Optical lattices & Quantum simulation

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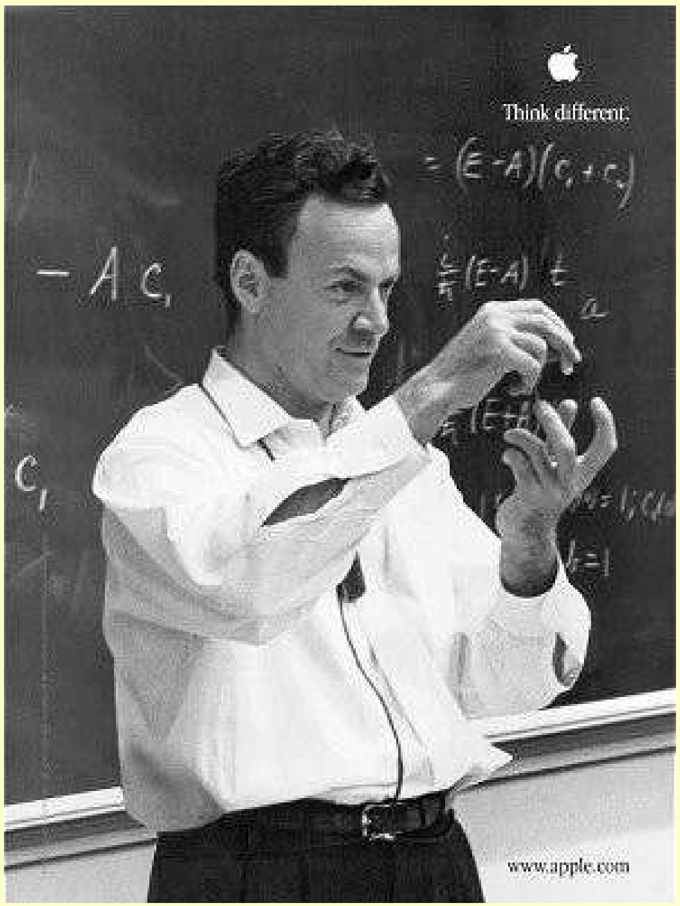
(17-4-2009)

Quantum simulation

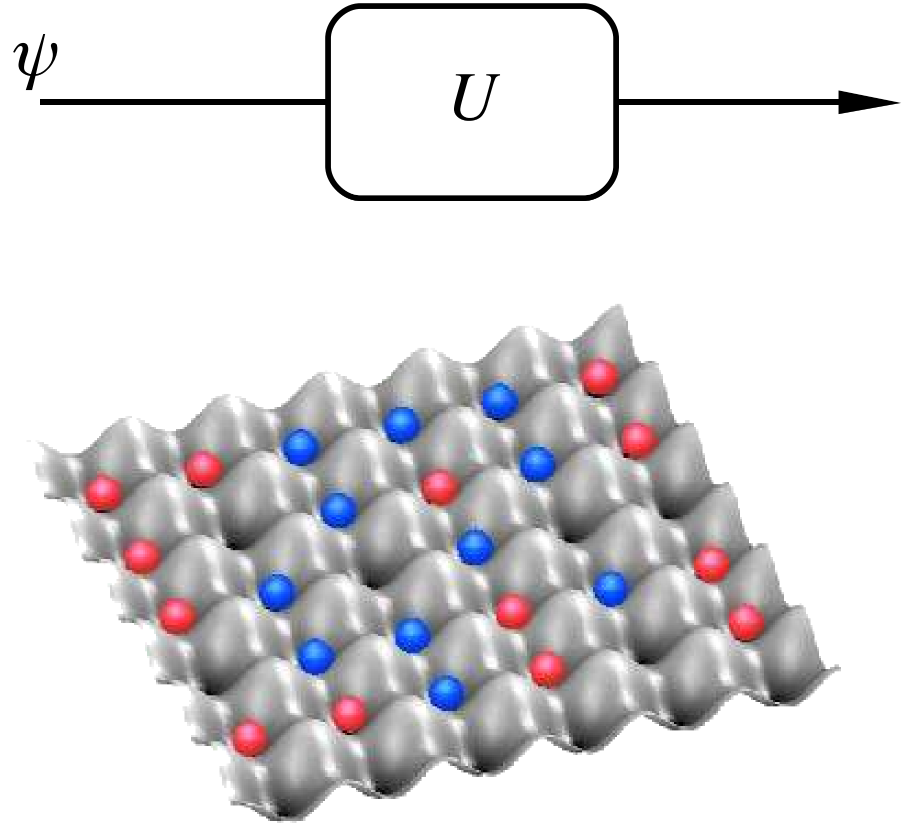
The full description of quantum mechanics is given by a function [with] too many variables [...] How can we simulate quantum mechanics? [...]



- Let the computer be built with quantum mechanical elements that obey quantum mechanical laws
- Let the computer be a logical universal automaton, can we imitate [quantum mechanics]?

R. P. Feynmann, *Simulating Physics with Computers*,
Int. Jour. Theor. Phys. 21, 1982

Why is this important?

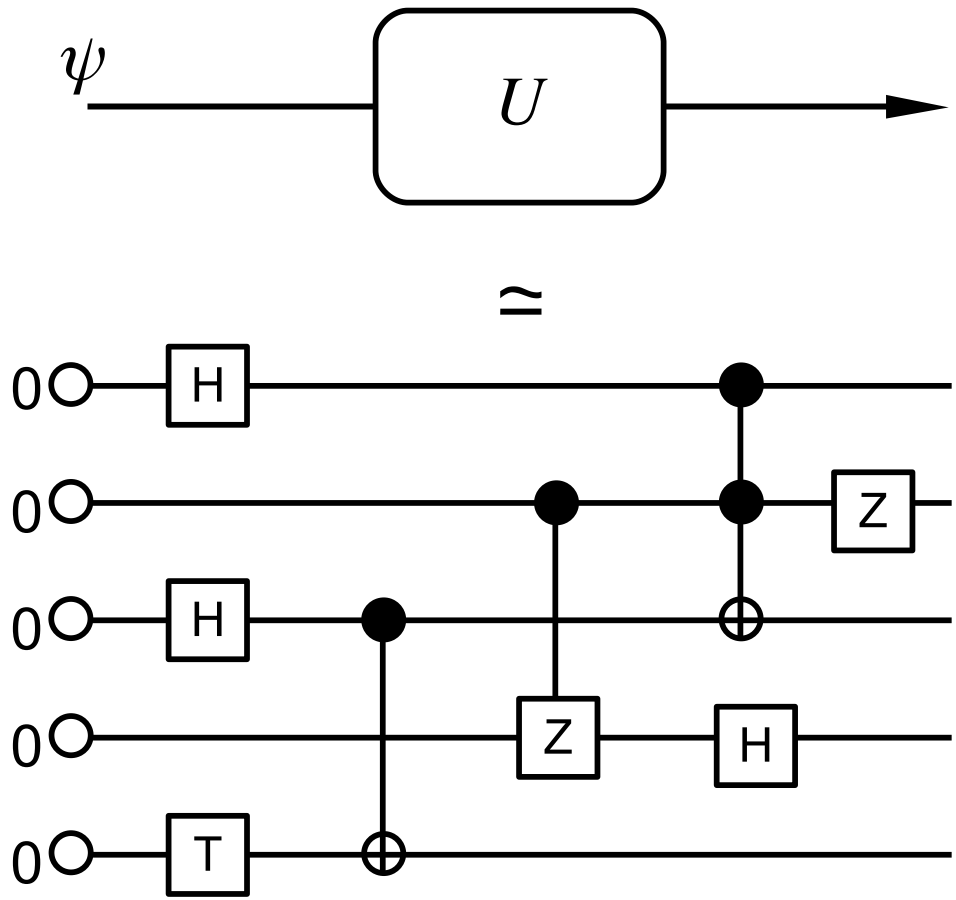


- Huge dimensionality of Hilbert space

$$n \ \ spins \rightarrow 2^n \ \ states$$

- Many new systems are described by many-body quantum mechanics
- Real experiments do not implement cleanly our physical models.
- Quantum information may shed new light in the models, simulations and methods.

Universal quantum simulation



- A universal quantum computer can approximately implement any unitary operation.
- Challenges:
  - Encoding of physical states.
  - Decomposition of unitaries.
  - **Implementation of quantum computer!**
  - Measurements at the end.

Trotter decomposition

- A fundamental tool in quantum simulation.
- It consists on approximations of the evolution operator

$$\exp(-i(H_1+H_2)\Delta t) \simeq \exp(-iH_1\Delta t/2)\exp(-iH_2\Delta t)\exp(-iH_1\Delta t/2)+ +O(\Delta t^2)$$

- Higher order approximations are possible

$$\exp(-i(H_1+H_2)\Delta) \simeq \exp(-iH_1\theta\Delta/2)\exp(-iH_2\theta\Delta)\exp(-iH_1(1-\theta)\Delta/2)\times \exp(-iH_2(1-2\theta)\Delta)\times \exp(-iH_1(1-\theta)\Delta/2)\exp(-iH_2\theta\Delta)\exp(-iH_1\theta\Delta/2)+ +O(\Delta^5)$$

but not much further.

Simulating spin models

- Using controlled collisions we implement the unitary

$$U_z = \prod_{\langle ij \rangle} \exp(-i\phi(\sigma_i^z - 1)(\sigma^z + 1)) = \exp(-i H_{zz} \Delta t)$$

with the effective Hamiltonian.

$$H_{zz} = \sum_{\langle i\, j \rangle} J \sigma_i^z \sigma_j^z, \quad J = \frac{\phi}{\Delta t}$$

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- With a global, single-qubit rotation we can change the direction

$$U_{z \to x} U_z U_{z \to x}^{+} = U_{xx} = \exp(-i H_{xx} \Delta t)$$

$$U_{z \to x} = \sqrt{i} (\sigma^y)^{\otimes N}$$

Simulating spin models

- Combining several steps

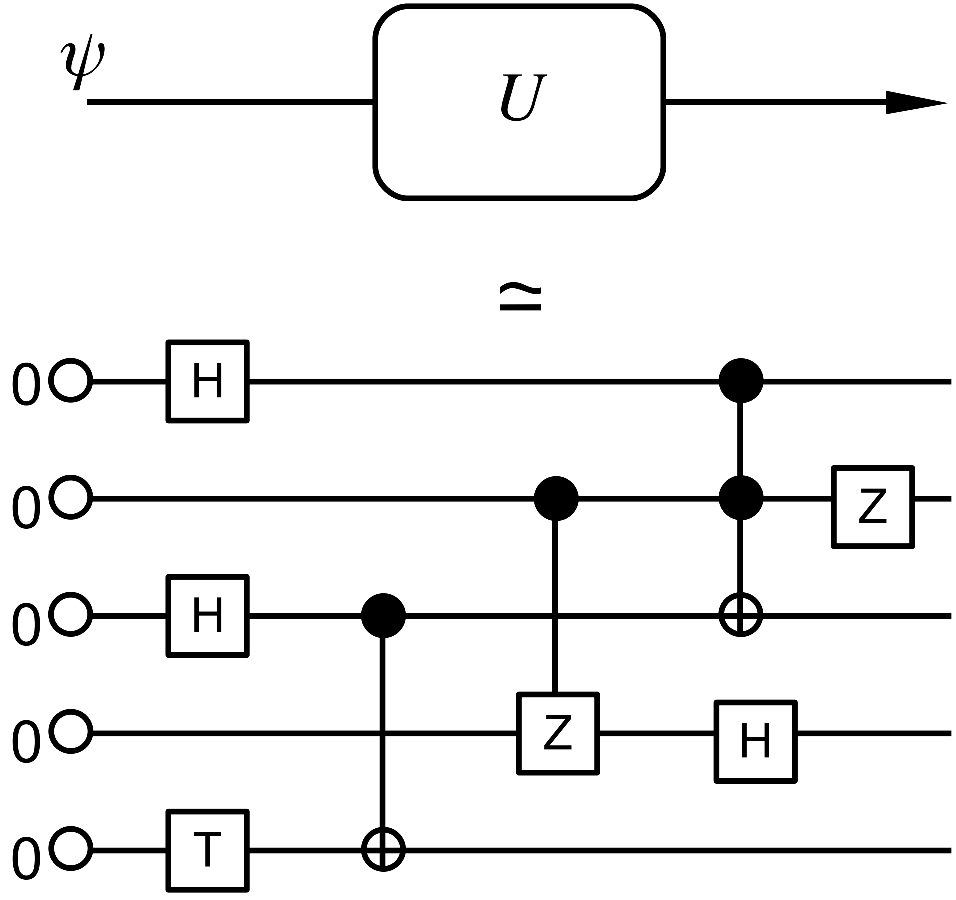
$$U_{Heis} = U_{z\to y} U_z U^{+}_{z\to y} U_{z\to x} U_z U^{+}_{z\to x} U_z + O(\Delta t)$$

we approximate the evolution with a Heisenberg Hamiltonian

$$U_{Heis} = \exp(-i H_{Heis} \Delta t)$$

$$H_{Heis} = \sum_{\langle i\, j\rangle} J \vec{\sigma}_i \cdot \vec{\sigma}_j = \sum_{\langle i\, j\rangle} J \left[\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y + \sigma_i^z \sigma_j^z \right]$$

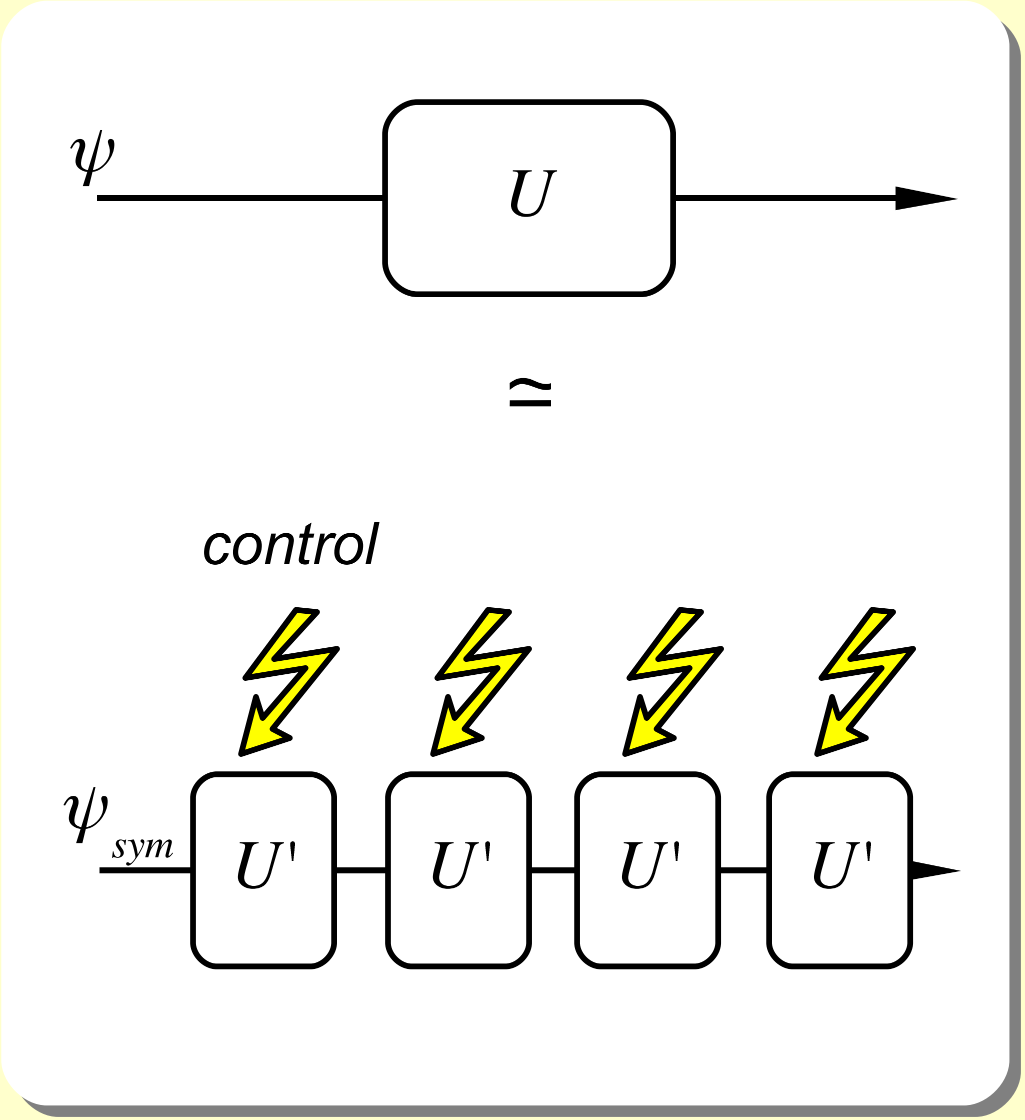
Universal quantum simulation



- Problems of universal simulators:
  - We need huge accuracy, as much as for building a quantum computer.
  - Not all Hamiltonians can be efficiently simulated by our qubit computer
    - Fermionic models
  - Quantum register not adapted to topology of the problem.

A simpler approach

Quantum simulators

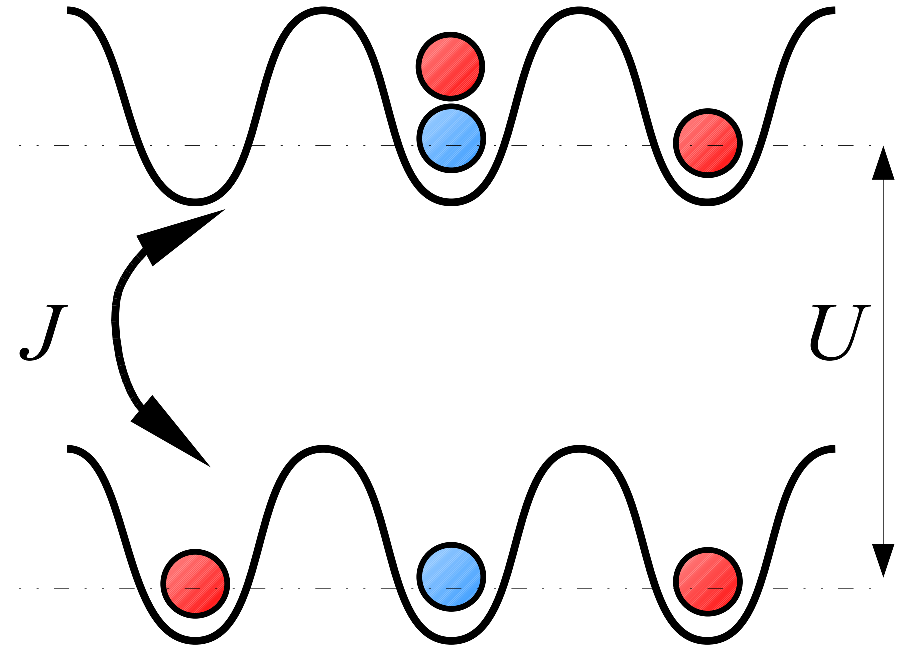


- Definition
  - A flexible quantum system.
  - Its evolution can be controlled in a time dependent fashion.
  - It can be measured.
  - By piecewise control the dynamics of the system resembles many other models.

Optical lattices

Introducing hopping

- We have a parameter that has not been used
  - lattice depth
- When lattice is lower, atoms may hop.
- The effective model is a lattice gas with hopping J and on-site interaction
  - Hubbard model



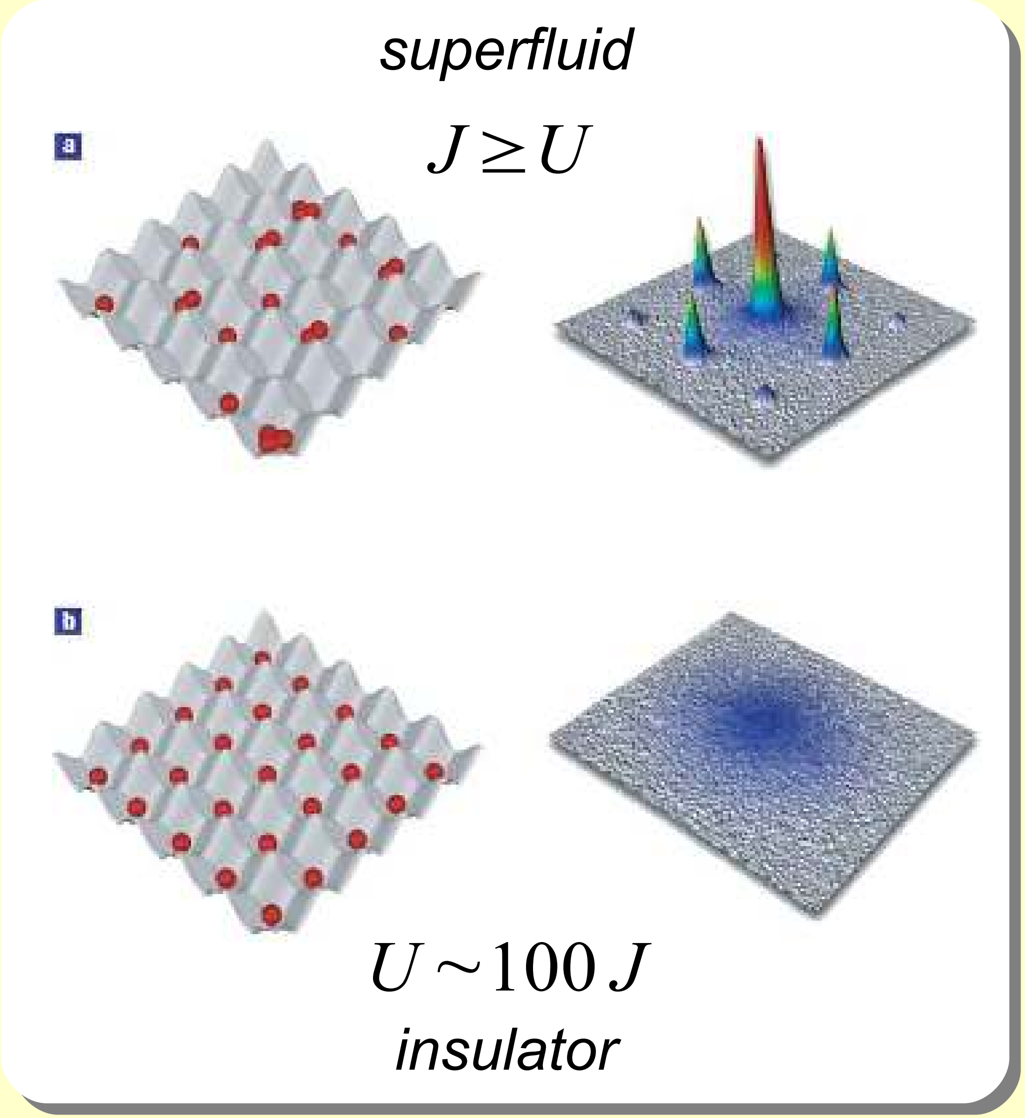
$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + U \sum_i n_i^2$$

Introducing hopping

- With bosons, we emulate the Bose-Hubbard model with
  - Superfluid regime
  - Insulator regime
- With fermions, the model is the Hubbard model

$$H = -J \sum_{\langle i,j \rangle, \sigma} a_{i\sigma}^{+} a_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

  - A model for high-Tc superconductor



Immanuel Bloch, Nature Physics **1**, 23 - 30 (2005)

Effective spin interactions

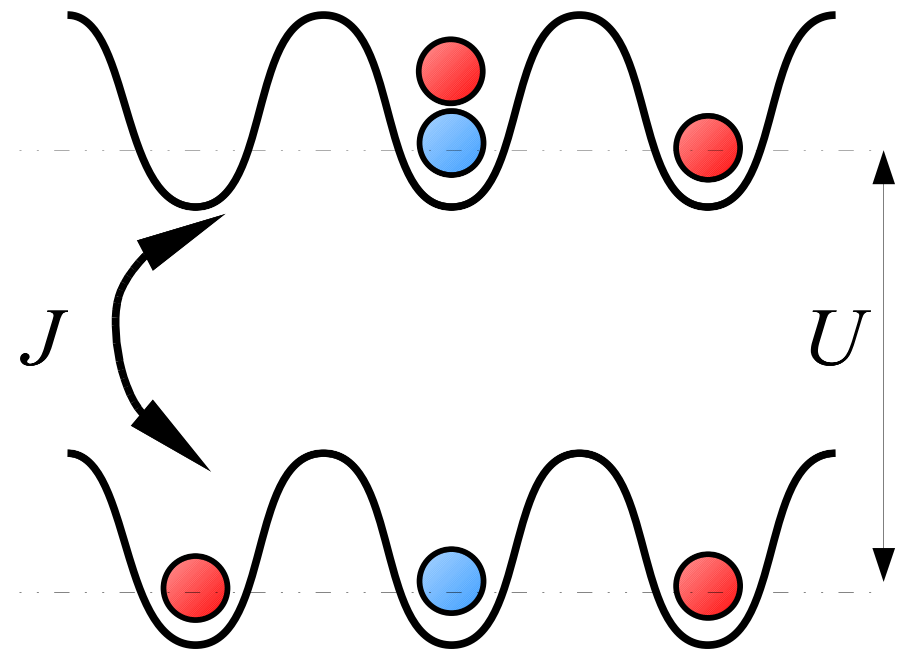
Weak hopping limit

- If the on-site interaction is large

$$U \gg J$$

  it is not favorable for atoms to come together.

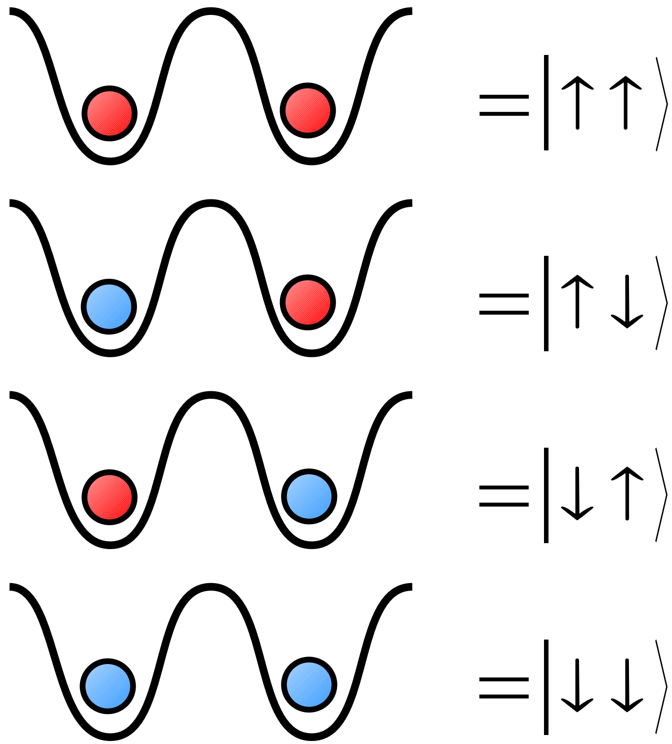
- We can then treat hopping as a perturbation.
- The unperturbed states have 1 atom / site.



$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + U \sum_i n_i^2$$
$$= J V + H_0$$

Effective spin interactions

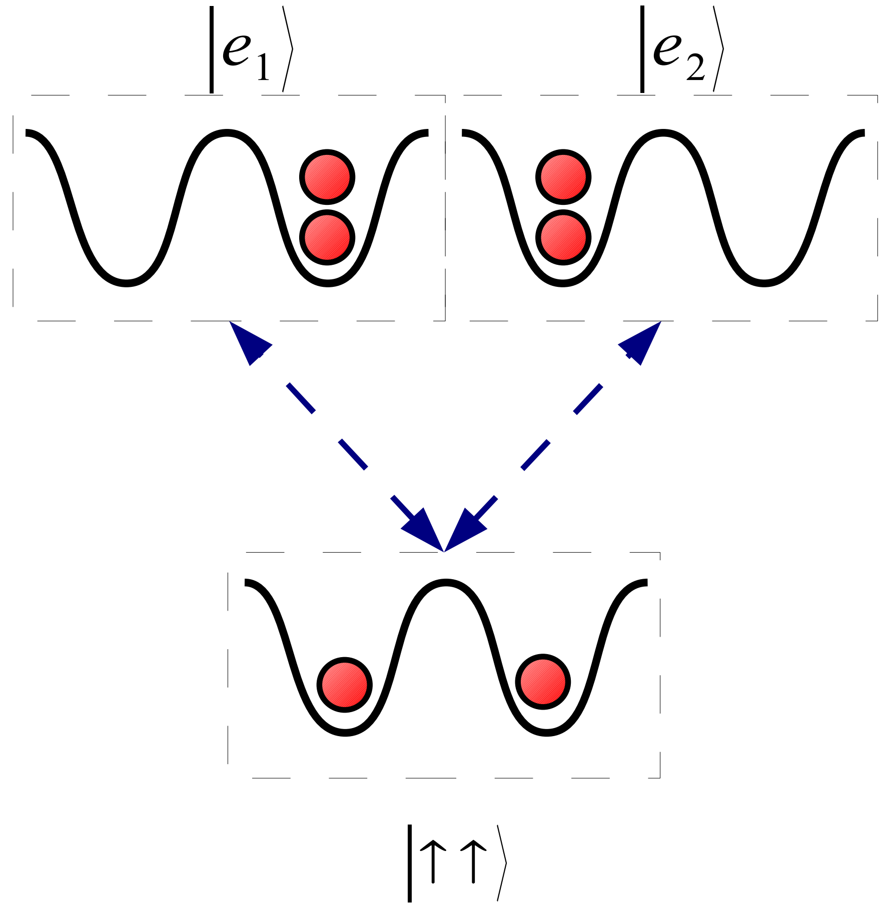
- We will adopt an encoding of spins based on atom internal state.



Effective spin interactions

- We will adopt an encoding of spins based on atom internal state.
- Only need to consider second order perturbation theory.

$$H_{eff} \sim -J^2 \sum_{g,g'} \sum_{e} |g\rangle\langle g|V|e\rangle \times$$
$$\times \langle e|V|g'\rangle\langle g'| \times$$
$$\times \frac{1}{2}\left[\frac{1}{E_g - E_e} + \frac{1}{E_{g'} - E_e}\right]$$



Effective spin interactions

- First case: hopping to the right to a cell with an indistinguishable particle

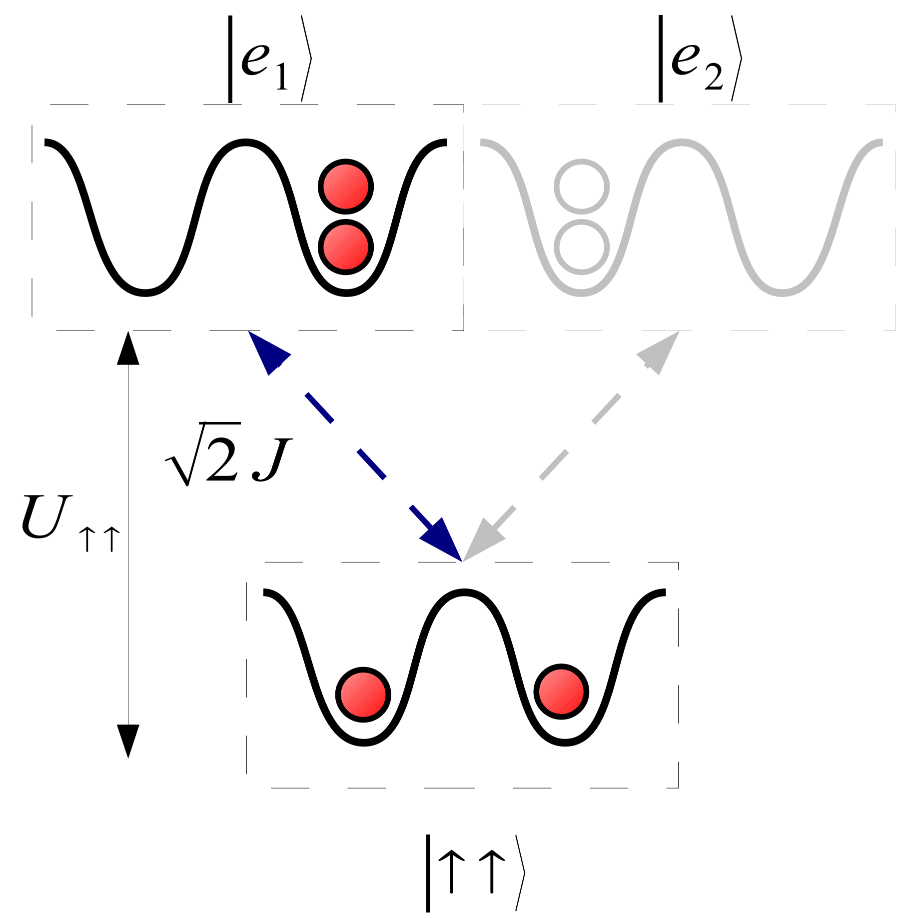
$$\langle e_1 | V | \uparrow\uparrow \rangle = \sqrt{2}$$

$$E_{e_1} = \frac{1}{2} U_{\uparrow\uparrow} 2(2-1) = U_{\uparrow\uparrow}$$

$$E_{\uparrow\uparrow} = 0$$

final contribution

$$\frac{2J^2}{-U_{\uparrow\uparrow}} |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$



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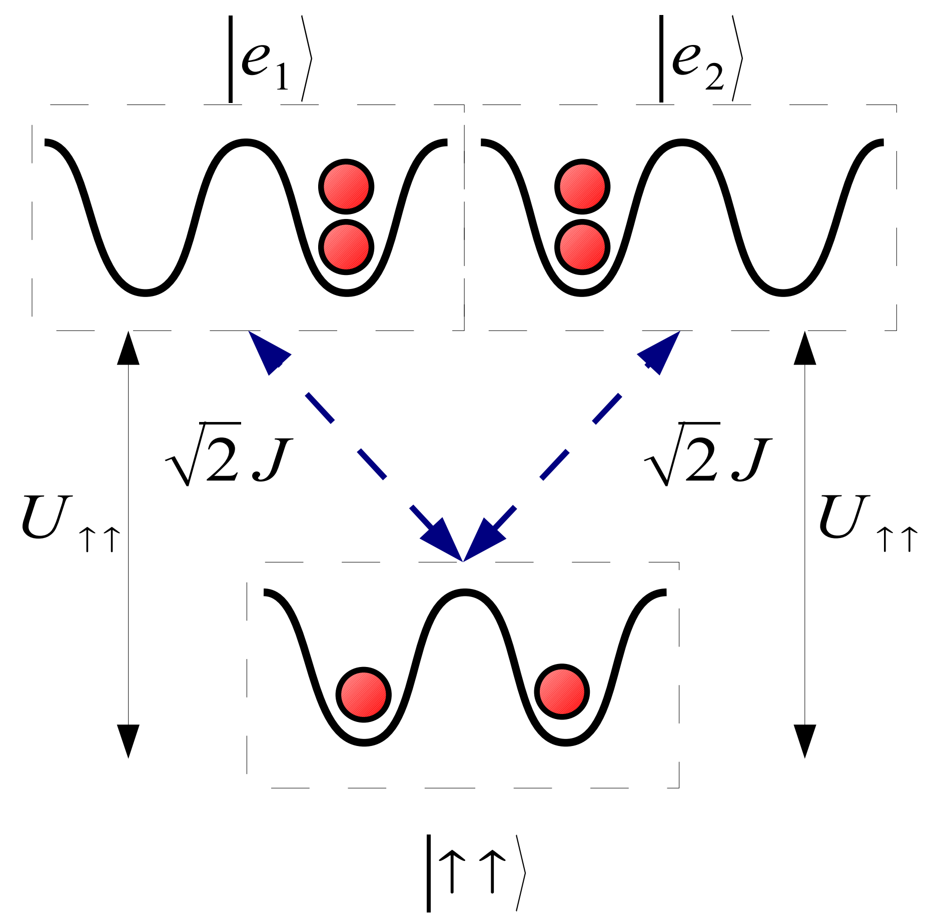
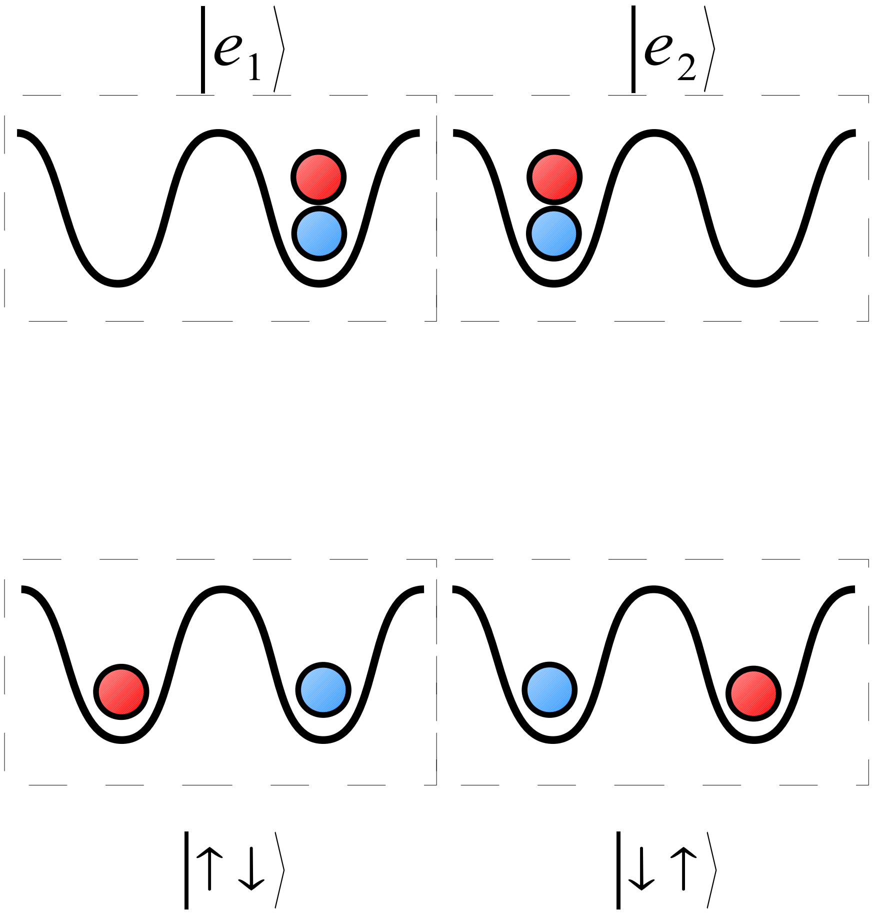
- With both paths

$$2 \times \frac{2J^2}{-U_{\uparrow\uparrow}} |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$



Effective spin interactions

- The situation changes slightly when having **distinguishable** particles.

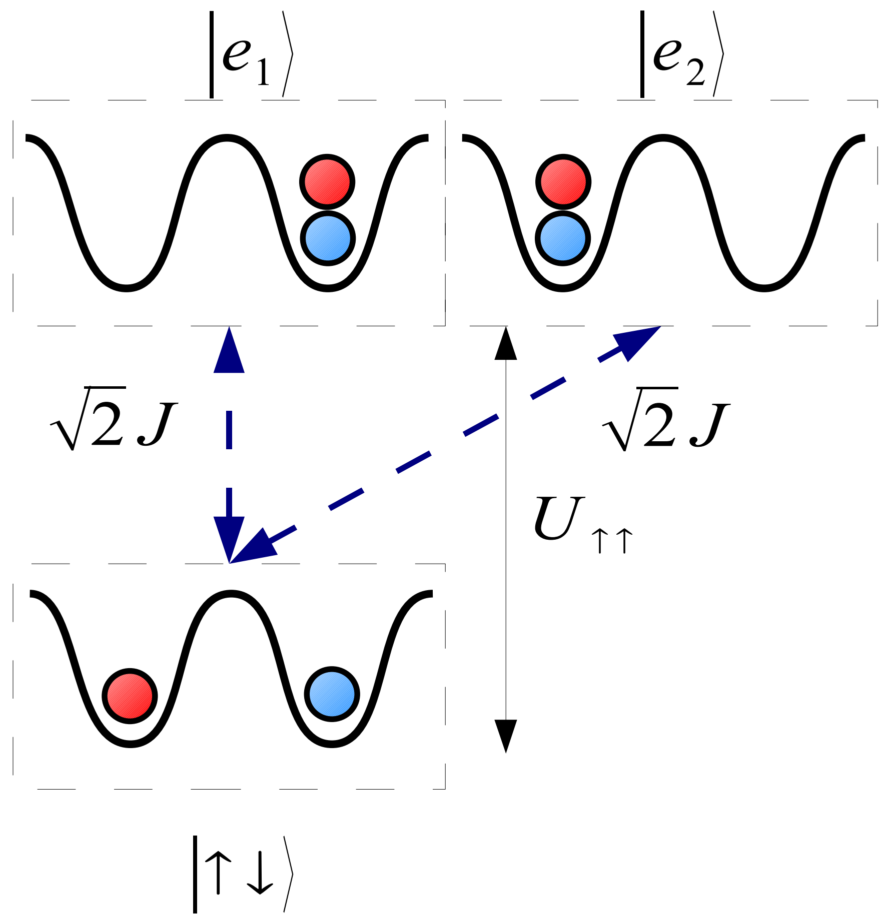


Effective spin interactions

- The situation changes slightly when having **distinguishable** particles.
- We have paths returning to the same initial state

$$2\times\frac{J^2}{-U_{\uparrow\downarrow}}|\uparrow\downarrow\rangle\langle\uparrow\downarrow|$$

with a factor 2 missing because particles are different on the same site



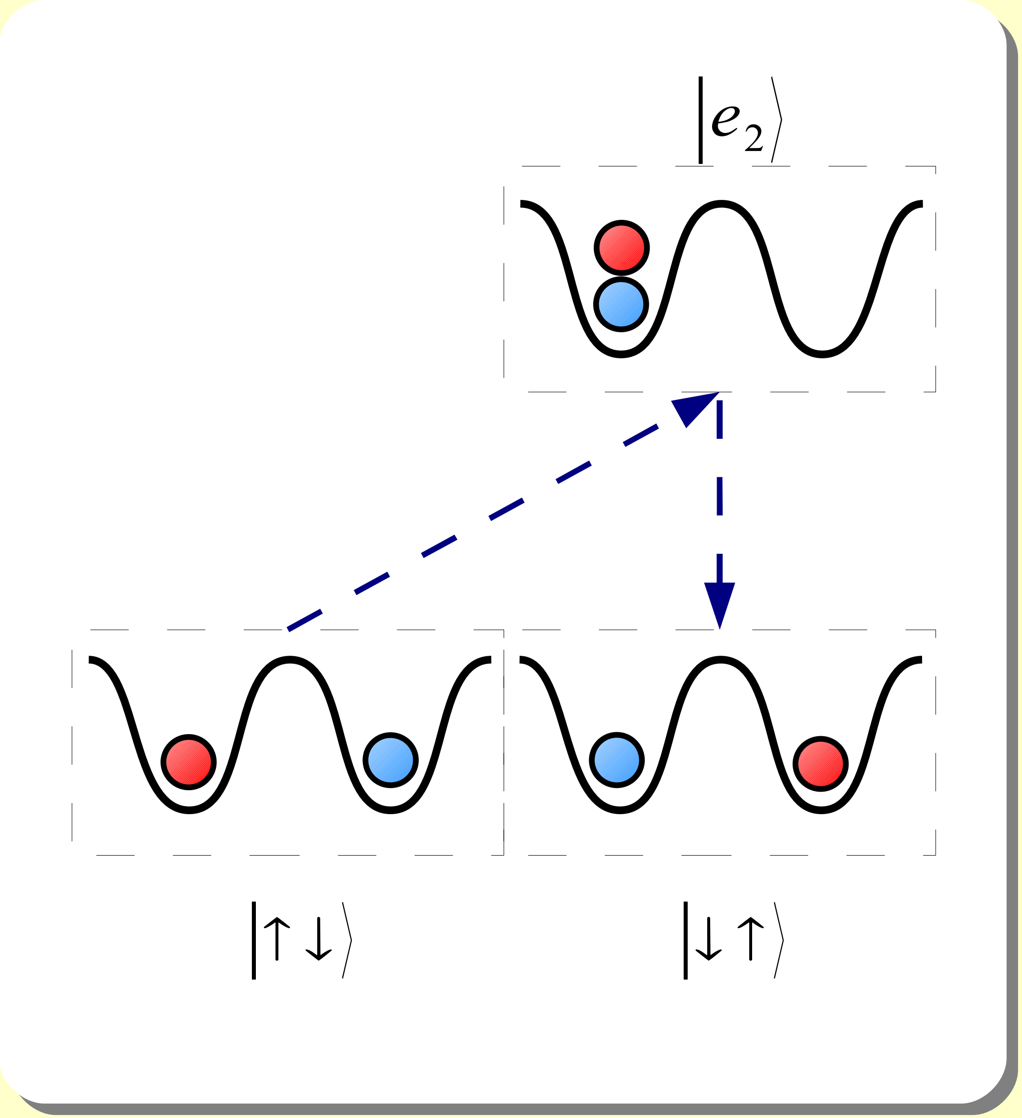
Effective spin interactions

- And we have also the possibility of exchanging neighboring spins

$$2\times\frac{J^2}{-U_{\uparrow\downarrow}}\left(|\uparrow\downarrow\rangle\langle\downarrow\uparrow|+|\downarrow\uparrow\rangle\langle\uparrow\downarrow|\right)$$

- These are the so called XY interactions

$$t\left(\sigma_i^x\sigma_j^x+\sigma_i^y\sigma_j^y\right)=$$
$$=t\,2\left(\sigma_i^+\sigma_j^-+\sigma_i^-\sigma_j^+\right)$$



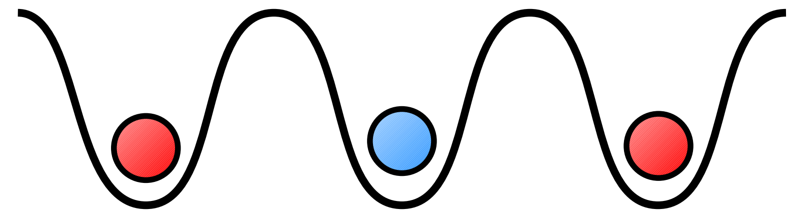
Everything together

- All in all, we can simulate any XYZ Hamiltonian, where the constants are given by

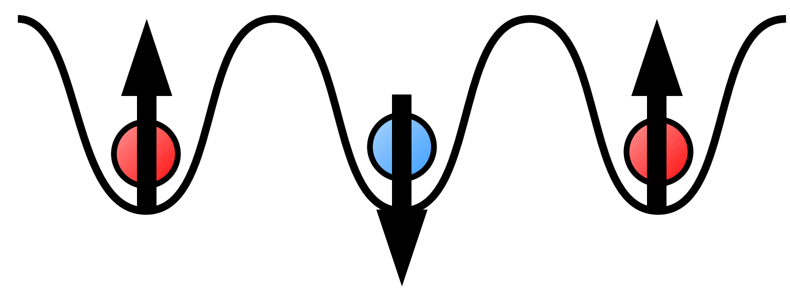
$$\lambda_z = J^2\left[\frac{1}{2U_{\uparrow\downarrow}} - \frac{1}{U_{\uparrow\uparrow}} - \frac{1}{U_{\downarrow\downarrow}}\right]$$

$$\lambda_{xy} = -\frac{2J^2}{U_{\uparrow\downarrow}}$$

$$h_z = -\frac{J^2}{U_{\downarrow\downarrow}} - \frac{J^2}{U_{\uparrow\uparrow}}$$



$$H = \sum_{\langle i,j\rangle} \lambda_{xy}\left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y\right) + \sum_{\langle i,j\rangle} \lambda_z \sigma_i^z \sigma_j^z + \sum_i h_z \sigma_i^z$$



J. J. García-Ripoll, J. I. Cirac, New. Jour. Phys, **5**, 76 (2003)

We can do even more

- Apply AC Stark shifts or magnetic fields to the atoms

$$\vec{\sigma}\cdot\vec{B}_{eff}$$

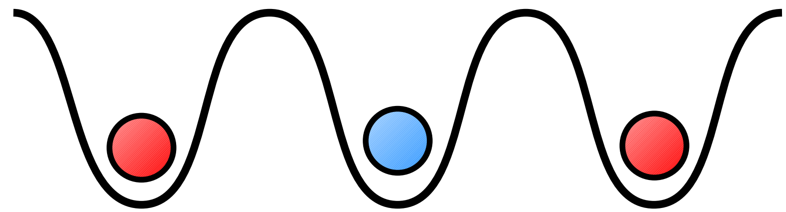
- Apply a global tilt of the lattice

$$\sum_i V(x)(a_i^+ a_i + b_i^+ b_i)$$

which changes the constants

$$\lambda_{xy} \rightarrow \frac{2J^2 U_{\uparrow\downarrow}}{U_{\uparrow\downarrow}^2 - \Delta^2}$$

etc, etc...



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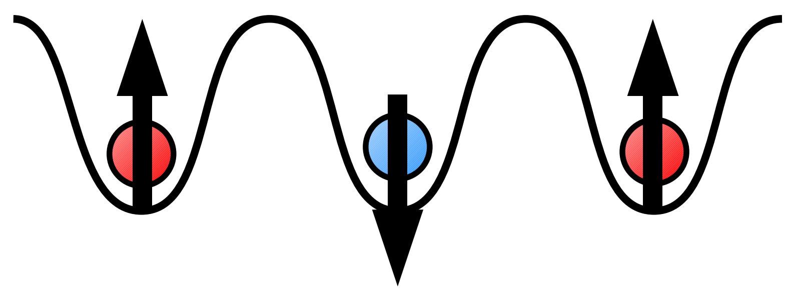

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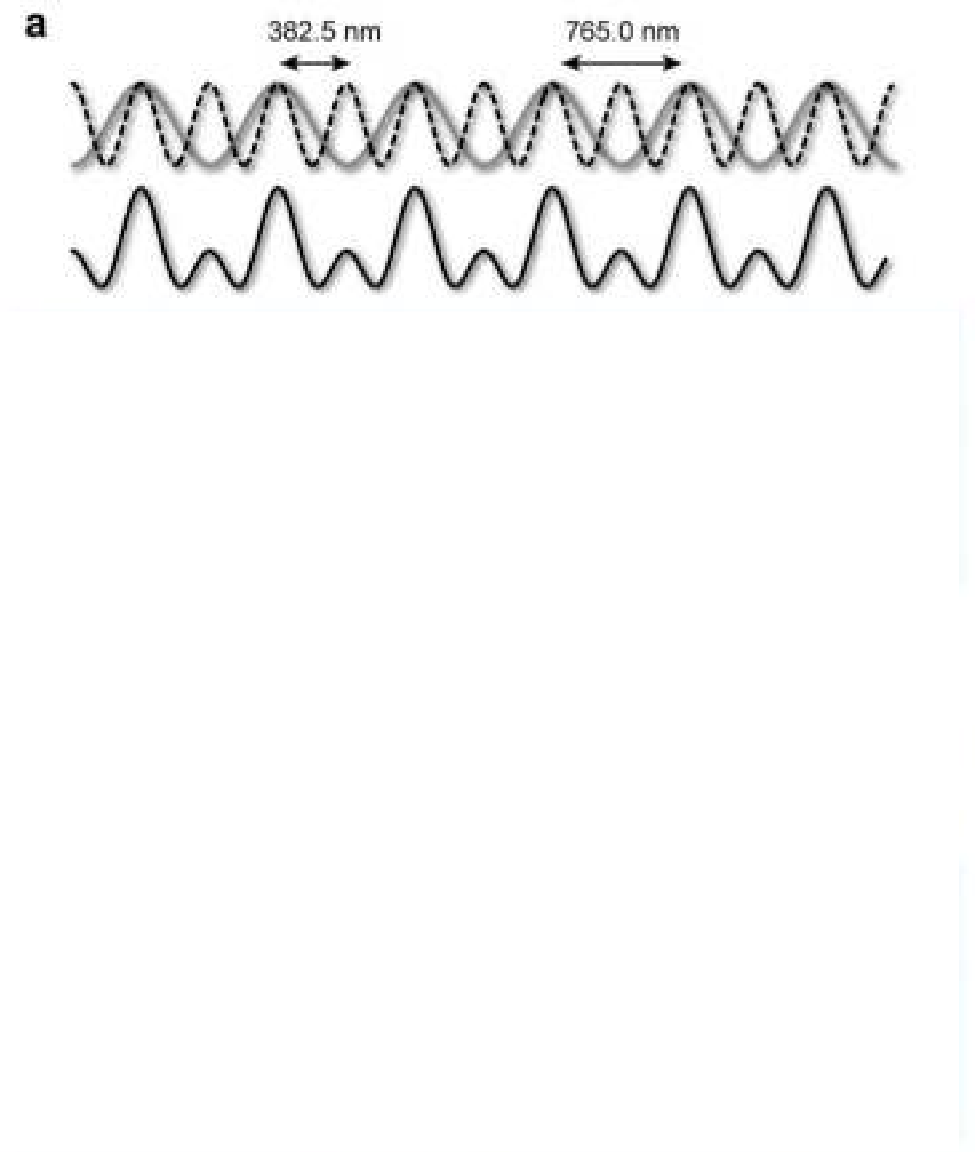
Experiments

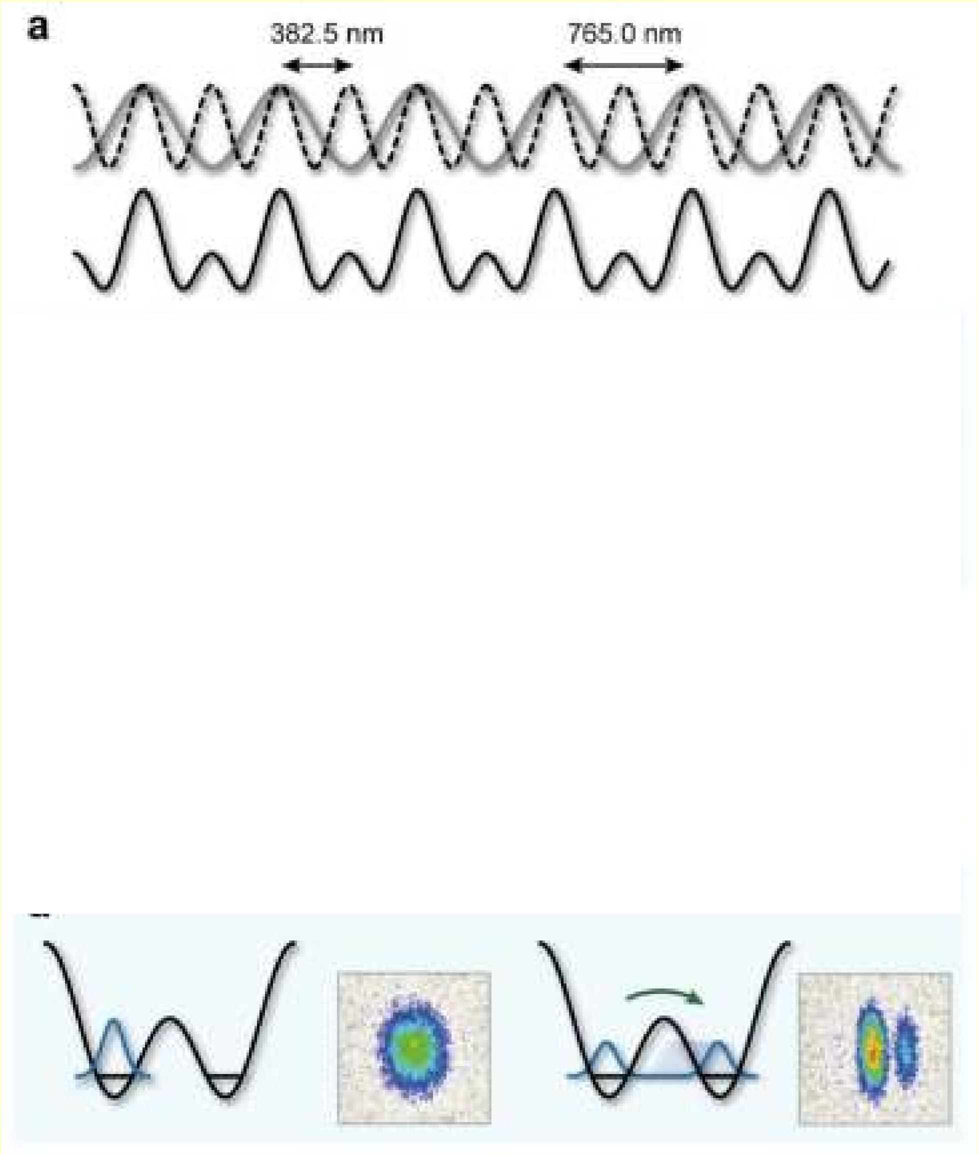
Superlattices



S Fölling et al, Nature **448**, 1029 (2007)

- Using two frequencies, one may create a complex lattice
  - Superlattice

Superlattices



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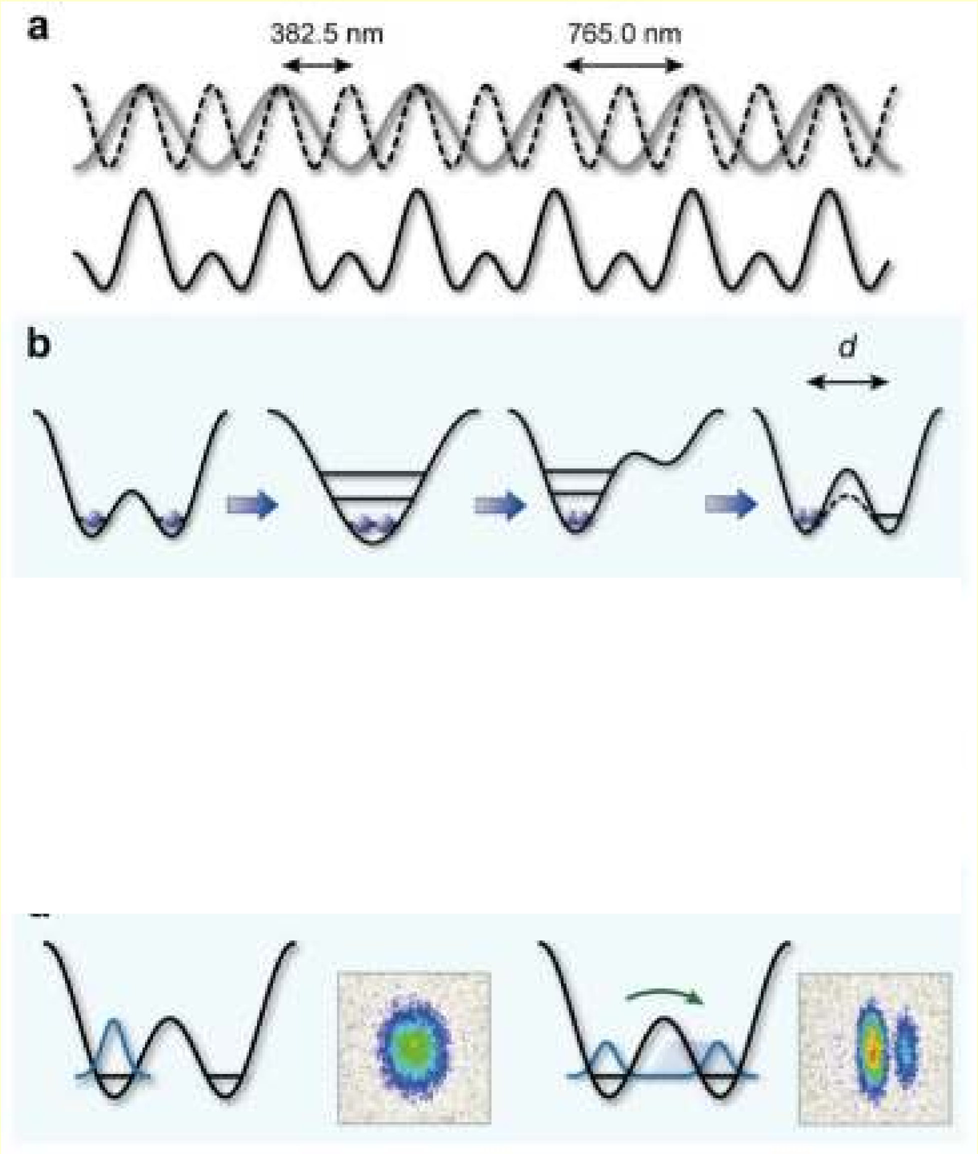
- A particle sitting on one well may tunnel to a neighboring site
  - This is the principle behind effective spin interactions.

Superlattices



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- Using two frequencies, one may create a complex lattice
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- A particle sitting on one well may tunnel to a neighboring site.
- If two particles are together, tunneling may only happen through virtual processes.
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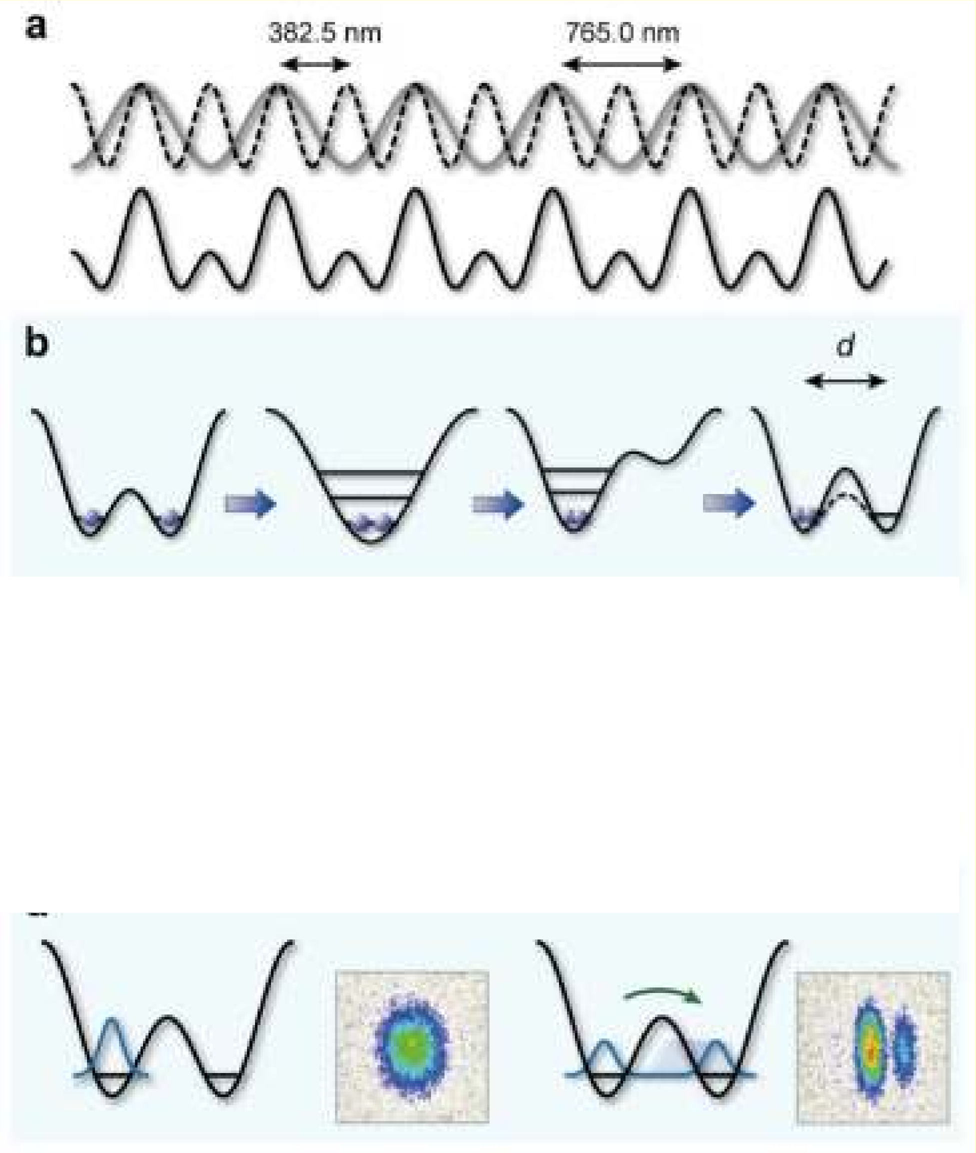
Superlattices



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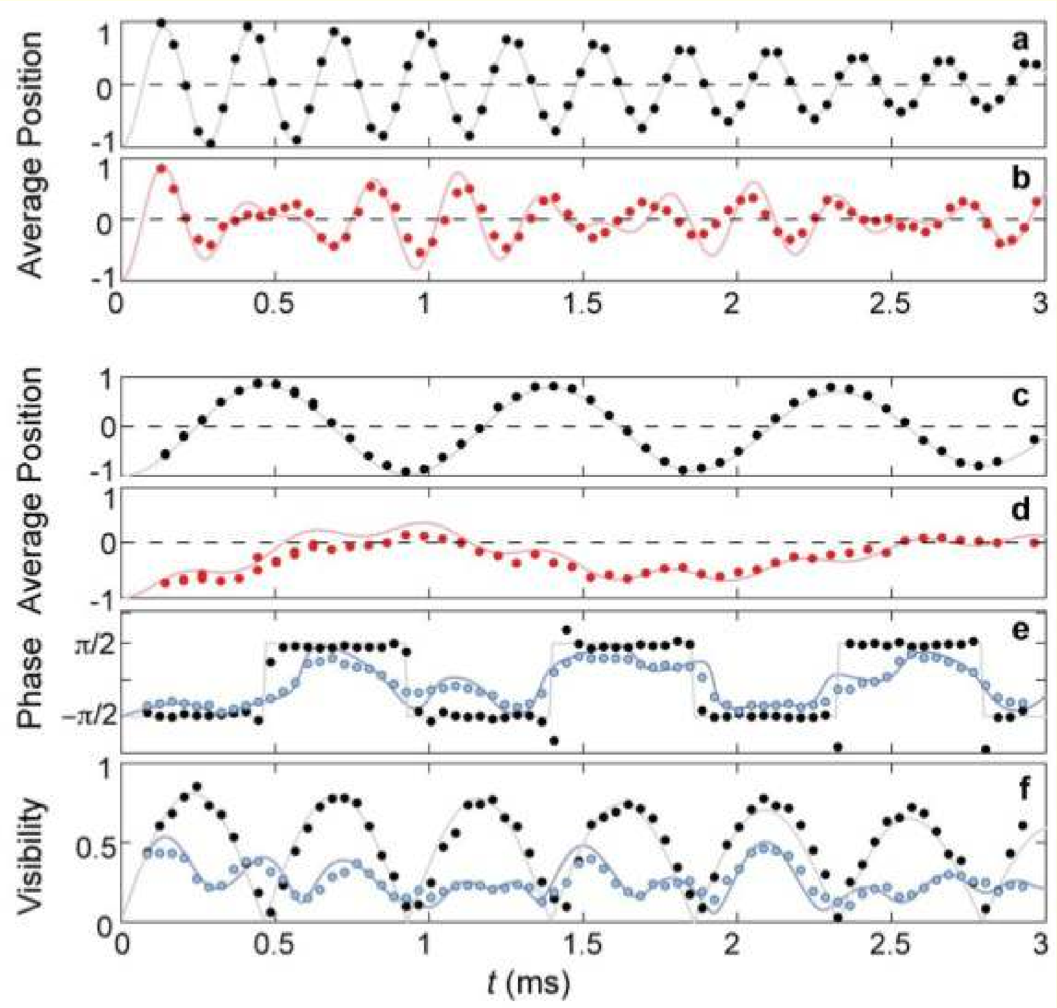

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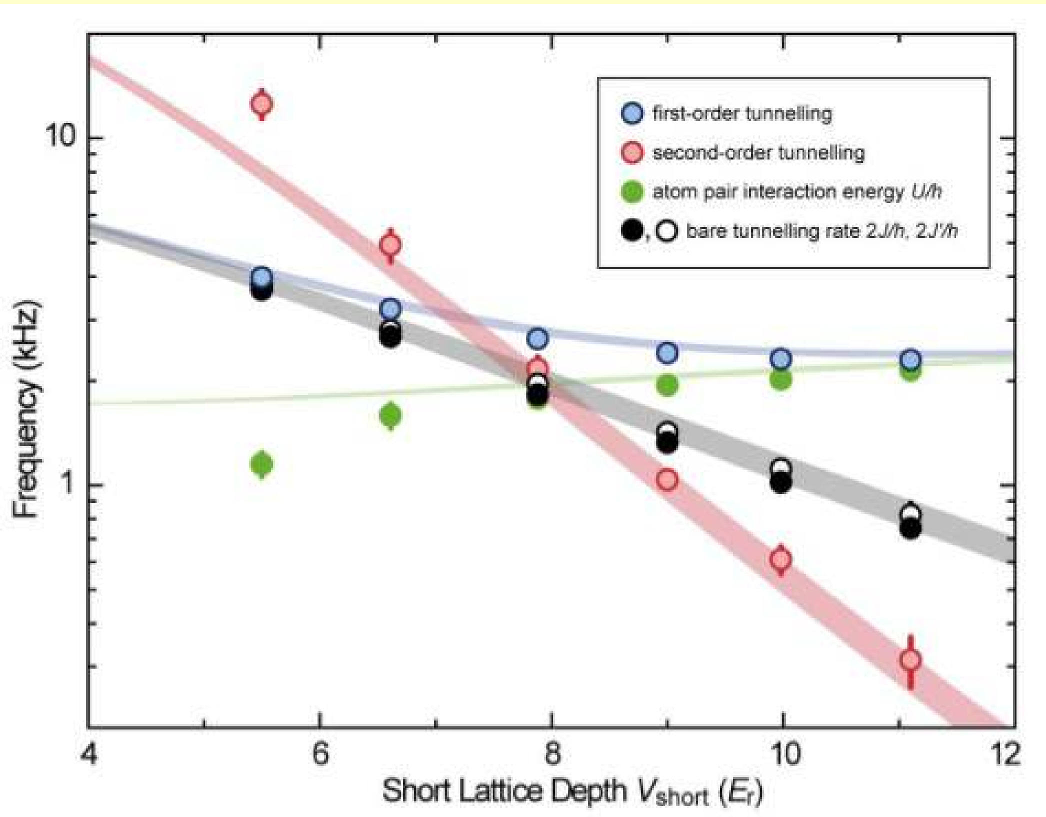

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- Perfectly matched theory

S Fölling et al, Nature **448**, 1029 (2007)

Testing effective interactions

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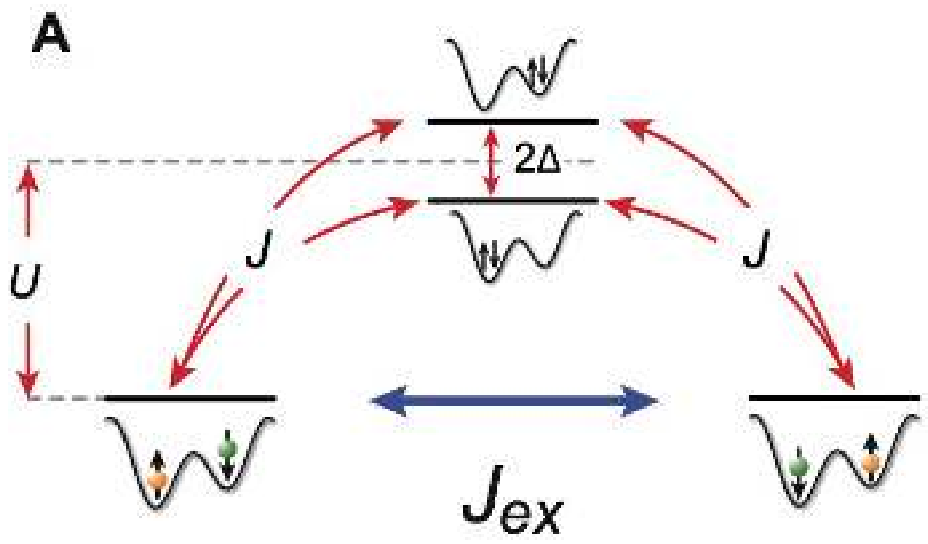
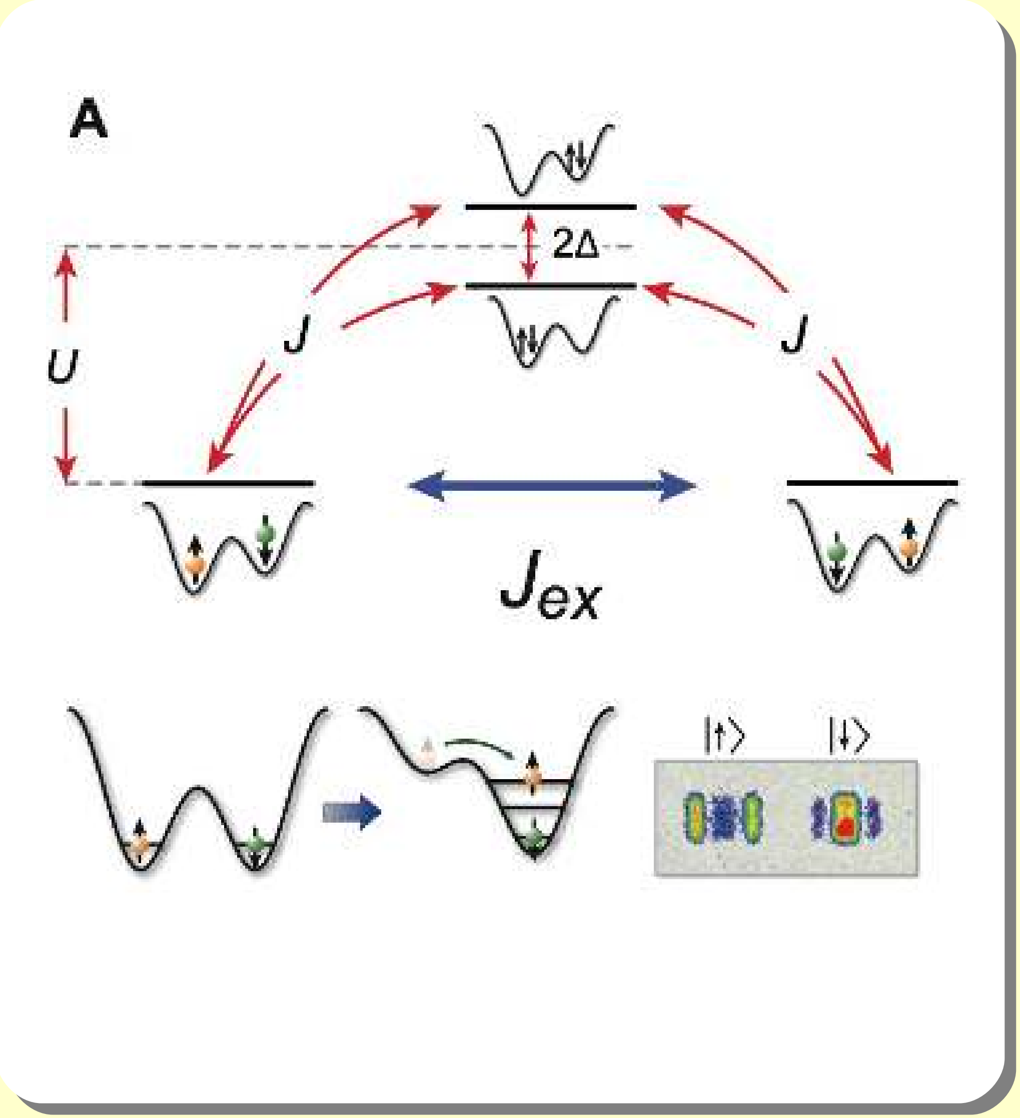


- Store atoms in neighboring sites, isolated by superlattice.
- The lattice sites suffer some imbalance, Δ
- We have similar virtual processes as before.

S Trotzky et al, Science **319**, 5861 (2007)

Testing effective interactions



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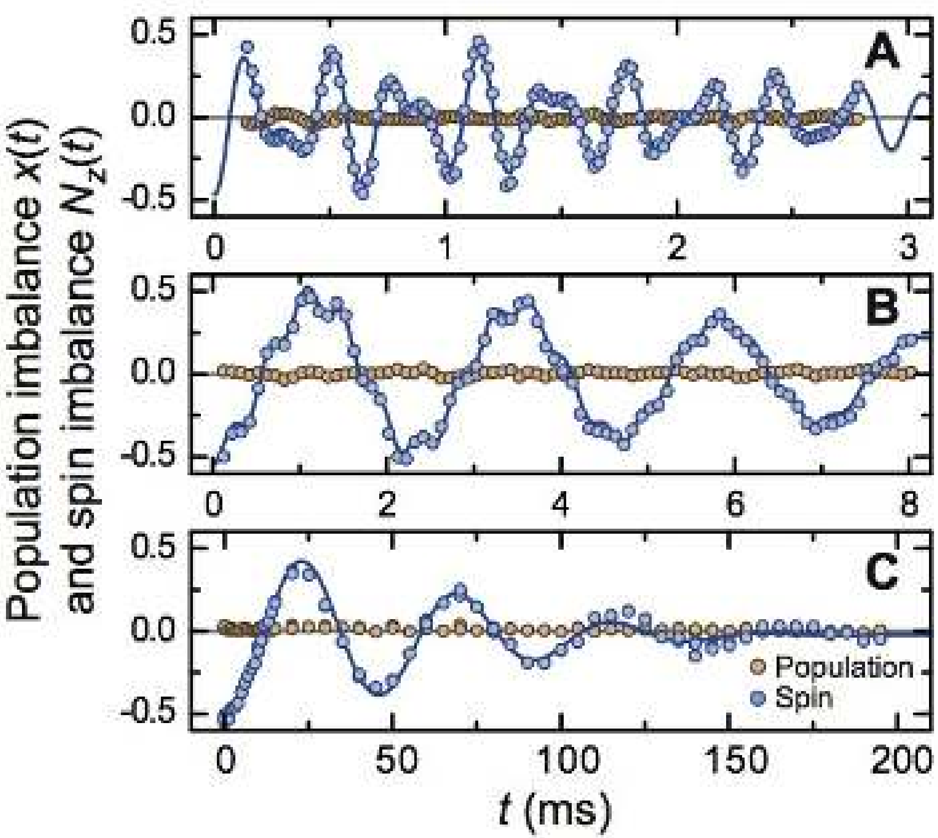

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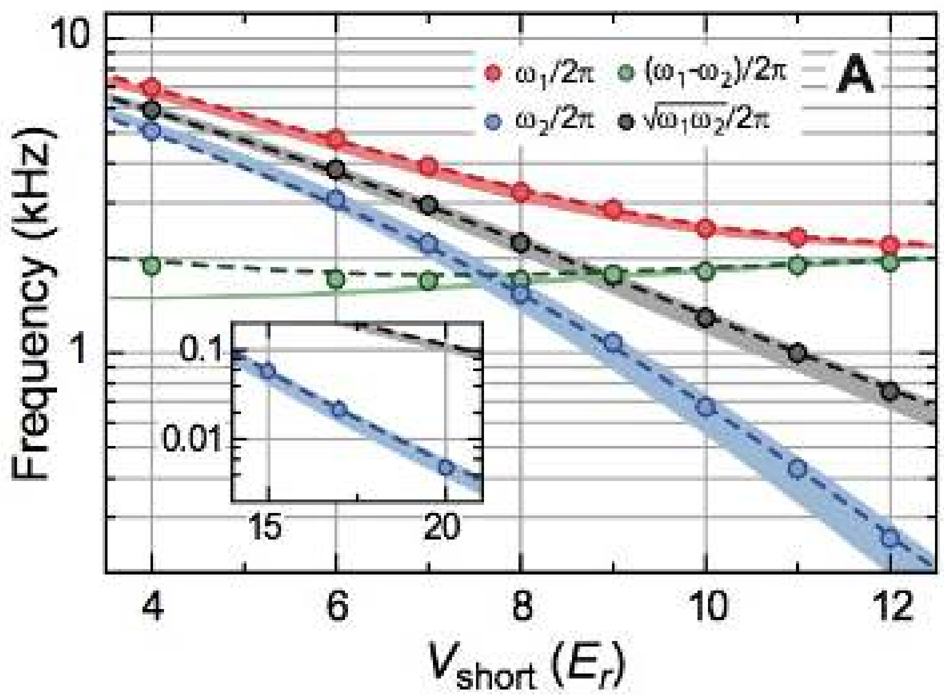

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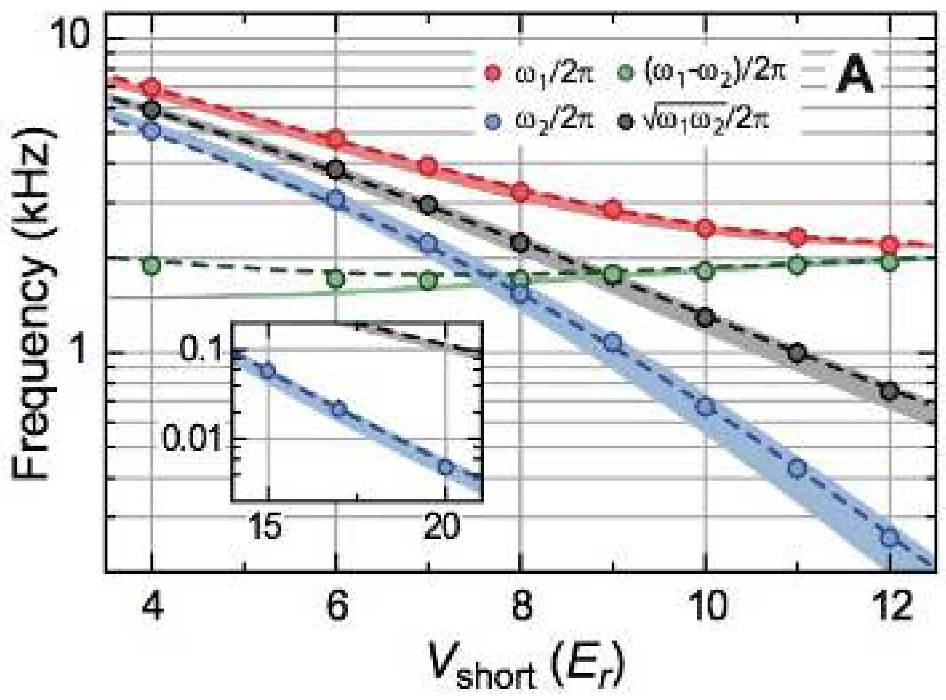
- Store atoms in neighboring sites, isolated by superlattice.
- The lattice sites suffer some imbalance, Δ
- We have similar virtual processes as before.
- We can resolve the population and spin of the left and right lattice wells.
- From the spectra of oscillation we can get the energy levels.
- **Perfect match with theory!**

Testing effective interactions



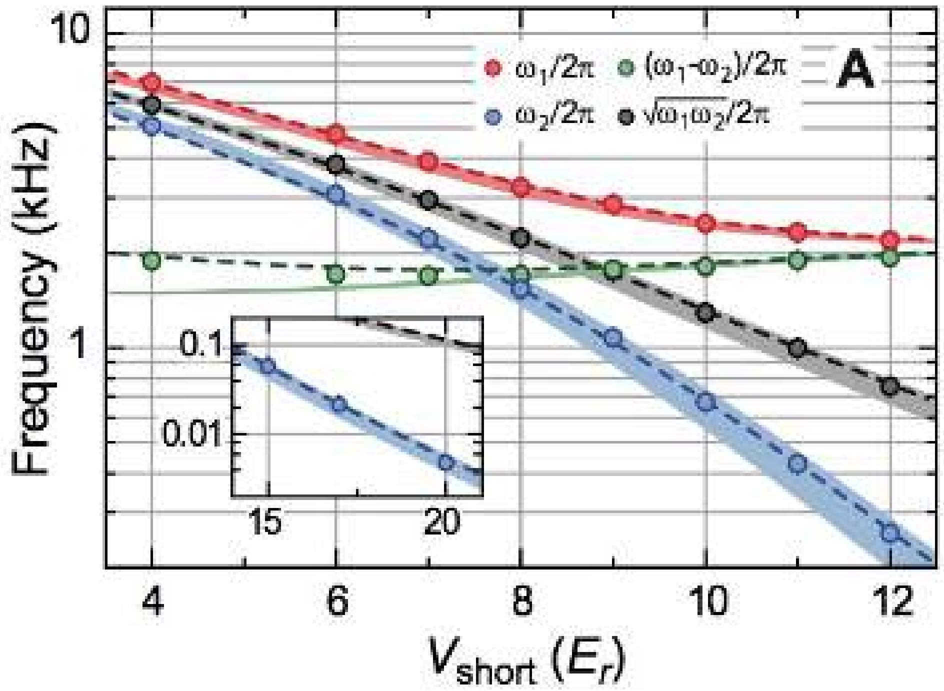
- Note that we now have a parameter to play with, Δ

$$H \sim \frac{-2J^2}{U-\Delta}$$

- Normally, U > 0 and we have ferromagnetic interaction.
- With Δ, we can change the sign of interactions.

S Trotzky et al, Science **319**, 5861 (2007)

Testing effective interactions



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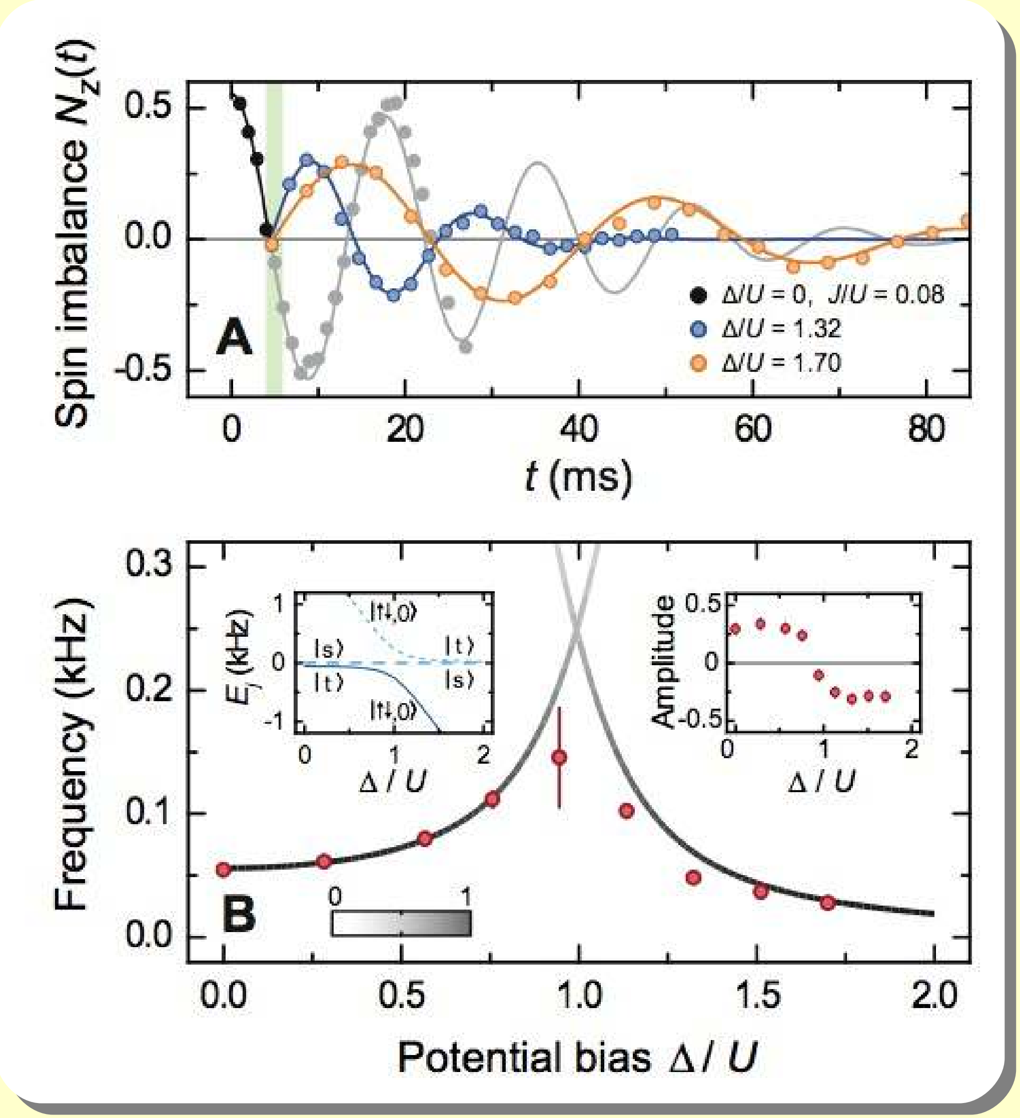

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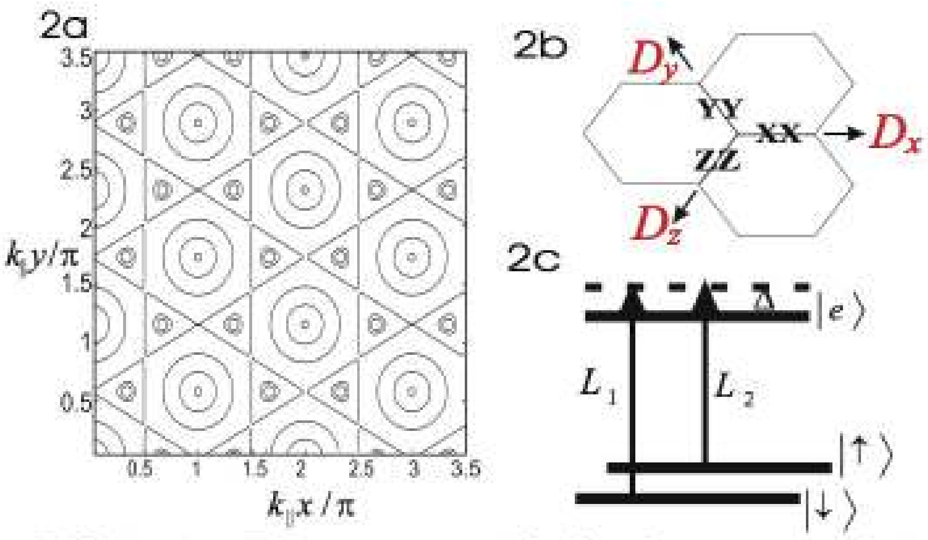
$$H \sim \frac{-2J^2}{U-\Delta}$$

- Normally, U > 0 and we have ferromagnetic interaction.
- With Δ, we can change the sign of interactions.
- This can even be done at **real time**.

S Trotzky et al, Science **319**, 5861 (2007)

Sophisticated models

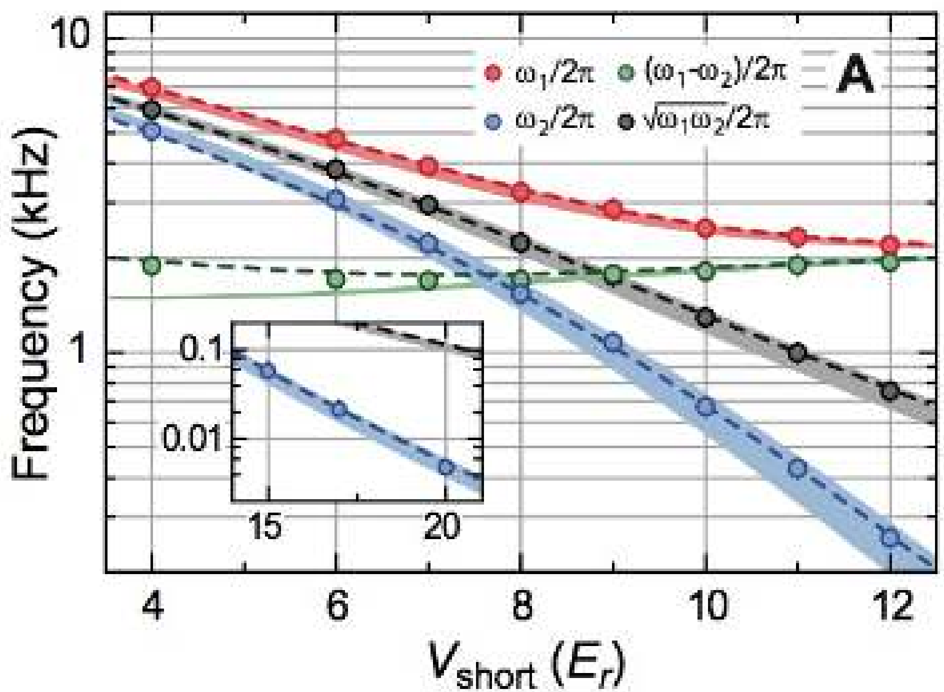


- Proposals to engineer honeycomb lattices.
- Using Trotter decompositions, simulate different XX, YY, ZZ along bonds.
- Kitaev model for topological quantum computation.

L-M Duan et al, PRL **91**, 090402 (2003)

Problems

Problems



- Very weak interactions
  - 100-200 Hz in the region where perturbation theory really applies
- Very sensitive to the control parameters.
- Solutions?
  - Use molecules
  - Use ions

S Trotzky et al, Science **319**, 5861 (2007)