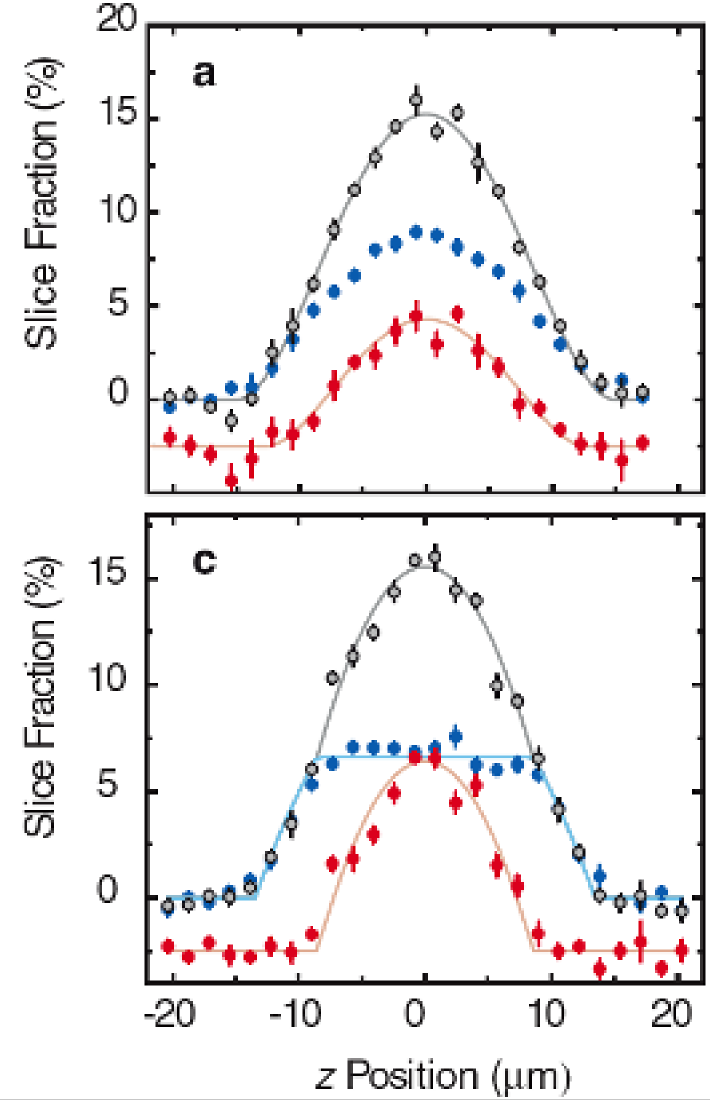
- Applying EM field gradients, the energy levels of the atoms change.
- Only some of them become sensitive to the lasers.
- Similar in principle to NMR techniques:
  - spectrally resolved qubits.



Each point a single frequency tuning.

Single-qubit manipulations

- Applying EM field gradients, the energy levels of the atoms change.
- Only some of them become sensitive to the lasers.
- Similar in principle to NMR techniques:
  - spectrally resolved qubits.
- But not **enough spatial resolution** yet
  - More ideas later!



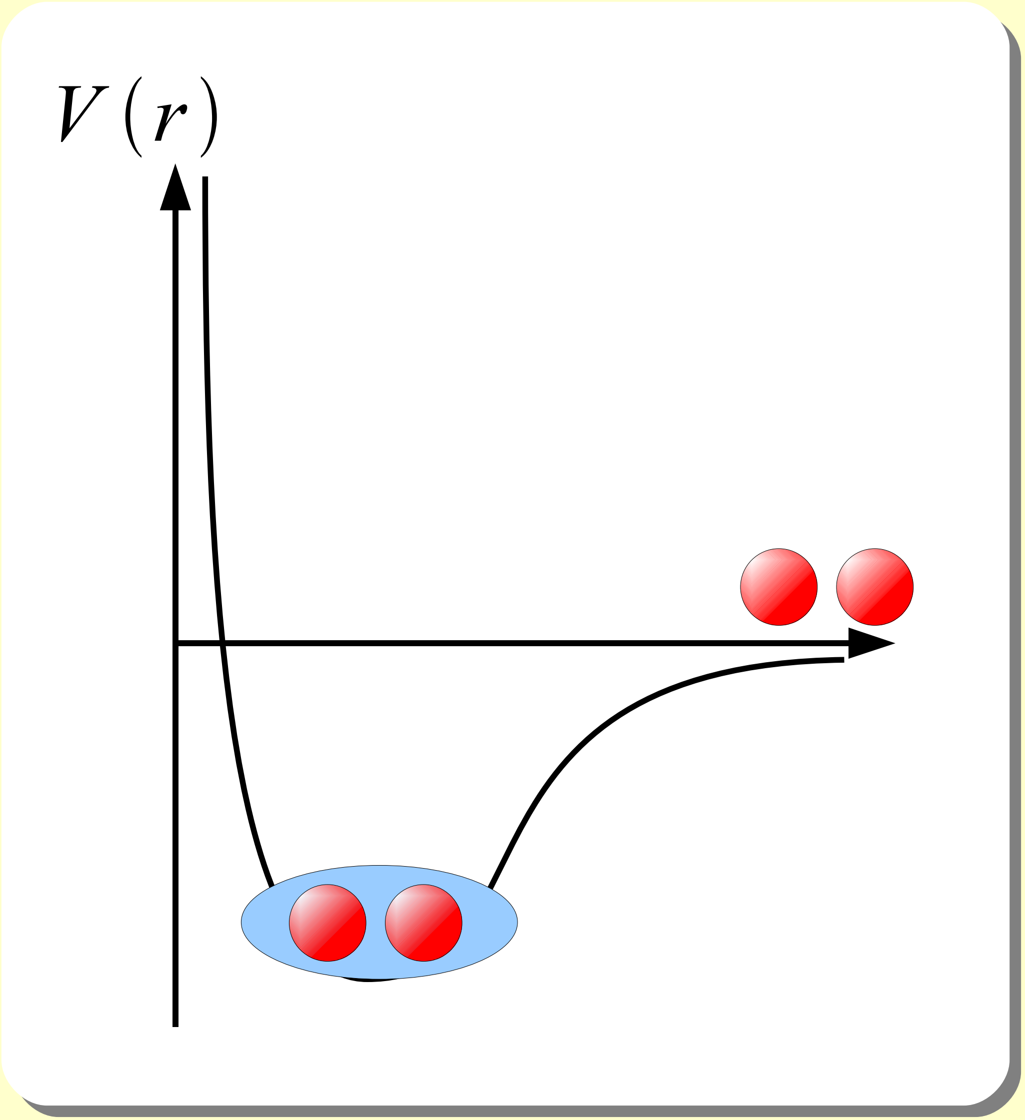
Interactions

Interactions

- No Coulomb interaction.
- No significant dipolar interaction.
- The state of free atoms is metastable
  - Ground state has molecules or a solid

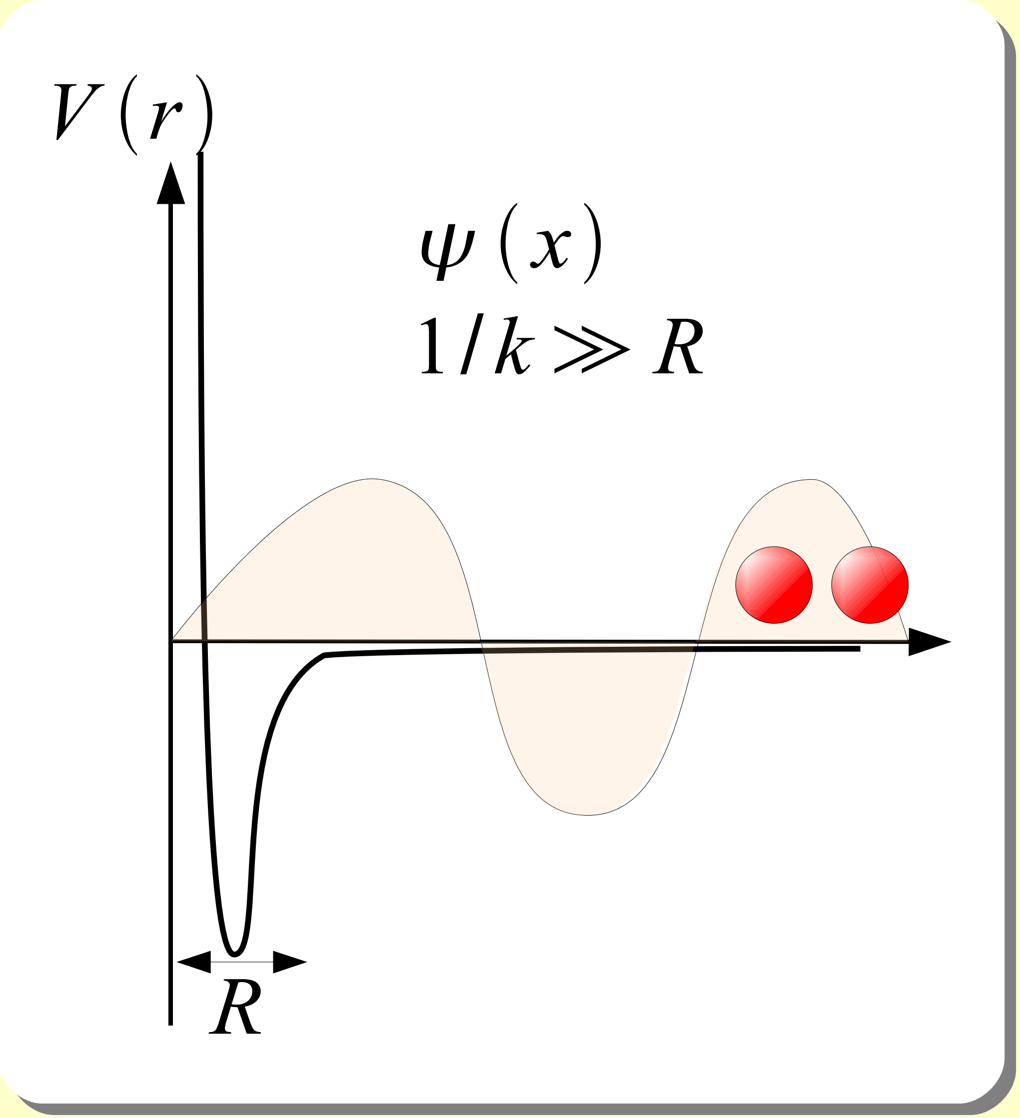
Interactions

- No Coulomb interaction.
- No significant dipolar interaction.
- The state of free atoms is metastable
  - Ground state has molecules or a solid
  - There is some short-range binding potential
  - Too much energy to get actually bound.



Interactions

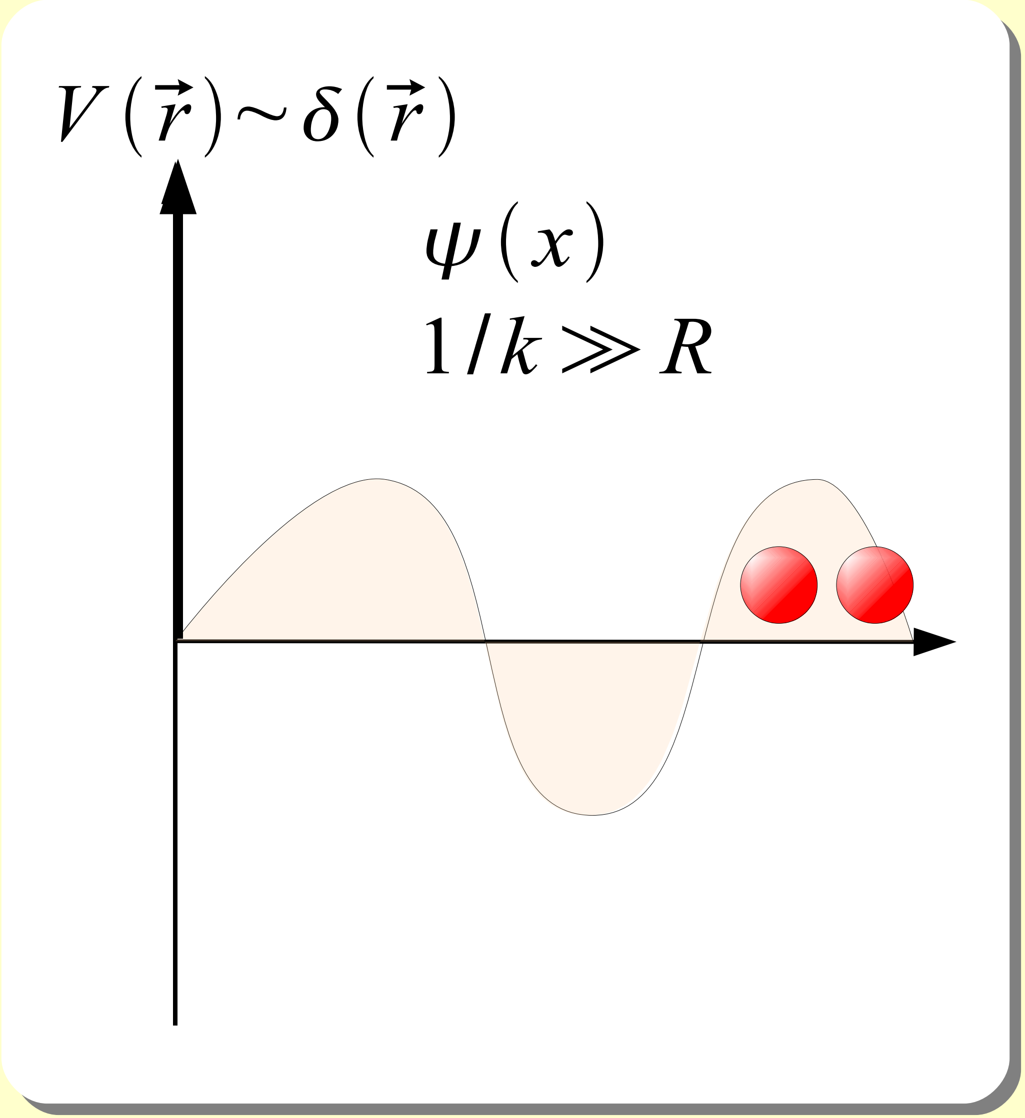
- When atoms have little kinetic energy, they do not approach much.



Interactions

- When atoms have little kinetic energy, they do not approach much.
- We can replace the potential by one with similar scattering properties

$$V(r) \sim \frac{4\pi\hbar^2 a_0}{m}\delta(r)$$



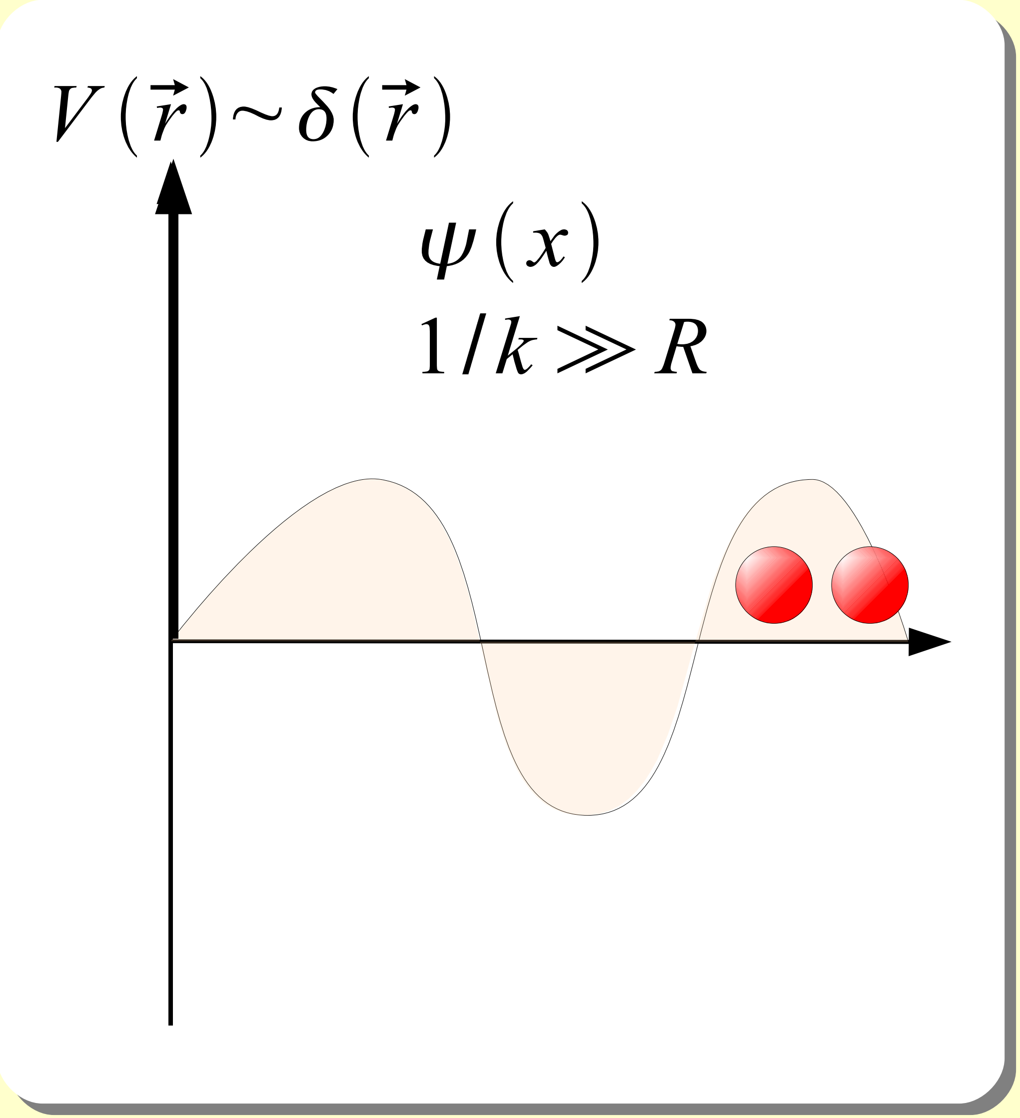
Interactions

- When atoms have little kinetic energy, they do not approach much.
- We can replace the potential by one with similar scattering properties

$$V(r) \sim \frac{4\pi\hbar^2 a_0}{m}\delta(r)$$

- Second quantization language

$$\frac{g}{2}\int \psi^{+}(x)^2\psi(x)^2\,dx$$



Interactions in the lattice

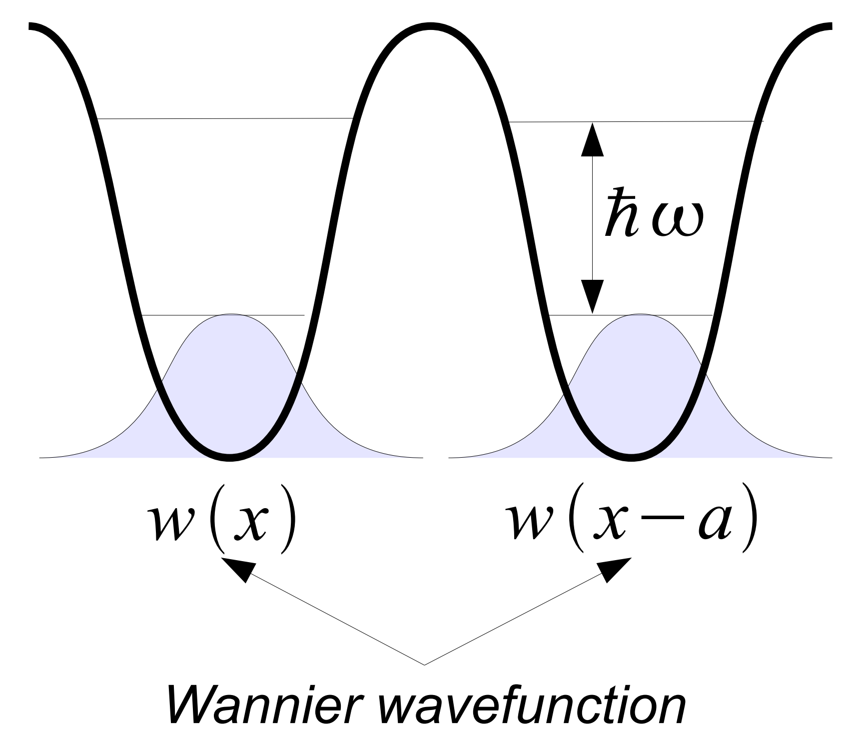
- Atoms are localized in the approximate grounds states of the lattice wells.
- **Tight binding** approximation

$$\psi^+(x) = \sum_i a_i^+ w(x - x_i)$$

- In the interaction we get

$$H_{int} \sim \sum_i \frac{U}{2} a_i^{+2} a_i^2$$

$$= \sum_i \frac{U}{2} n(n-1)$$



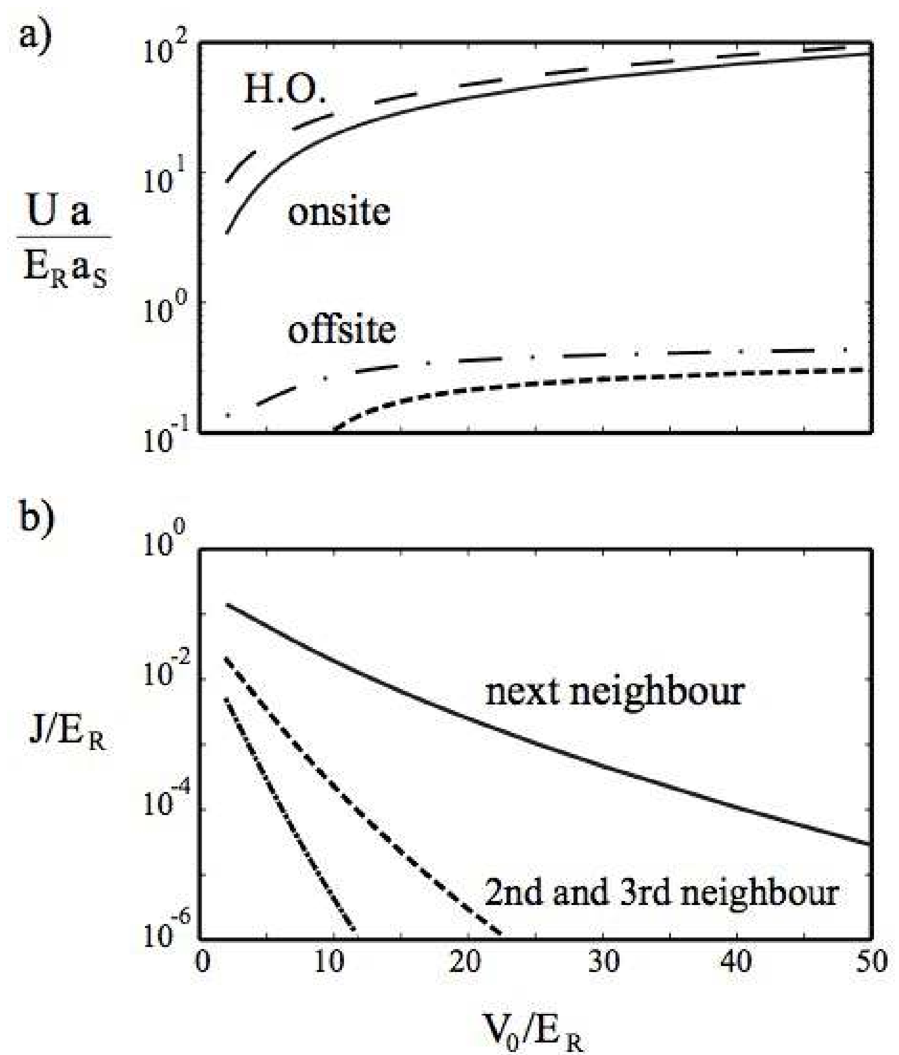
Interactions in the lattice

- The on-site interaction is dominant

$$H_{int} \sim \sum_i \frac{U}{2} a_i^{+2} a_i^2$$

$$= \sum_i \frac{U}{2} n(n-1)$$

- It changes very slowly with the trapping potential.
- Other interaction terms, between neighbors, are negligible.



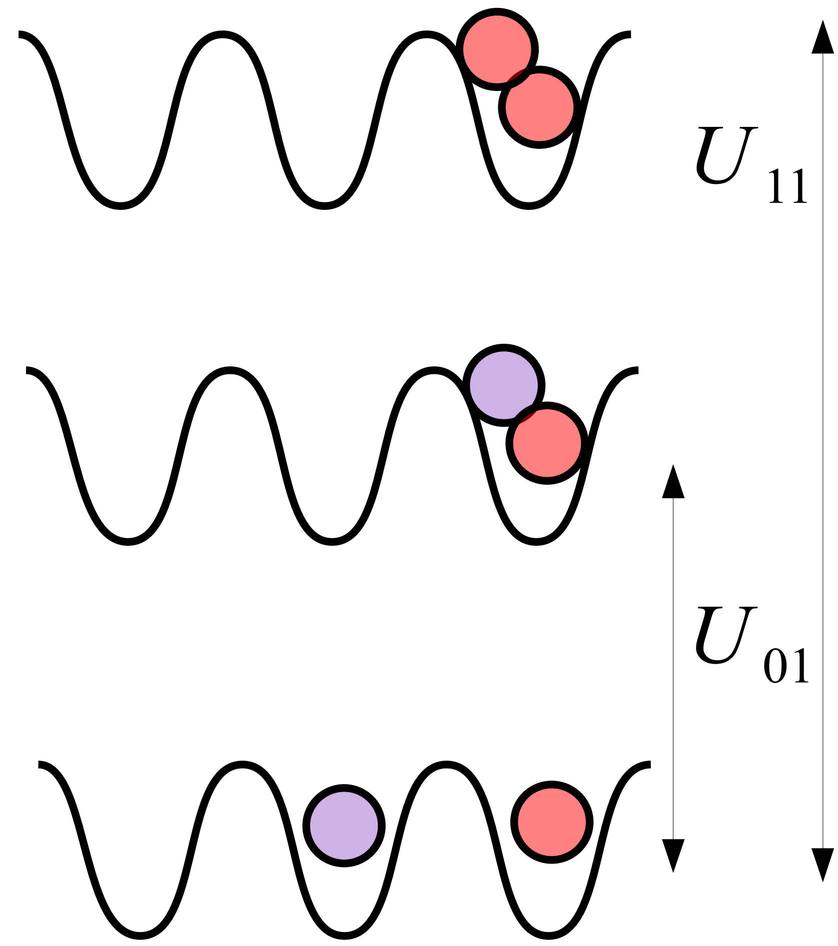
Interactions in the lattice

Summing up:

- Atoms in the same lattice site interact.
- Interaction is "contact", and too weak to change wavefunction
- Interaction may depend on internal state

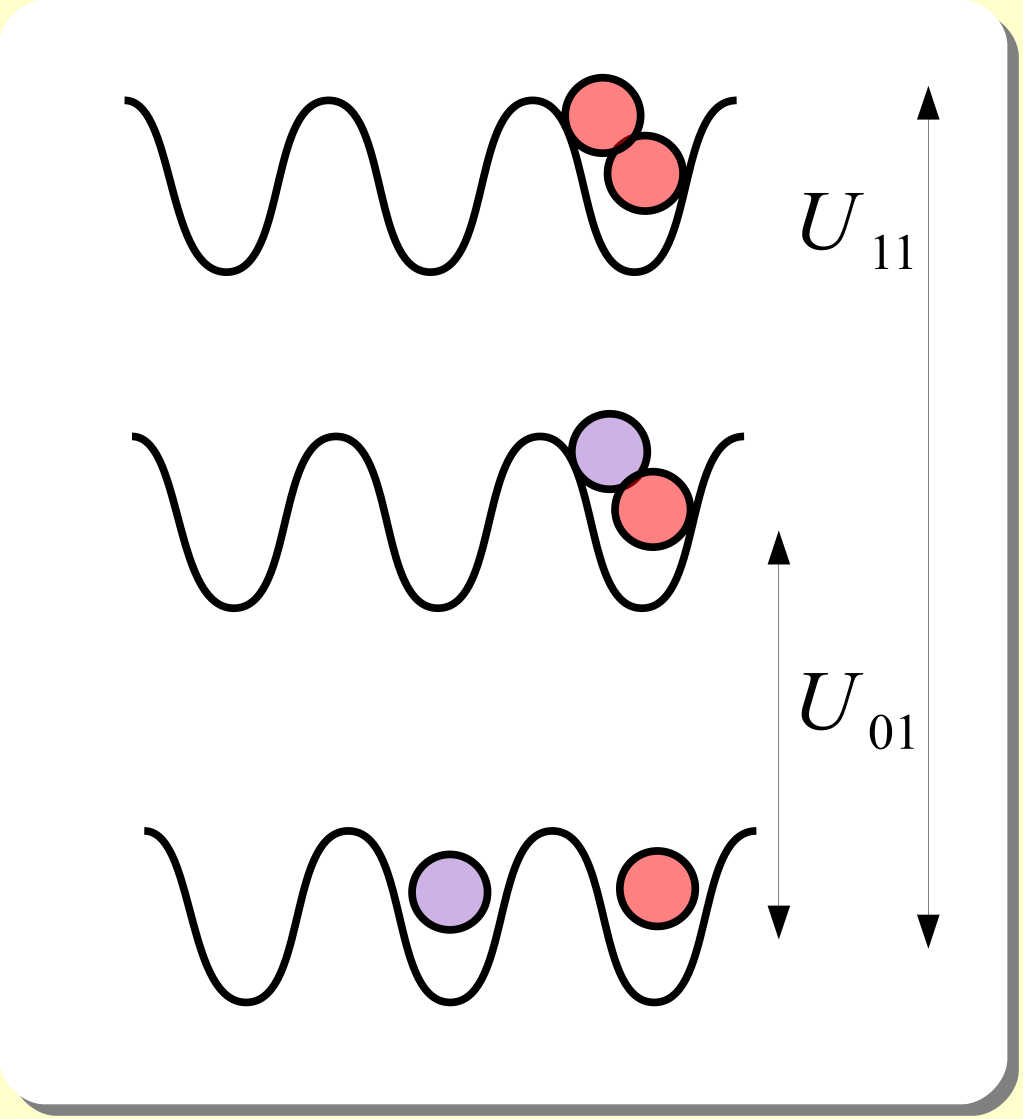
$$H \sim \sum_{\alpha,\beta=0,1} \frac{1}{2} U_{\alpha\beta} n_\alpha n_\beta$$

- All other terms are negligible



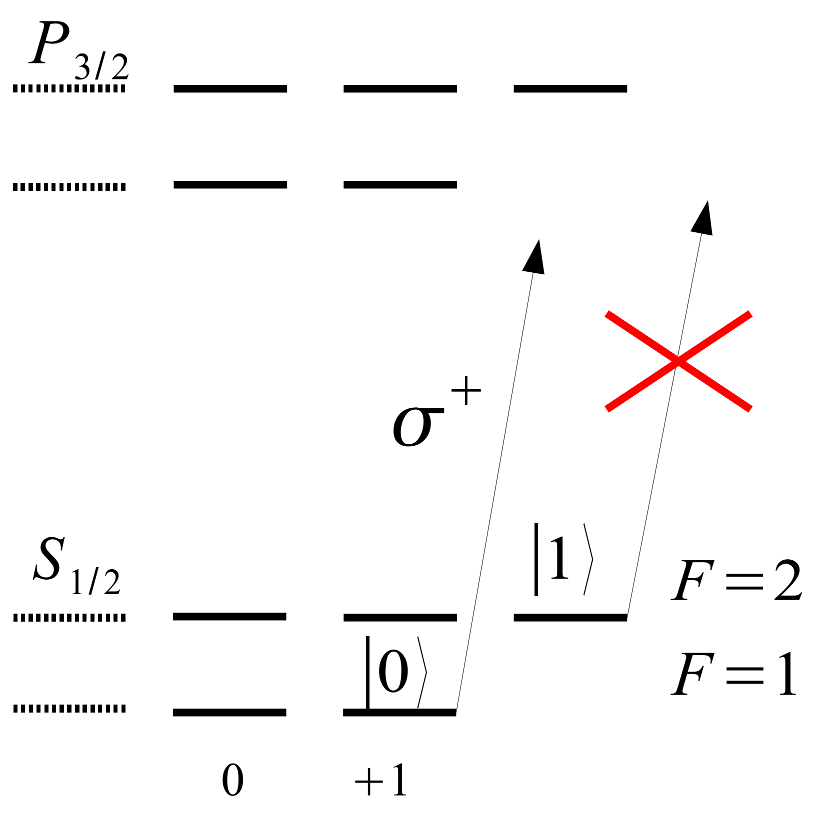
Interactions in the lattice

How dow we bring the qubits together to interact?



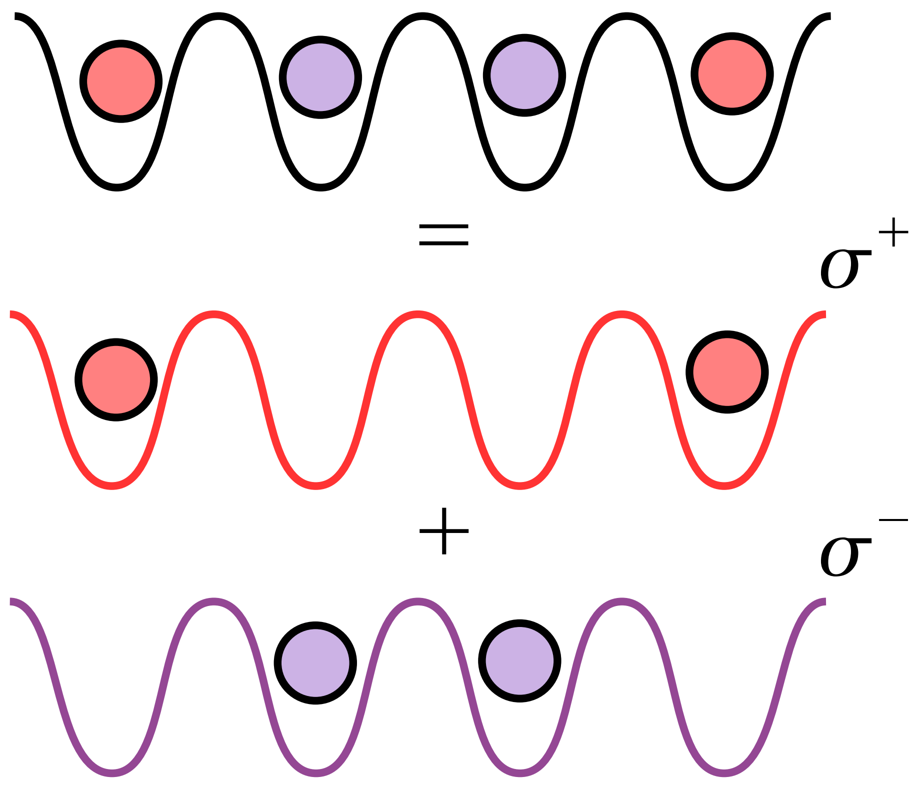
State-dependent lattices

State-dependent lattices



- Not all atomic states react the same to different polarizations.
  - Some transitions are forbidden or suppressed

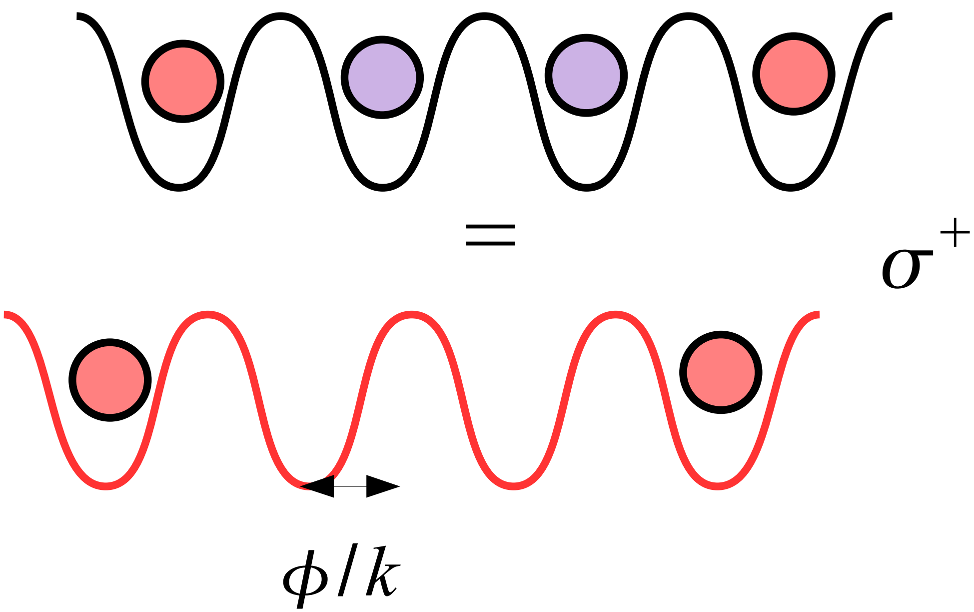
State-dependent lattices



O Mandel et al, Nature **425**, 937-940 (2003)

- Not all atomic states react the same to different polarizations.
  - Some transitions are forbidden or suppressed
- We can use this to create state-dependent lattices

State-dependent lattices

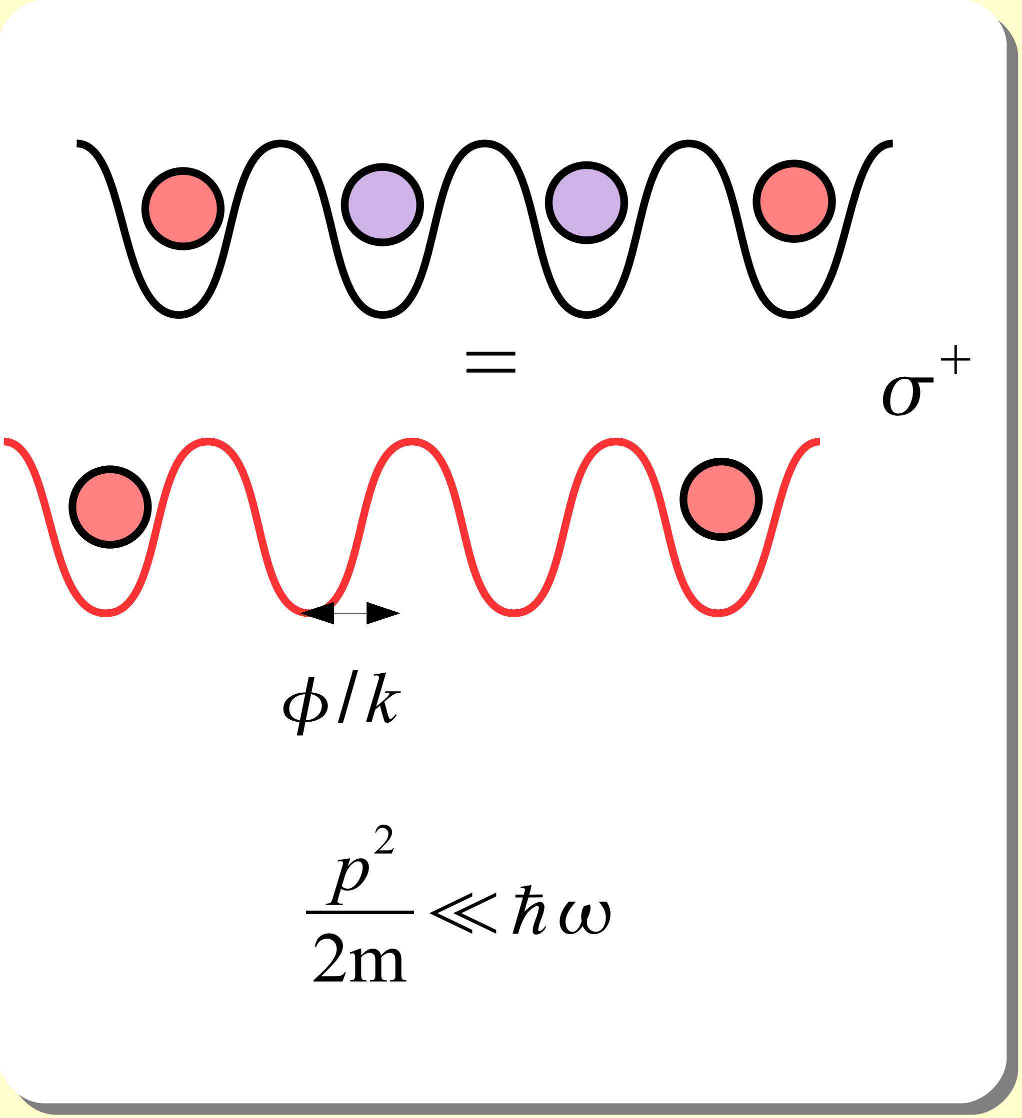


$$e^{ikx - i\phi} + e^{-ikx + i\phi} \sim \cos(kx - \phi)$$

- Not all atomic states react the same to different polarizations.
  - Some transitions are forbidden or suppressed
- We can use this to create state-dependent lattices.
- We can apply a phase shift (birrefrigence) to displace each lattice.

O Mandel et al, Nature **425**, 937-940 (2003)

State-dependent lattices

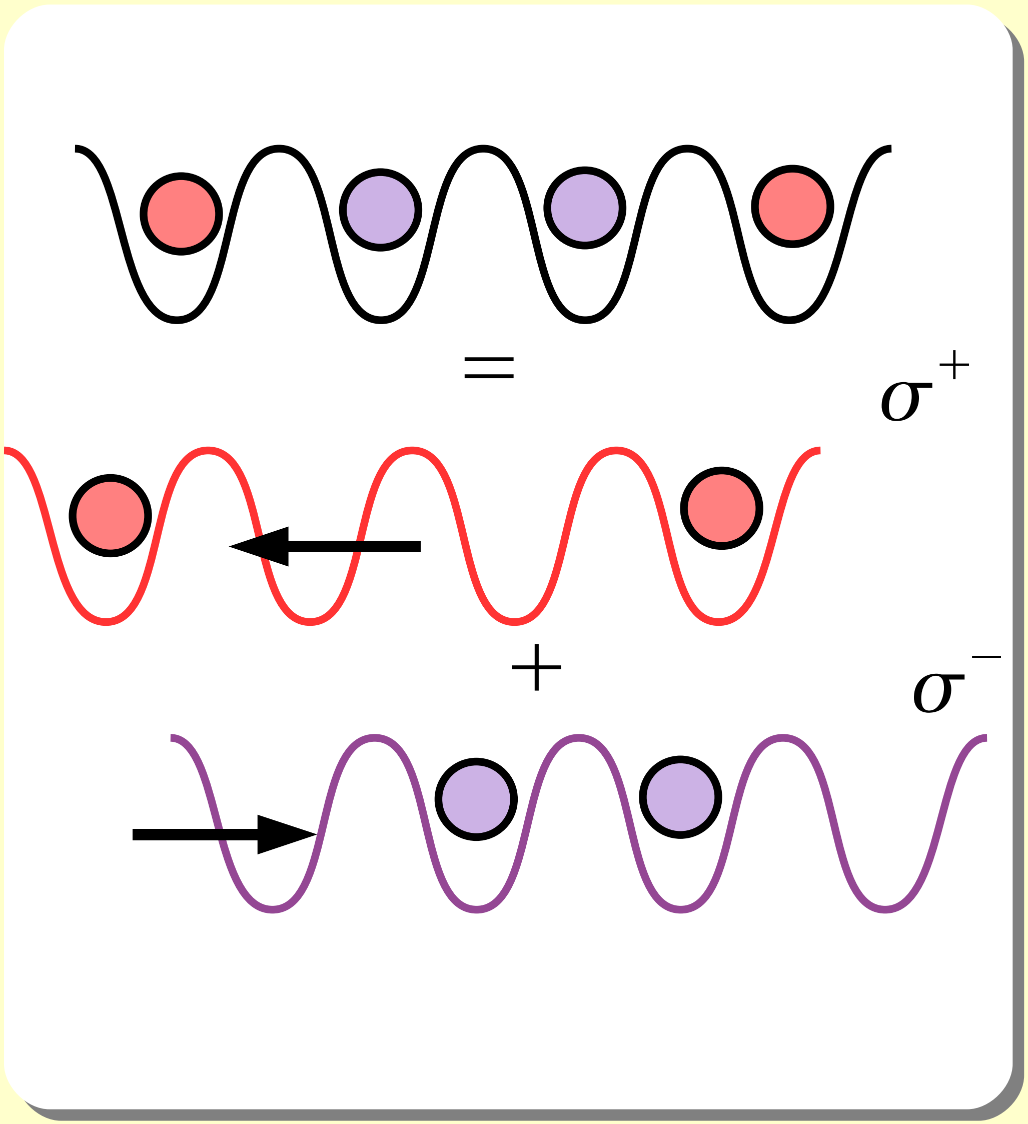


O Mandel et al, Nature **425**, 937-940 (2003)

- Not all atomic states react the same to different polarizations.
  - Some transitions are forbidden or suppressed
- We can use this to create state-dependent lattices.
- We can apply a phase shift (birrefrigence) to displace each lattice.
- The lattice has to be moved slowly, compared to the band gap.

Quantum gates

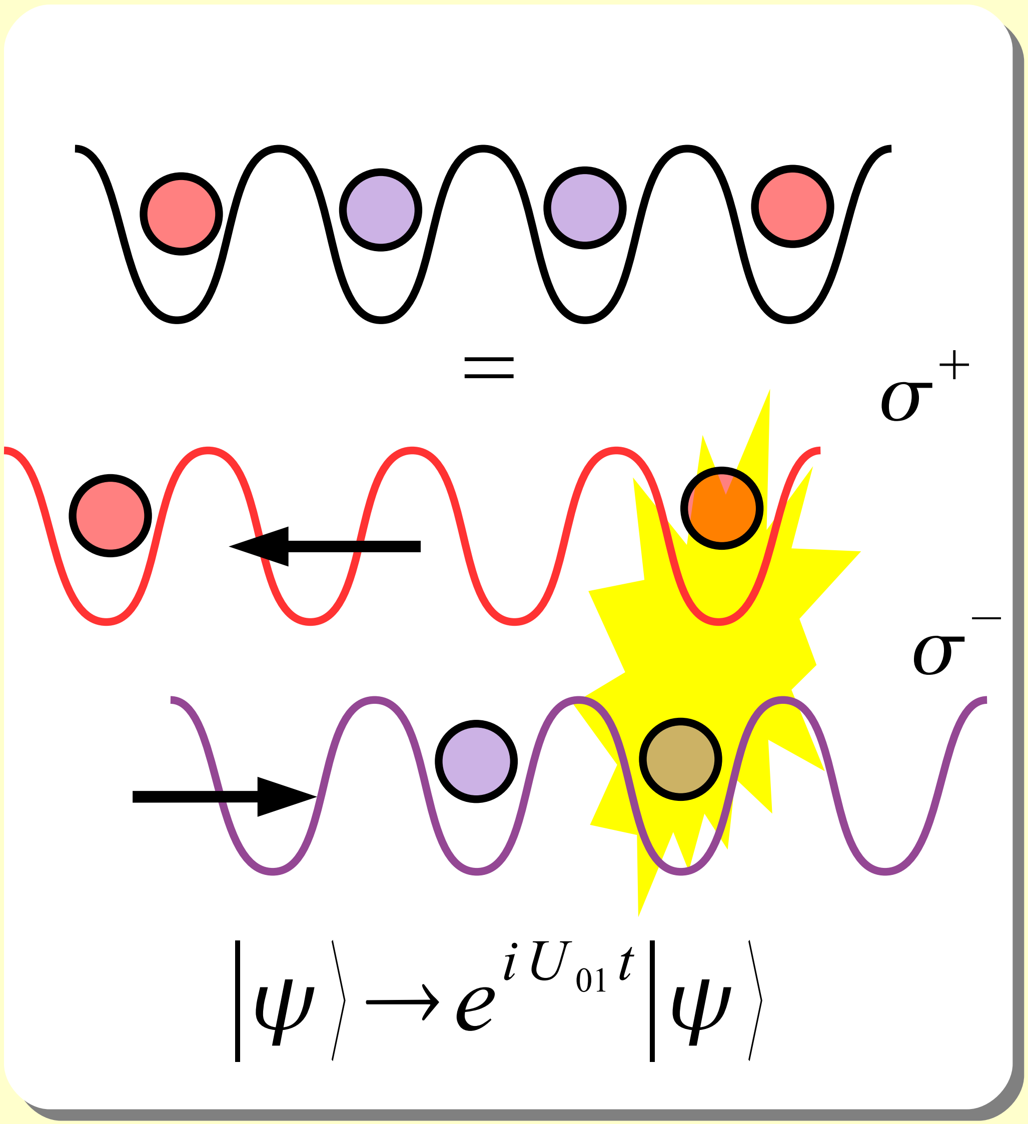
Controlled collisions



- We trap atoms in two states.
- Shift the lattices relative to each other.

O Mandel et al, Nature **425**, 937-940 (2003)

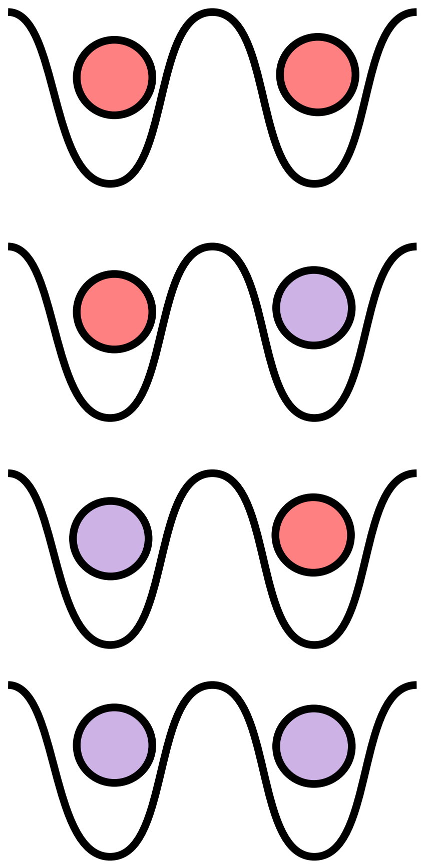
Controlled collisions



- We trap atoms in two states.
- Shift the lattices relative to each other.
- Neighboring atoms with different states collide
- During the time collision takes place, they gain a phase.

D. Jaksch, H. Briegel, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys. Rev. Lett. **82**, 1975 - 1978 (1999)

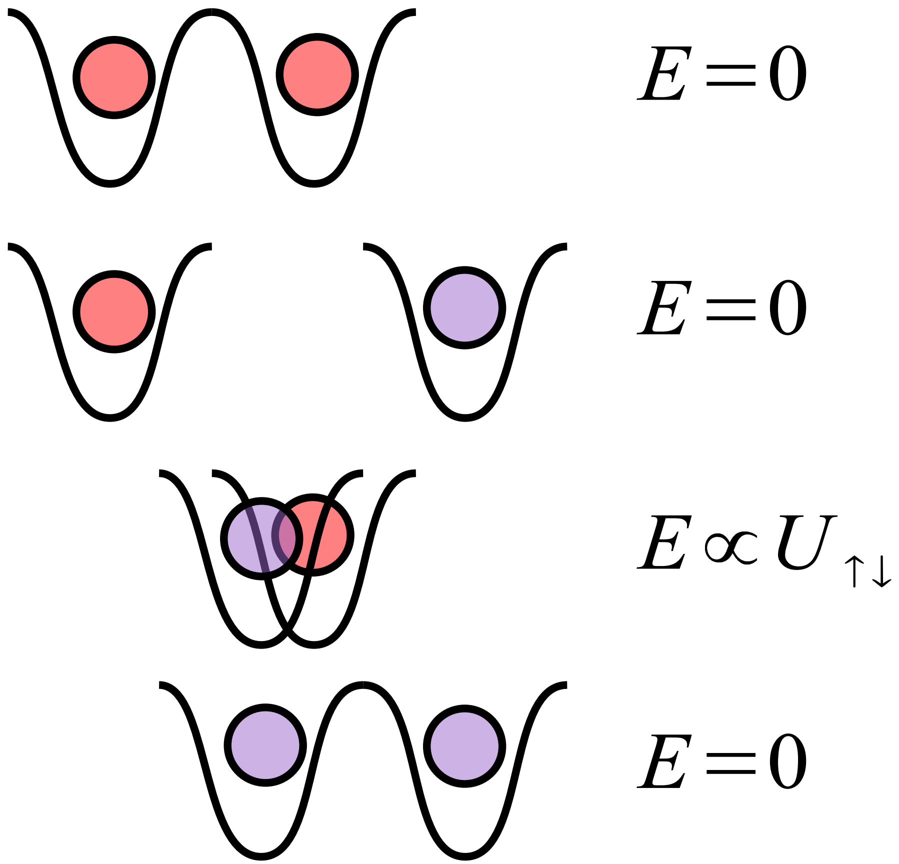
Controlled collisions



- We trap atoms in two states.
- Shift the lattices relative to each other.
- Neighboring atoms with different states collide
- During the time collision takes place, they gain a phase.
- This phase implements a quantum gate.

D. Jaksch, H. Briegel, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys. Rev. Lett. **82**, 1975 - 1978 (1999)

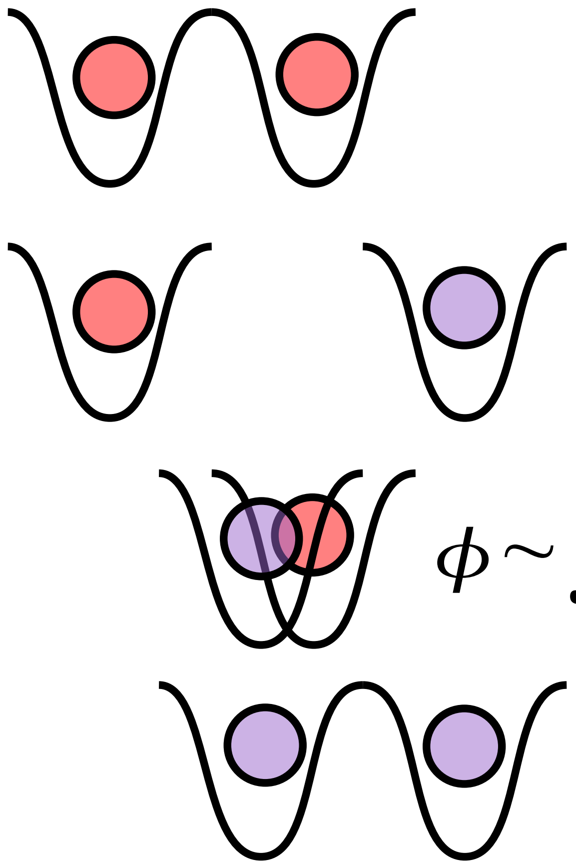
Controlled collisions



- We trap atoms in two states.
- Shift the lattices relative to each other.
- Neighboring atoms with different states collide
- During the time collision takes place, they gain a phase.
- This phase implements a quantum gate.

D. Jaksch, H. Briegel, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys. Rev. Lett. **82**, 1975 - 1978 (1999)

Controlled collisions

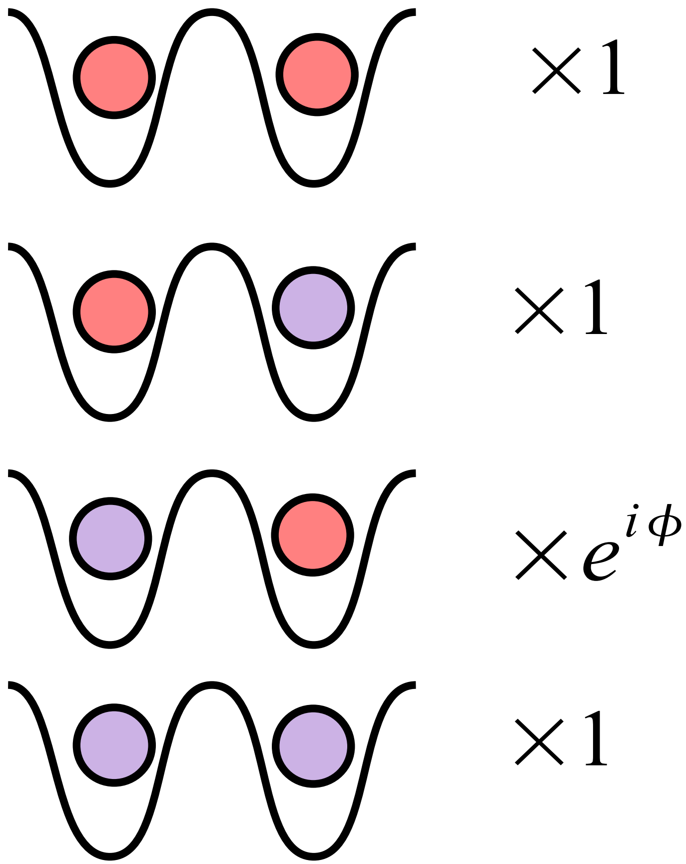


$$\phi \sim \int U_{\uparrow\downarrow}\, dt / \hbar$$

- We trap atoms in two states.
- Shift the lattices relative to each other.
- Neighboring atoms with different states collide
- During the time collision takes place, they gain a phase.
- This phase implements a quantum gate.

D. Jaksch, H. Briegel, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys. Rev. Lett. **82**, 1975 - 1978 (1999)

Controlled collisions



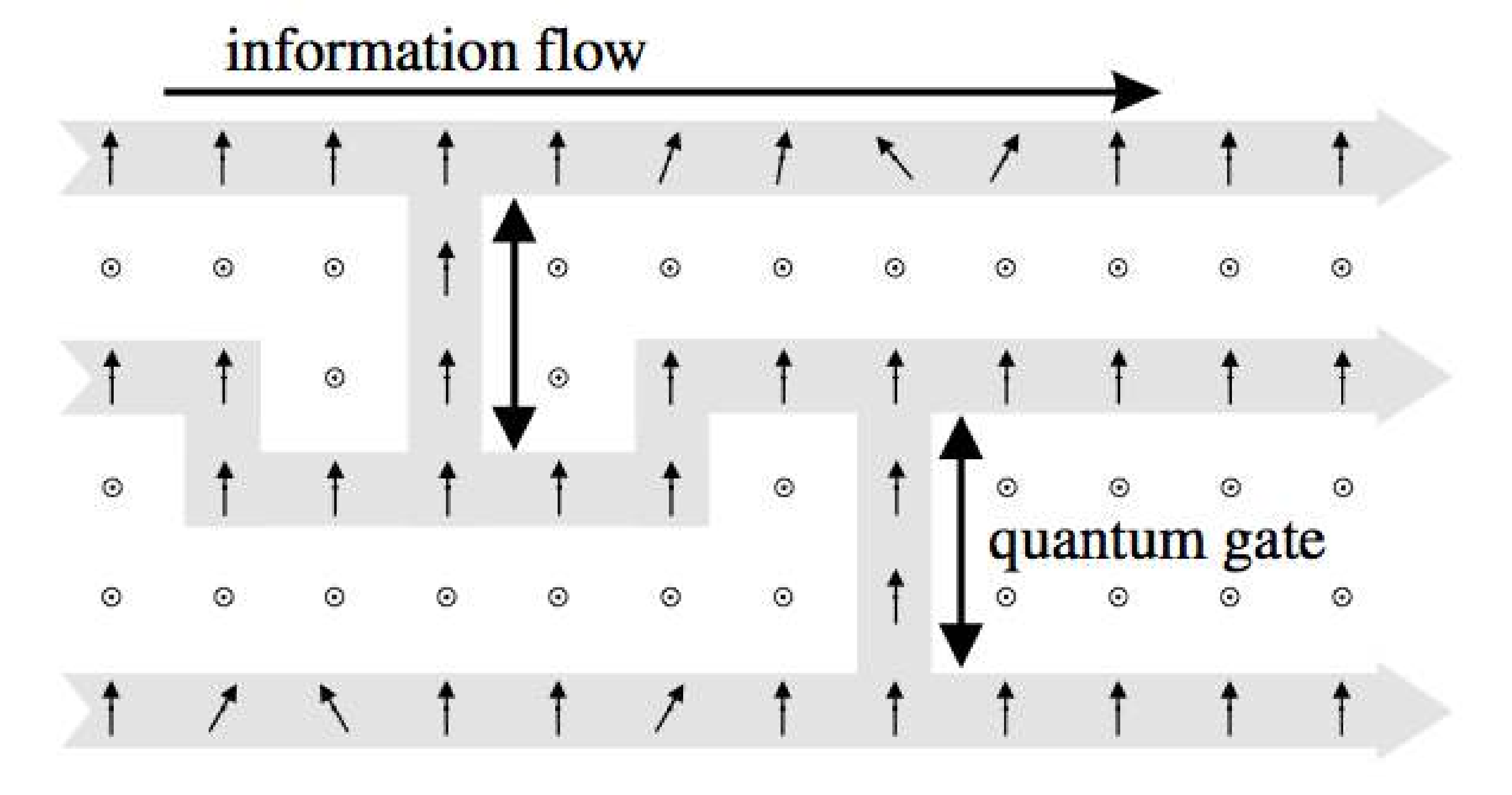
$$U_{gate} = e^{-i U_{01} t (\sigma_1^z - 1)(\sigma_2^z + 1)/4}$$

- We trap atoms in two states.
- Shift the lattices relative to each other.
- Neighboring atoms with different states collide
- During the time collision takes place, they gain a phase.
- This phase implements a quantum gate.

D. Jaksch, H. Briegel, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys. Rev. Lett. **82**, 1975 - 1978 (1999)

Measurement based QC

Measurement based QC



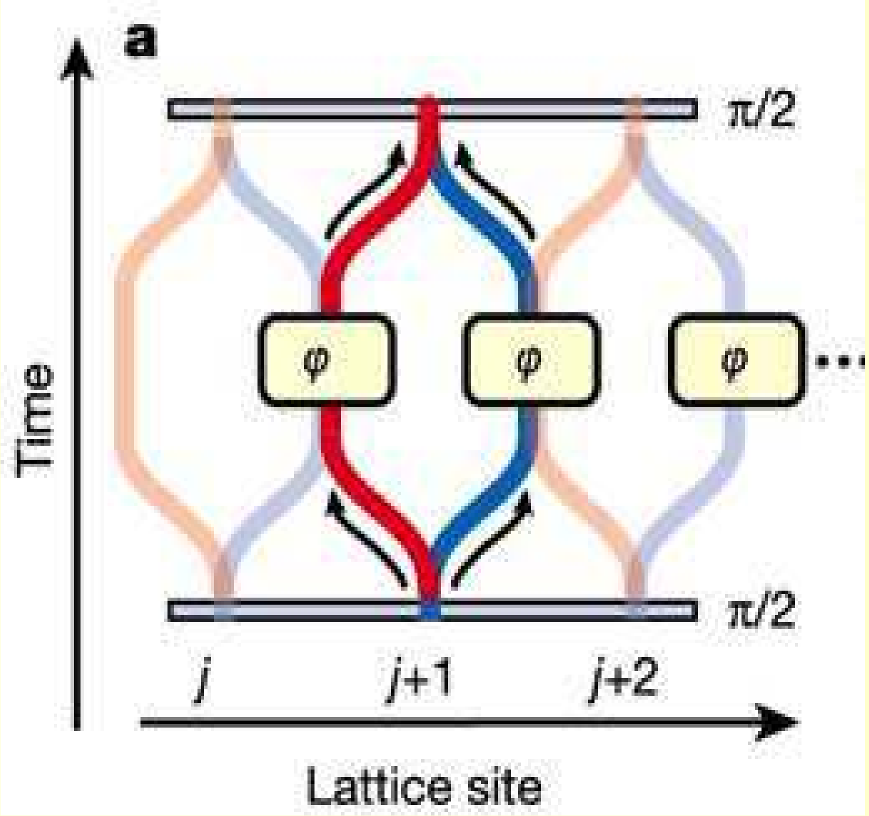
- It is possible to perform computations by measurements and local unitaries on a highly entangled state.
- This is called one-way quantum computing.

Cluster states

- In experiments this is done to **all atoms** at the same time.
- Massively parallelized quantum gate among nearest neighbors

$$U = \prod_{\langle i,j \rangle} e^{-i\phi[\sigma_i^z - 1][\sigma_j^z + 1]}$$

- Precisely what we need to make **cluster states**



$$U\left(|0\rangle + |1\rangle\right)^{\otimes N}$$

O Mandel et al, Nature **425**, 937-940 (2003)

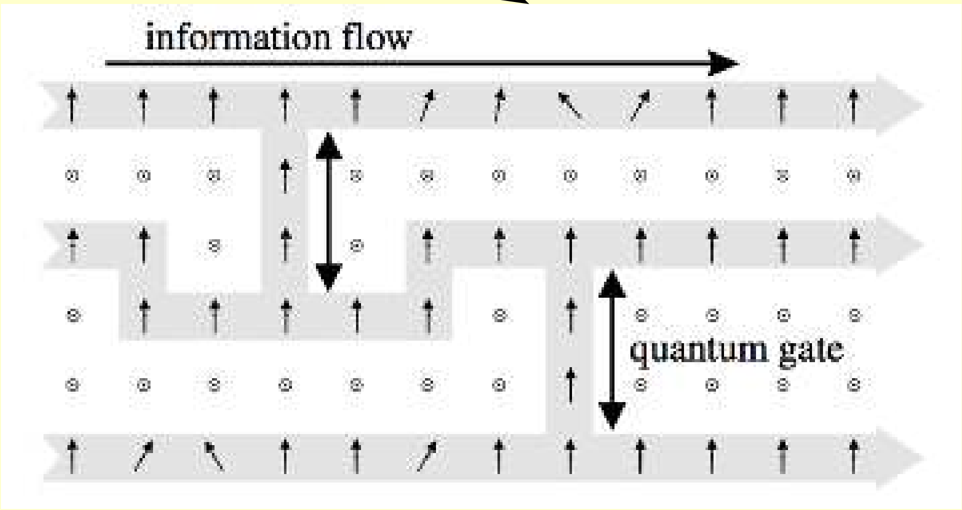
Cluster states

- In experiments this is done to **all atoms** at the same time.
- Massively parallelized quantum gate among nearest neighbors

$$U = \prod_{\langle i,j \rangle} e^{-i\phi[\sigma_i^z - 1][\sigma_j^z + 1]}$$

- Precisely what we need to make **cluster states**

Need individual addressing to make this useful



$$U\left(|0\rangle + |1\rangle\right)^{\otimes N}$$

O Mandel et al, Nature **425**, 937-940 (2003)