Other QC models

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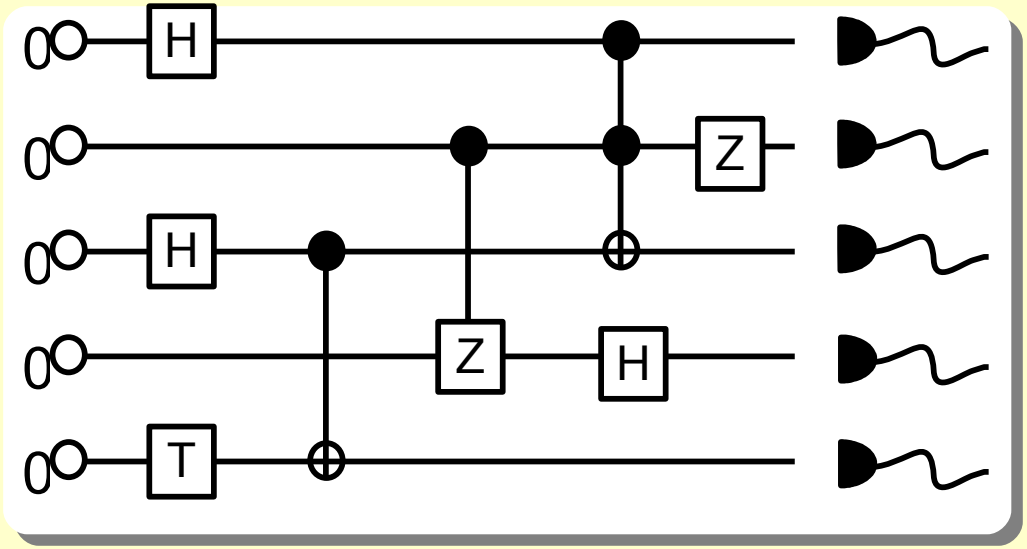
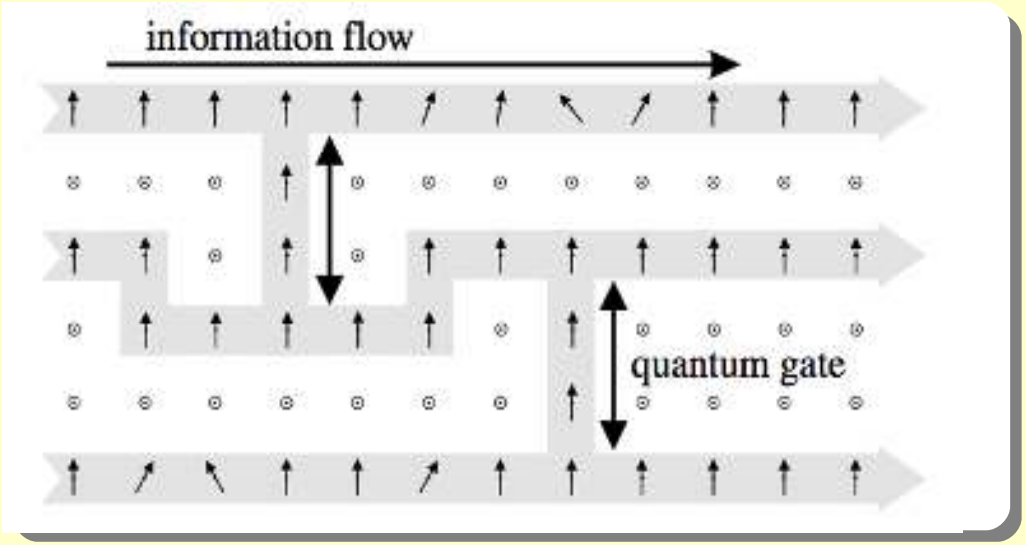
(20-4-2009)

QC models so far

- A model based on a quantum register, combining local and universal two-qubit unitaries.
- Conceptually simple, parallelized operations.

Circuit based QC



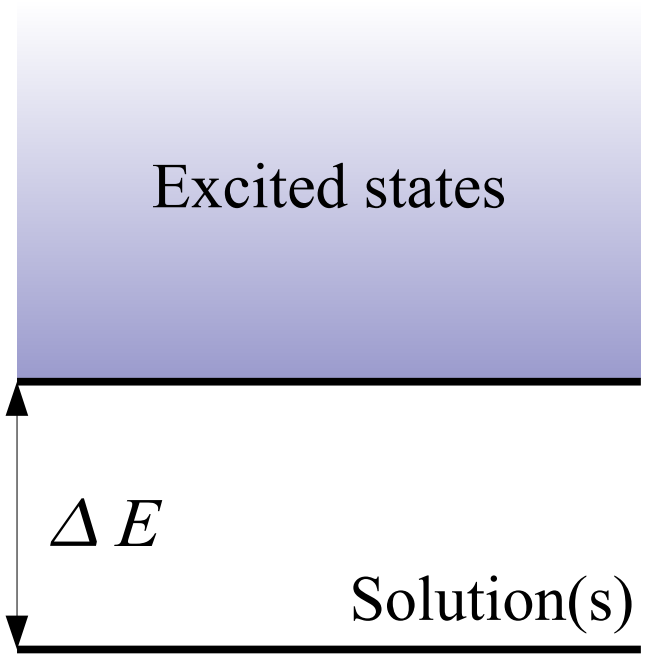


Measurement based QC

- Entangled state as a resource for QC.
- Only local operations and single-qubit measurements.
- Homogeneous, but same algorithms as circuit model

Adiabatic QC

Adiabatic QC

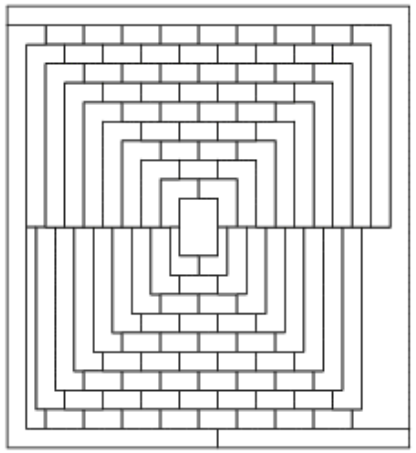


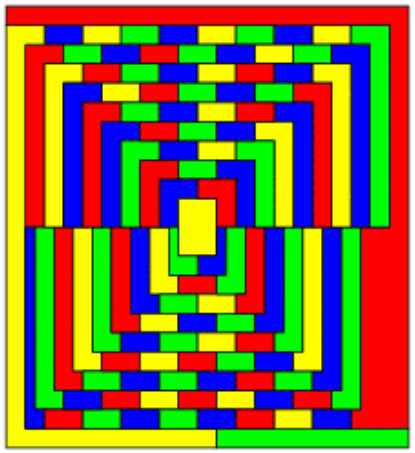
$$(H_{prob} - E_{min})\left|\psi_{sol}\right\rangle = 0$$

- Express the outcome of a computation as a ground state.
- Design the appropriate interactions.
- Prepare adiabatically that ground state.
- Measure to find the outcome.

E. Farhi et al, Science **20**, 472-475 (2001)

Utility?





Solve:

$$(-i_1)\vee i_3\vee i_5\wedge$$
$$i_2\vee i_3\vee(-i_5)\wedge$$
$$\ldots$$
$$i_k\in\{0,1\}$$

Microchip optimization...

- Equivalent to quantum circuits.
- Many problems can be formulated as Hamilt.
  - Travelling salesman, nSAT, exact cover sudokus...
  - H_{prob} are simple boolean functions
- Problems *NP-complete*

E. Farhi et al, Science **20**, 472-475 (2001)

How it works

$$H_0 = \sum_k \sigma_k^x$$

$$|\psi(0)\rangle \sim \left(|0\rangle + |1\rangle\right)^{\otimes N}$$

$$\downarrow$$

$$H(t) = t\,H_{prob} + (1 - t/T)\,H_0$$

$$\downarrow$$

$$H_{prob} = \sum_i f_i(\sigma_{i_1}^z, \sigma_{i_2}^z, \sigma_{i_3}^z)$$

$$|\psi(T)\rangle \sim \sum_{\vec{s} \in Sol} |s_1, s_2 \ldots s_N\rangle$$

- Begin with a state and H easy to prepare
- "Deform" adiabatically the effective interaction.
- The final H encodes the problem.
- We reach the final ground state if the instantaneous gap is large

$$\Delta E \gg \hbar / T$$

E. Farhi et al, Science **20**, 472-475 (2001)

Example: Grover

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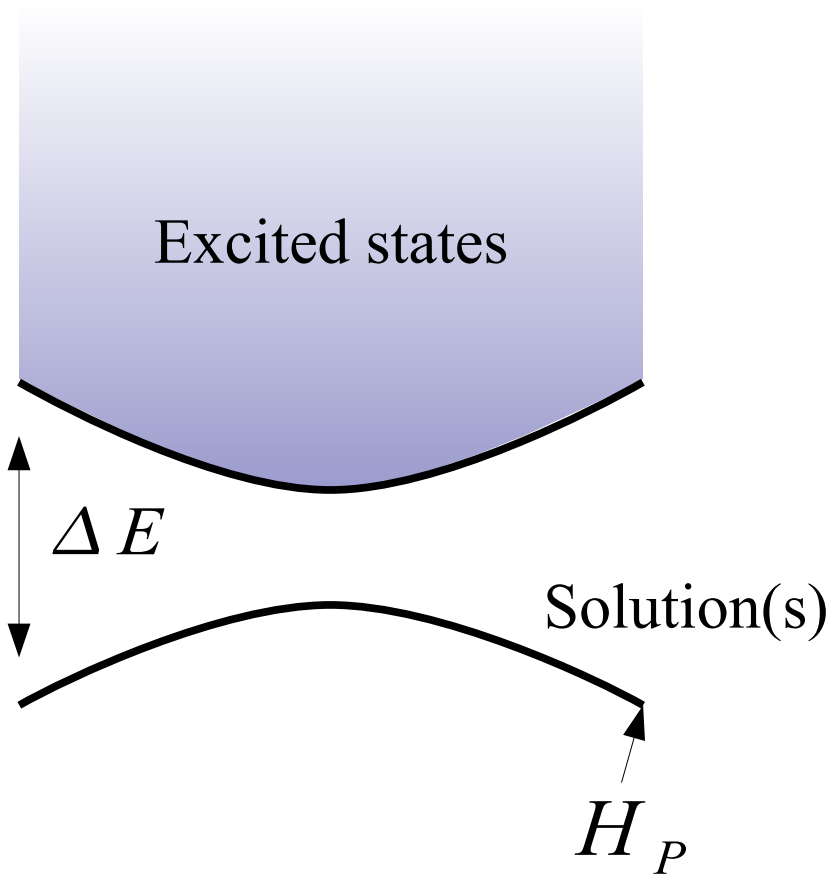


- We want to search a database, that is to solve

$$f(w)=1$$

- Our problem Hamiltonian can be simply

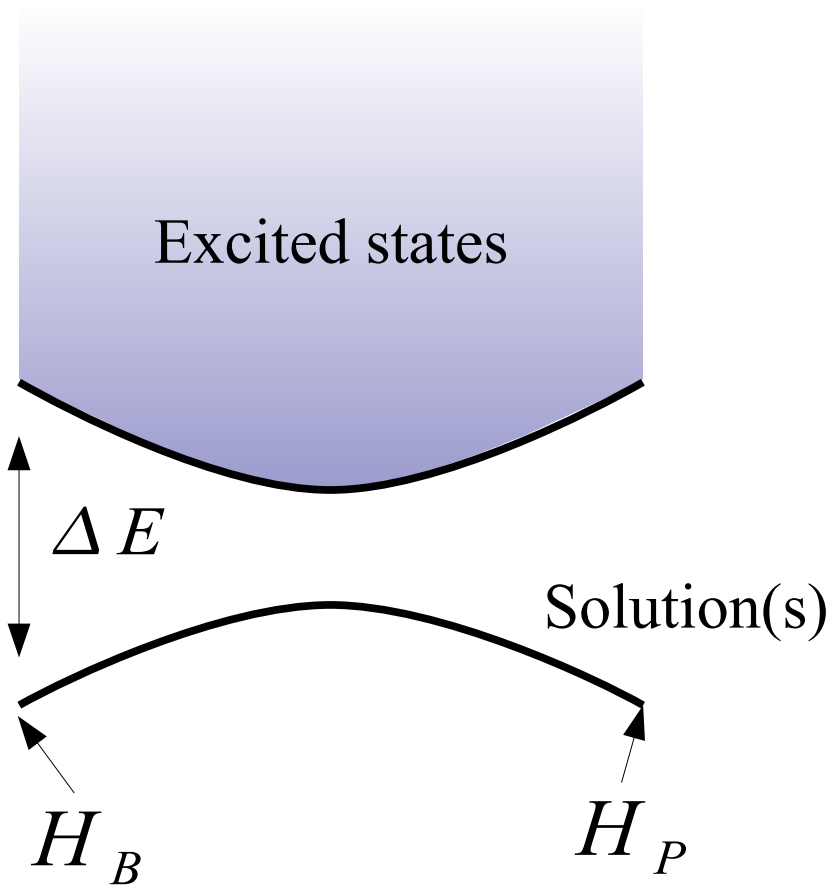
$$H_P=-|w\rangle\langle w|$$

with ground state $|w\rangle$

- Our Hamiltonian must end there

$$H(T)=H_P$$

Example: Grover

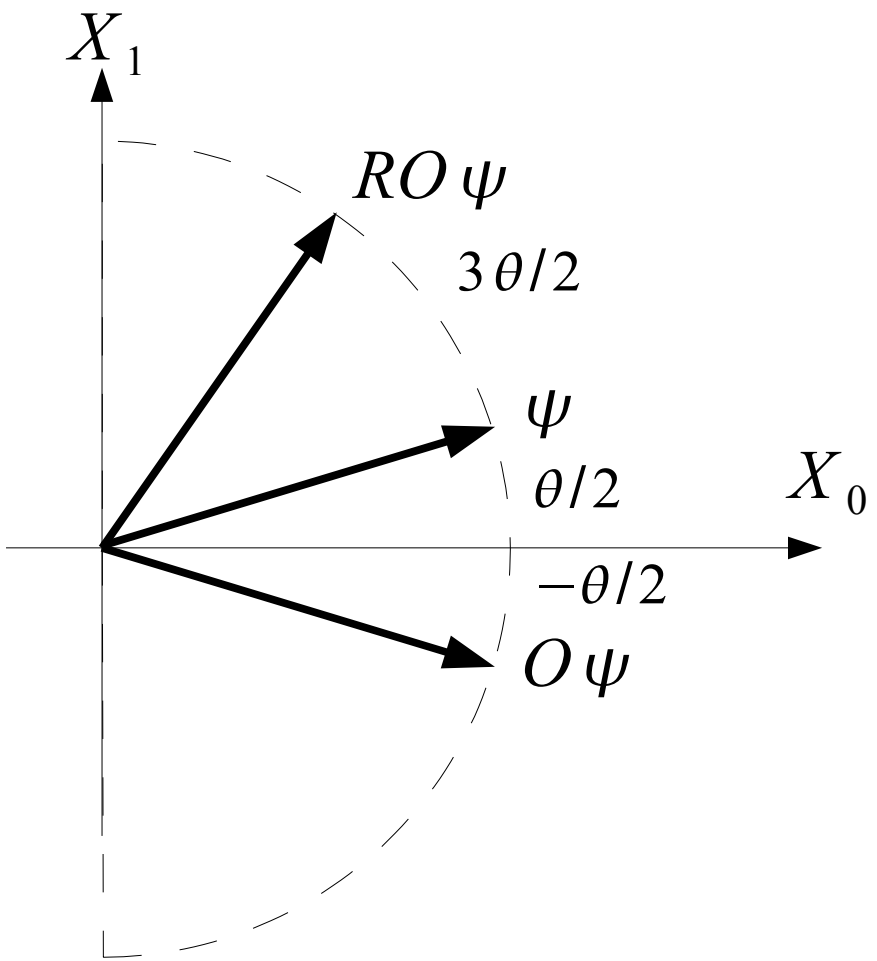


- The problem Hamiltonian has to be combined with a "mixing" term.
  - Allows transitions between different qubit states.
  - Easily prepared initial ground state.
- The interpolation among both may be a simple as

$$H(t) = (1 - \frac{t}{T}) H_B + \frac{t}{T} H_P$$

- This determines the gap and thus the alg. speed.

Mixing term



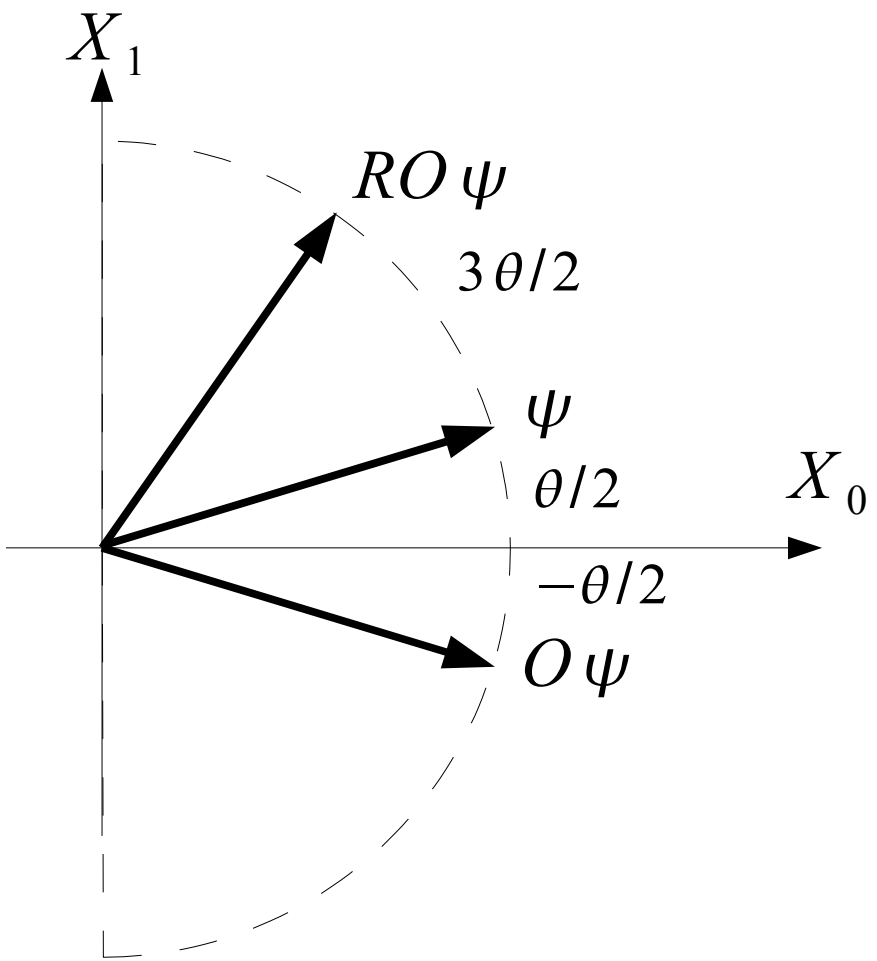
- We remind the fundamental operation in Grover

$$R \sim 2\left|\psi\right\rangle\left\langle\psi\right| - I$$

where

$$\left|\psi\right\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N} \left|x\right\rangle$$

Mixing term



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$$R \sim 2|\psi\rangle\langle\psi| - I$$

where

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N} |x\rangle$$

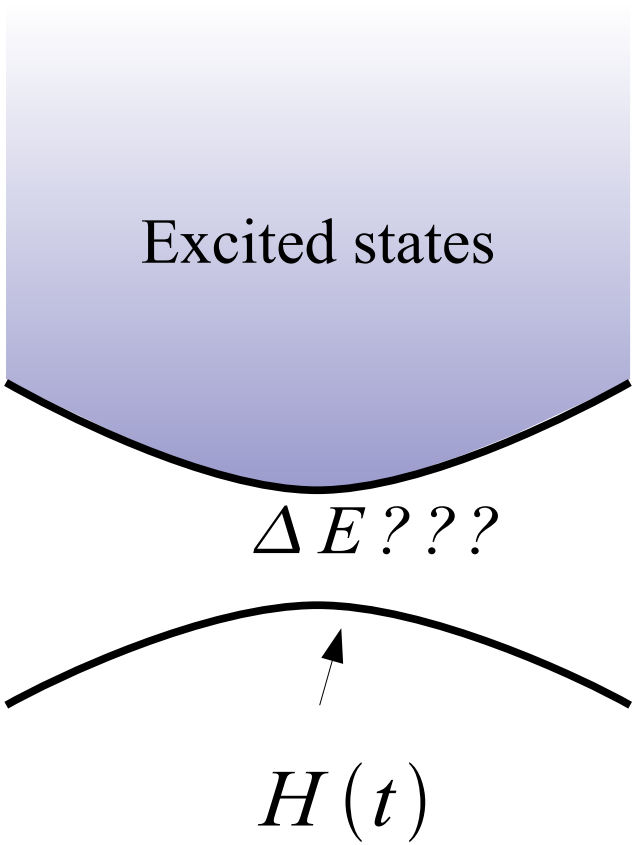
- We introduce thus a simple Hamiltonian

$$H_B = |\psi\rangle\langle\psi|$$

- Note that the initial state can be easily prepared.

$$|\psi\rangle \sim |+\rangle^{\otimes n}$$

Full Hamiltonian

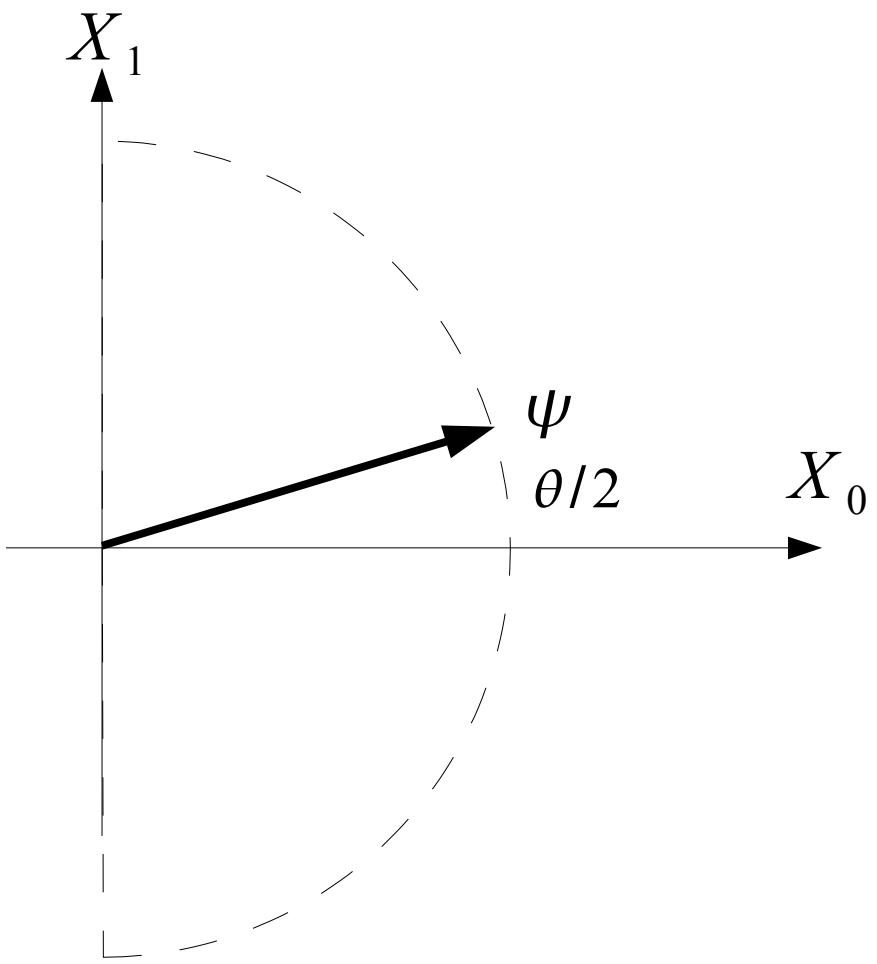


- We combine both terms

$$H = (1-\lambda)\frac{1}{N}|\psi\rangle\langle\psi| - \lambda|w\rangle\langle w|$$

- Note that the energy is bound by O(1)
  - We could arbitrarily increase speed by enlarging ||H||

Full Hamiltonian



- We reintroduce the idea from Grover

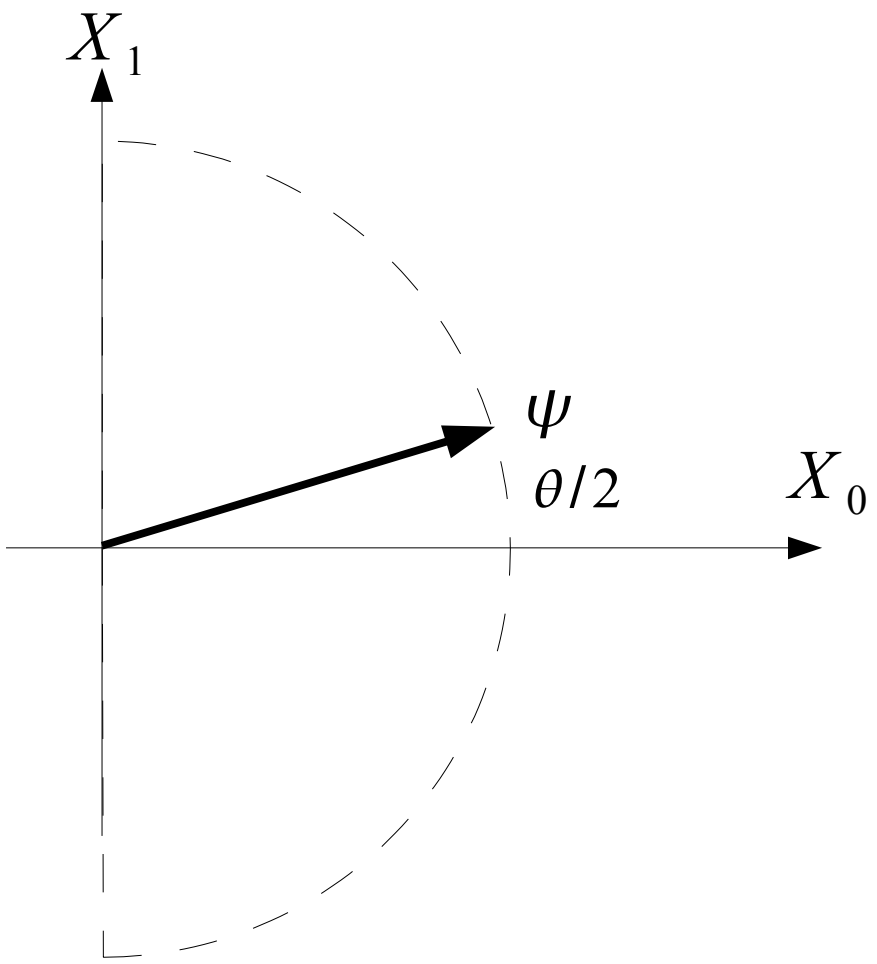
$$|\psi\rangle = \cos(\theta/2)|y\rangle + \\ + \sin(\theta/2)|w\rangle$$

so that

$$H_B = -\cos(\theta/2)^2|y\rangle\langle y| + \\ -\sin(\theta/2)^2|w\rangle\langle w| + \\ -\sin(\theta/2)\cos(\theta/2)\left[|w\rangle\langle y| + \right.$$

$$H_P = -|w\rangle\langle w|$$

Full Hamiltonian



- Easier to read

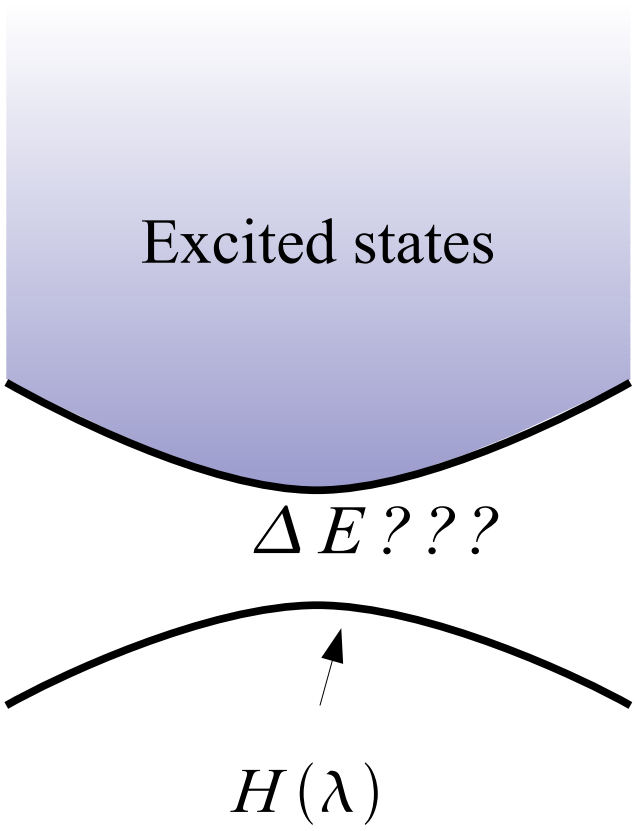
$$H_B \sim -\frac{1}{2} - \cos(\theta)\sigma_z - \\ -\frac{1}{2}\sin(\theta)\sigma_x$$

$$H_P \sim -\frac{1}{2} + \frac{1}{2}\sigma_z$$

using Pauli operators

$$\sigma_z = |y\rangle\langle y| - |w\rangle\langle w|$$
$$\sigma_x = |y\rangle\langle x| + |w\rangle\langle w|$$

Full Hamiltonian



- We can rewrite in the form

$$H = -\frac{1}{2} - \frac{1}{2}\vec{v}\cdot\vec{\sigma}$$

where

$$\vec{v} = [(1-\lambda)\cos(\theta) - \lambda, (1-\lambda)\sin(\theta)]$$

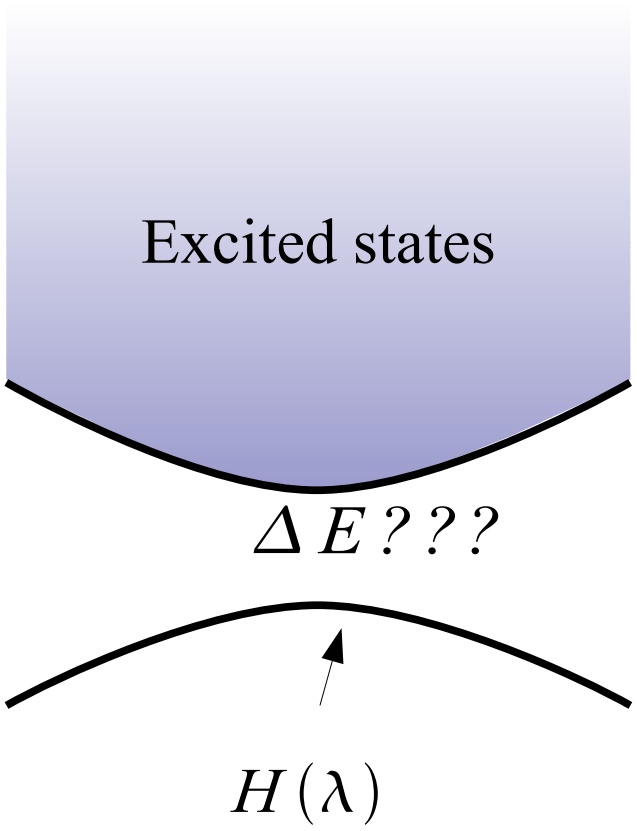
- The energy levels are

$$E = -\frac{1}{2} \pm \frac{1}{2}|\vec{v}|$$

with energy gap

$$\Delta E = |\vec{v}|$$

Full Hamiltonian



- From the expression

$$\vec{v}=[(1-\lambda)\cos(\theta)-\lambda,(1-\lambda)\sin(\theta)]$$

the gap is

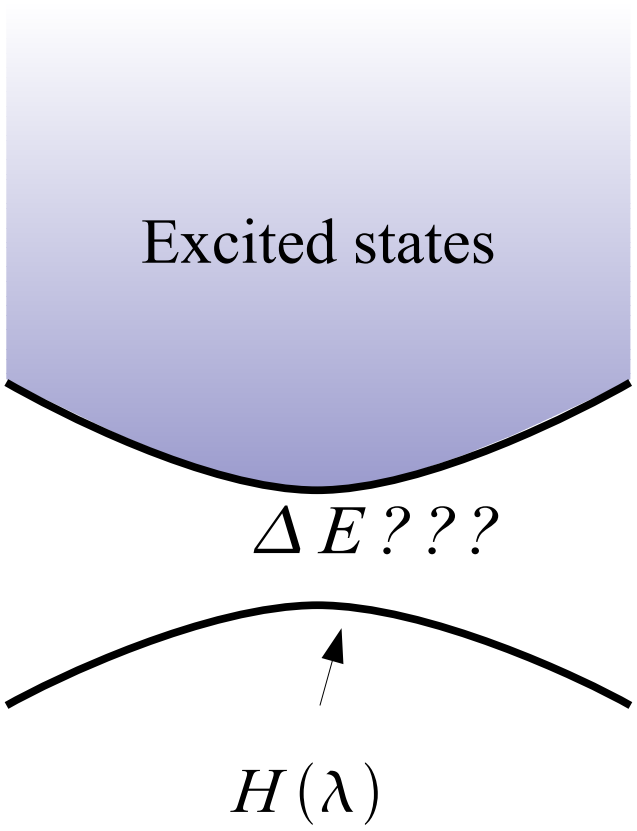
$$\Delta E=\sqrt{1-2\lambda(\lambda-1)(\cos\theta+1)}$$

- For our particular choice of state, the angle is

$$\cos(\theta/2)^2=\frac{N-1}{N}$$

$$\cos(\theta)=1-\frac{2}{N}$$

Full Hamiltonian



- Summing up

$$\Delta E = \sqrt{1 - 2\lambda(\lambda - 1)(\cos\theta + 1)}$$

$$\cos(\theta) = 1 - \frac{2}{N}$$

$$\lambda_{min} = \frac{1}{2}$$

$$\Delta E_{min} = \frac{1}{\sqrt{N}}$$

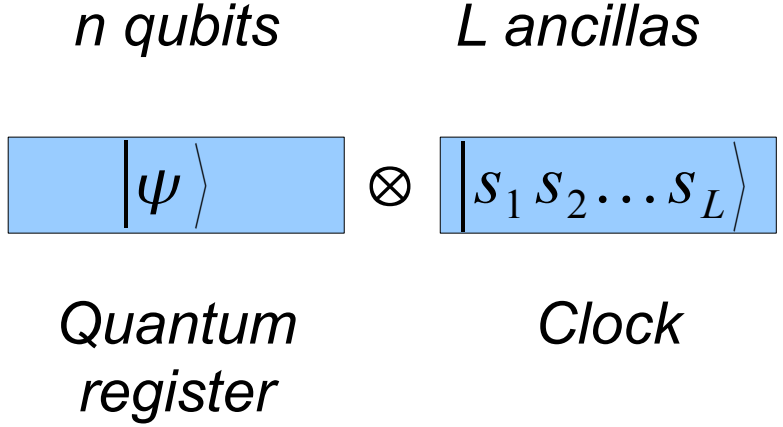
and time behaves like in Grover

$$T \geq O(\sqrt{N})$$

Why the interest?

1) Universality

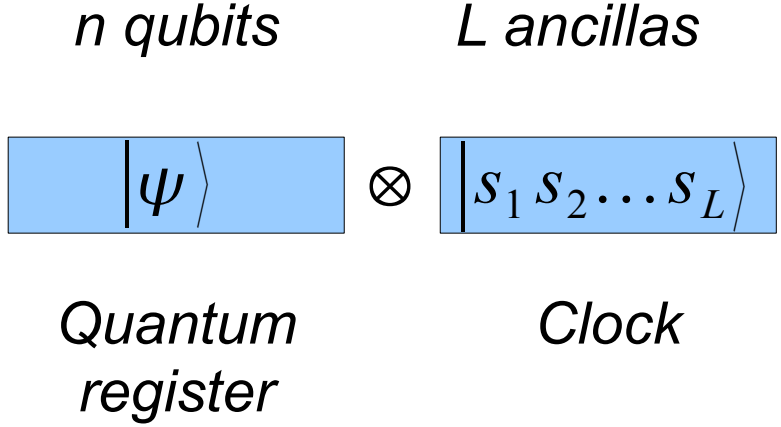
- Adiabatic quantum computation is as powerful as the circuit model.
- For every quantum circuit there exists an adiabatic Hamiltonian and viceversa.
- The overhead in the equivalence is polynomial
  - in time (adiab. resource)
  - in gates and ancillas (circuit resource)



D. Aharonov et al, Found. Comp. Science (2004)

1) Universality

- The proof in one sense uses Trotter decompositions
  - Approximate adiabatic evolution using quantum gates.
- On the other side we use
  - additional qubits to form a "clock"
  - a Hamiltonian that implements the unitaries
  - a Hamiltonian that forces the clock states to be consistent



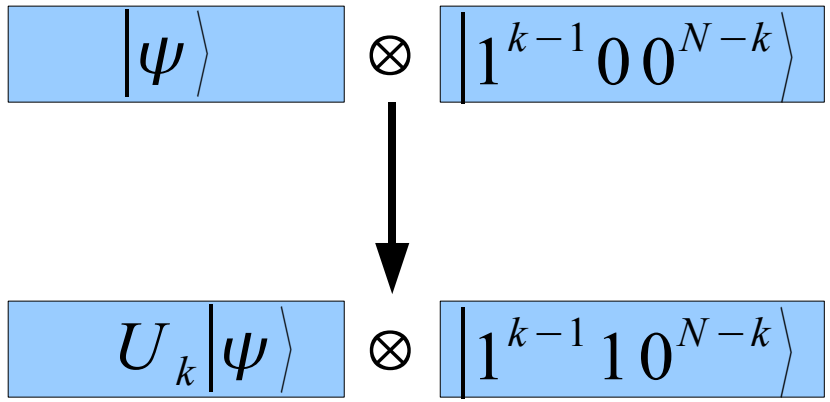
D. Aharonov et al, Found. Comp. Science (2004)

1) Universality

- The unitary Hamiltonian imposes that transitions only happen for the right clock states

$$
\begin{aligned}
H_k = & I \otimes |100\rangle\langle 100|_{k-1,k,k+1} \\
& + I \otimes |110\rangle\langle 110|_{k-1,k,k+1} \\
& + U_k \otimes |110\rangle\langle 100|_{k-1,k,k+1} \\
& + U_k^{+} \otimes |100\rangle\langle 110|_{k-1,k,k+1}
\end{aligned}
$$

k-th circuit step



D. Aharonov et al, Found. Comp. Science (2004)

1) Universality

- The unitary Hamiltonian imposes that transitions only happen for the right clock states

$$H_k = I \otimes |1\,0\,0\rangle\langle 1\,0\,0|_{k-1,k,k+1} + I \otimes |1\,1\,0\rangle\langle 1\,1\,0|_{k-1,k,k+1} + U_k \otimes |1\,1\,0\rangle\langle 1\,0\,0|_{k-1,k,k+1} + U_k^{+} \otimes |1\,0\,0\rangle\langle 1\,1\,0|_{k-1,k,k+1}$$

- Another term forbids wrong clock states

$$H_{clock} = \sum_k |0\,1\rangle\langle 0\,1|_{kk+1}$$

consistent clock

$|\psi\rangle \otimes |1\ldots01\ldots\rangle$

NO!

D. Aharonov et al, Found. Comp. Science (2004)

1) Universality

- Another term imposes the initial state at zero clock

$$H_{init} = \left[\sum_i |1\rangle\langle 1|_i \right] \otimes |0\rangle\langle 0|_0^{clock}$$

- And we have to impose initially the clock is zero

$$H_{clock0} = \sum_k |1\rangle\langle 1|_1^{clock}$$

initial state

$$|0 \ldots 0\rangle \otimes |0 \ldots 0\rangle$$

D. Aharonov et al, Found. Comp. Science (2004)

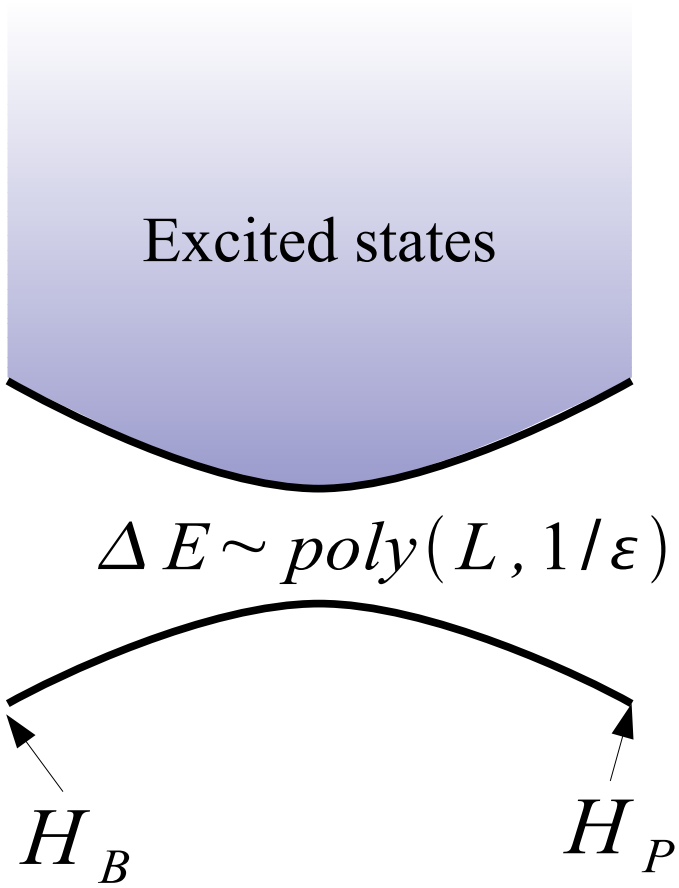
1) Universality

- Put together

$$H_B = H_{clock0} + H_{init} + H_{clock}$$

$$H_P = \sum_k H_k + H_{init} + H_{clock}$$

- The merit of the proof is to show that the gap is polynomial in the problem size, L, and inverse tolerance
  - Time increases with the number of gates still!



D. Aharonov et al, Found. Comp. Science (2004)

2) Robustness

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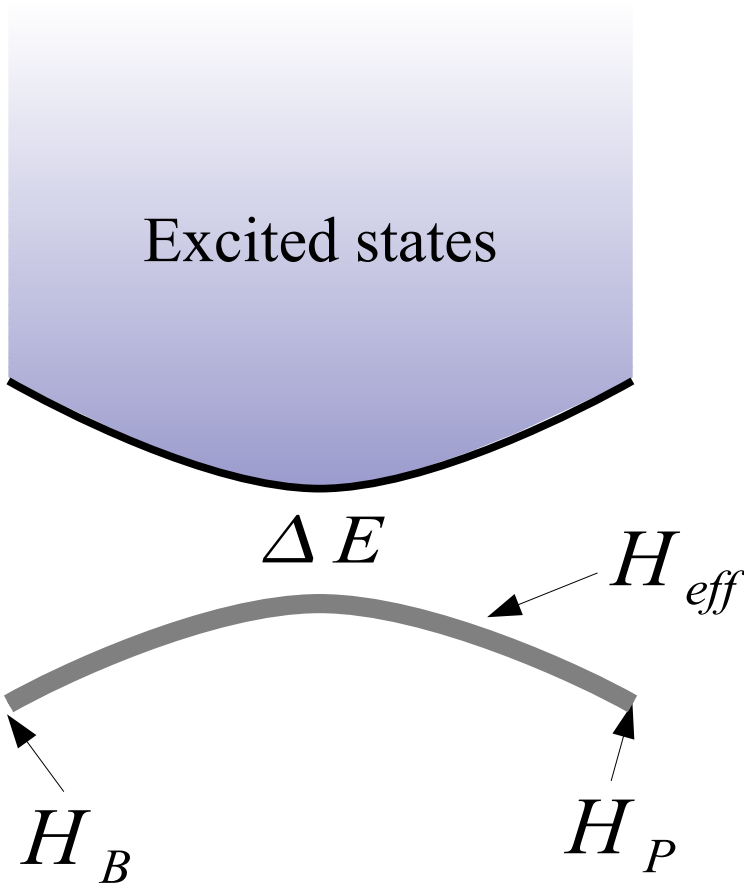


- Remember perturbation theory:
  - We have two separate regions in spectrum.
  - If perturbations are small, they remain disconnected.
  - At most a new effective Hamiltonian for the ground state manyfold

$$H_{eff} = (1-\lambda) H_B + \lambda H_P + \\ + \epsilon H_{eff}$$

  - At most mixing of ground states.

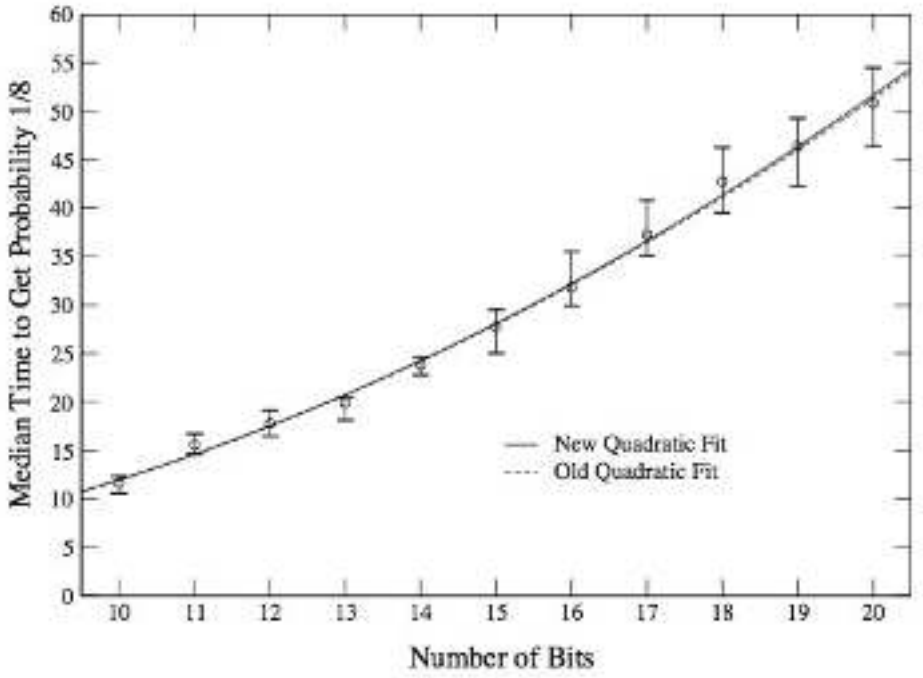
Real life

Local Hamiltonians

- What mixing Hamiltonians are experimentally realizable?

E. Farhi et al, Science **20**, 472-475 (2001)

Local Hamiltonians



- What mixing Hamiltonians are experimentally realizable?
- We have use a very nonlocal one

$$H_B = |\psi\rangle\langle\psi| = \frac{1}{N}\sum_{x,\,y=0}^{N} |x\rangle\langle y|$$

- An alternative would be

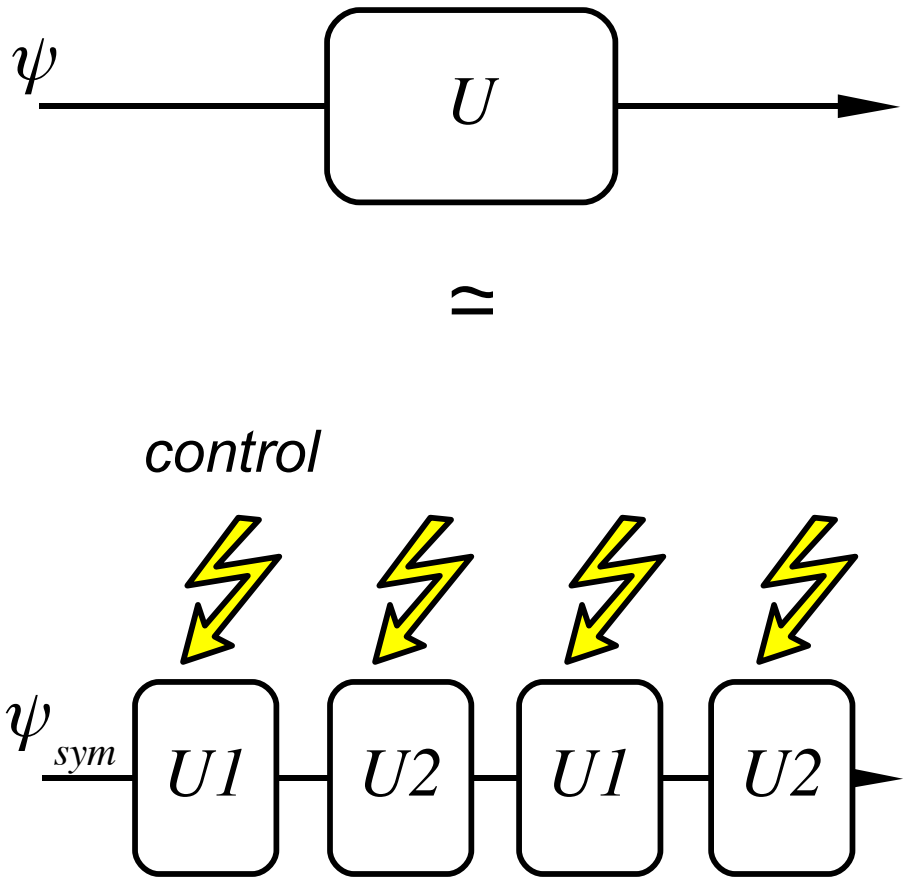
$$H_B = \sum_{i=0}^{n} \sigma_i^x$$

  - Computing gap is difficult
  - Little numerical evidence

E. Farhi et al, Science **20**, 472-475 (2001)

Practical implementations

Simulated implementation



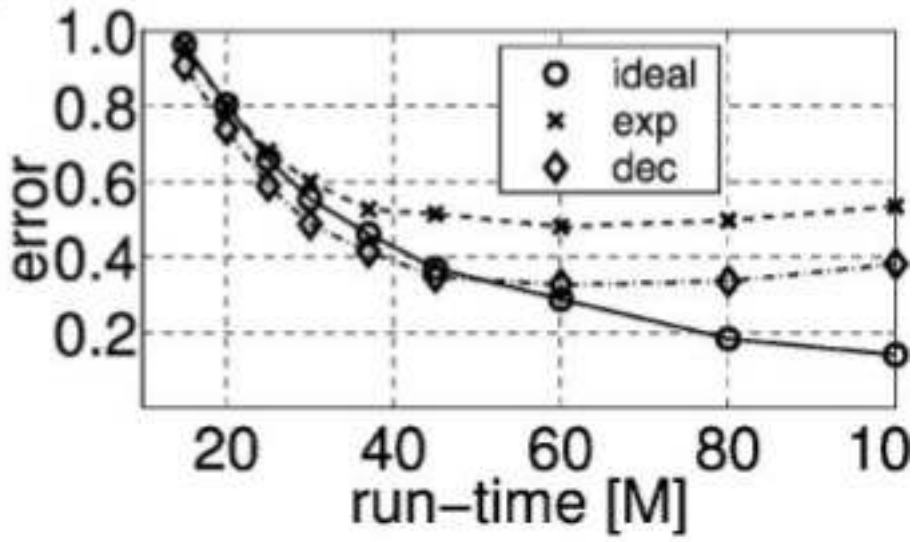
- An adiabatic evolution can be simulated using two alternating Hamiltonians

$$U(T) \sim \prod_n \exp\left\{ i \left[(1-\lambda_n) H_B + \lambda_n H_P \right] \right\}$$
$$\sim \prod_n \exp\left\{ i (1-\lambda_n) H_B \right\} \times$$
$$\times \exp\left\{ i \lambda_n H_P \right\}$$

- This introduces additional errors, but we expect the effective “gap” will suppress them.

Steffen et al, PRL **90** 067903 (2003)

NMR adiabatic algorithm



- Molecule: H F C Br2
- Effective spin-spin interaction between H, F and C nuclei

$$H = \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z + \sum_i \left[B \sigma_i^z + \vec{B}(t) \cdot \vec{\sigma} \right]$$

- With a suitable modulation of the transverse mag. field, we can simulate both the problem

$$H_P = \sum_{i,j} f(\sigma_i^z, \sigma_j^z)$$

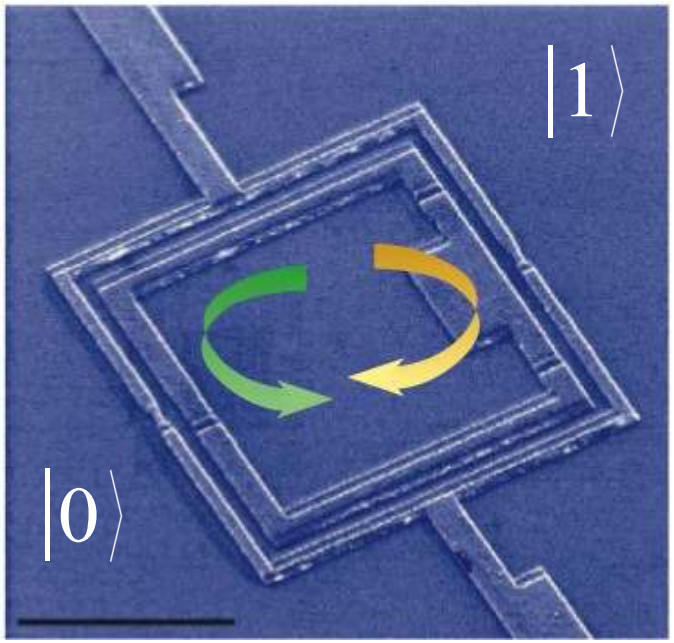
and mixing Hamiltonians

$$H_B = \sum_i \sigma_i^x$$

E. Farhi et al, Science **20**, 472-475 (2001)

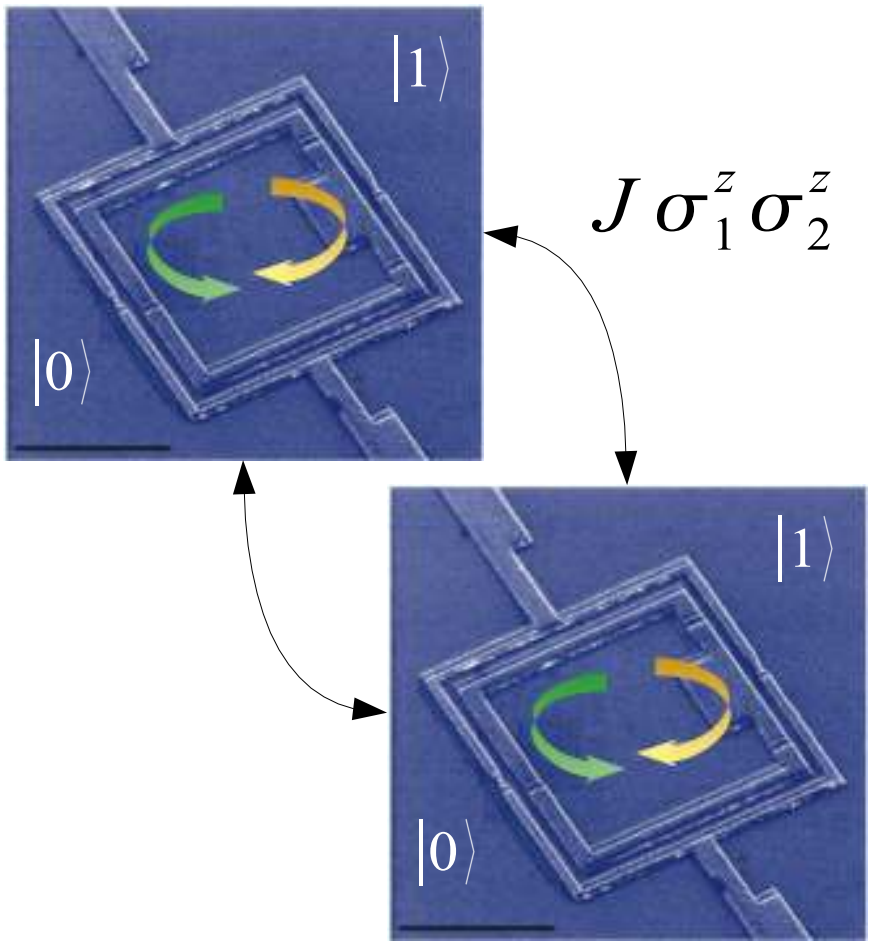
“Real” adiabatic computation

Superconducting circuits



- SQUIDS are superconducting loops.
- They can host long lived currents.
- The sense of a current can implement a qubit state

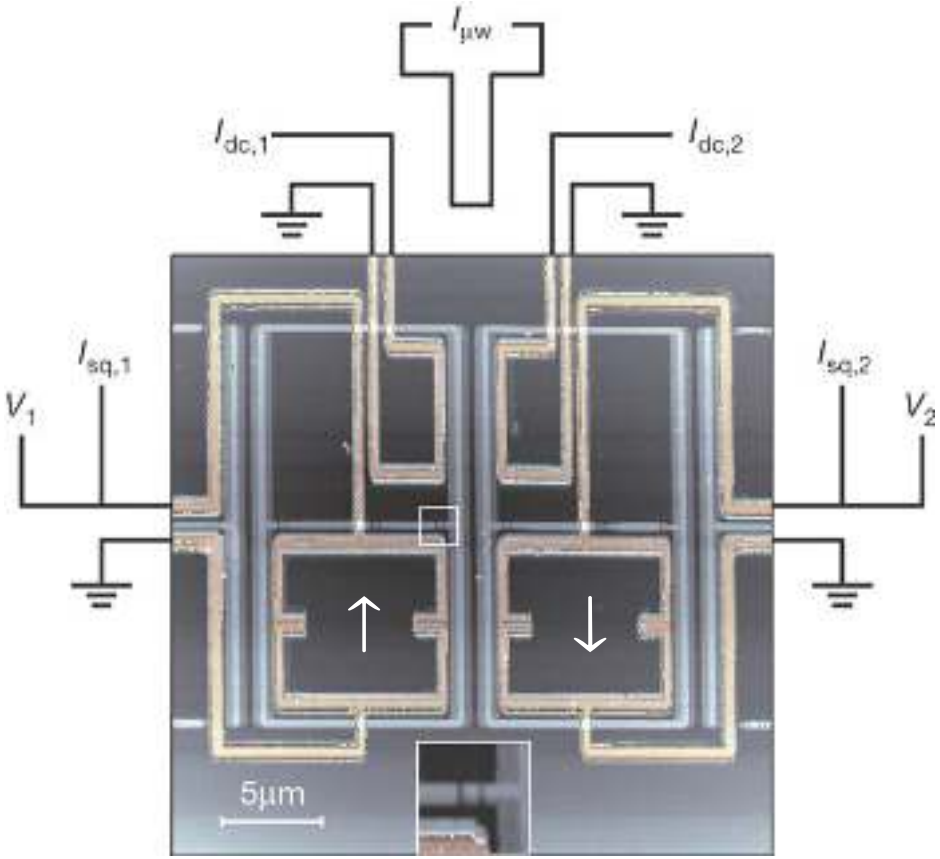
Superconducting circuits



- SQUIDS are superconducting loops.
- They can host long lived currents.
- The sense of a current can implement a qubit state
- These currents behave like small **magnets** giving rise to a magnetic interaction

$$H \sim \sum_{i,j} J_{i,j}\sigma_1^z\sigma_2^z + \sum h_i^{ext}\sigma_i^x$$

Superconducting circuits



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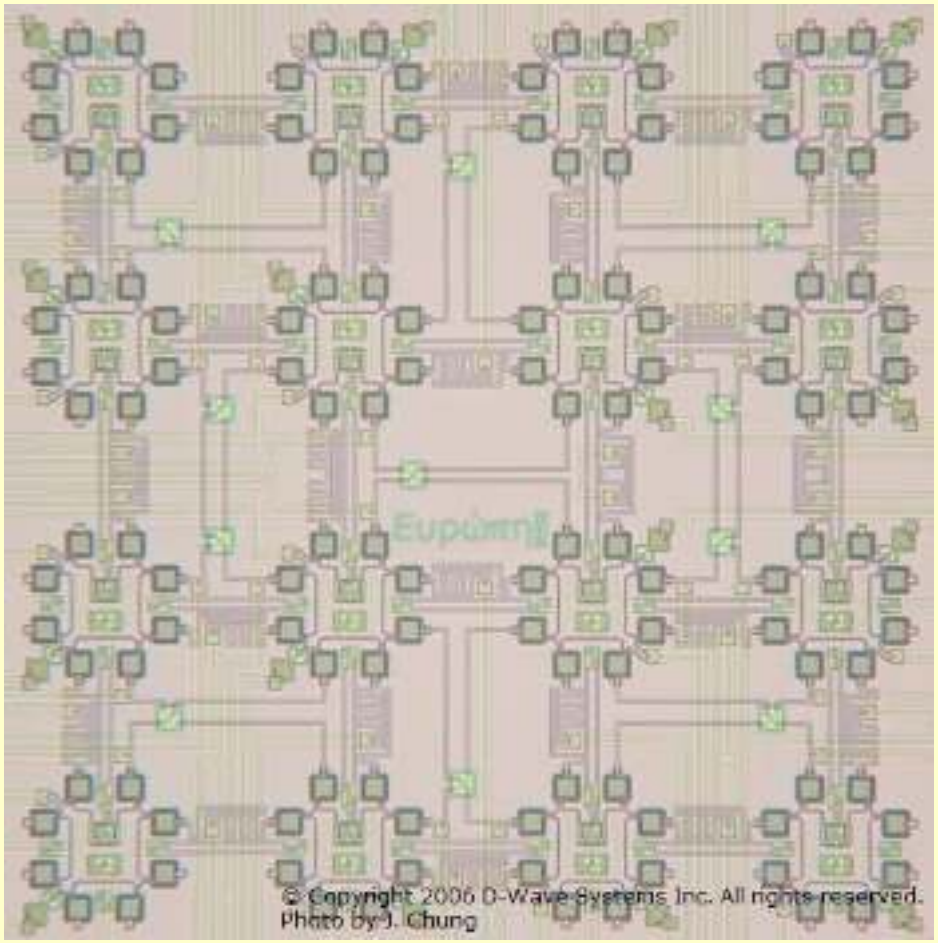
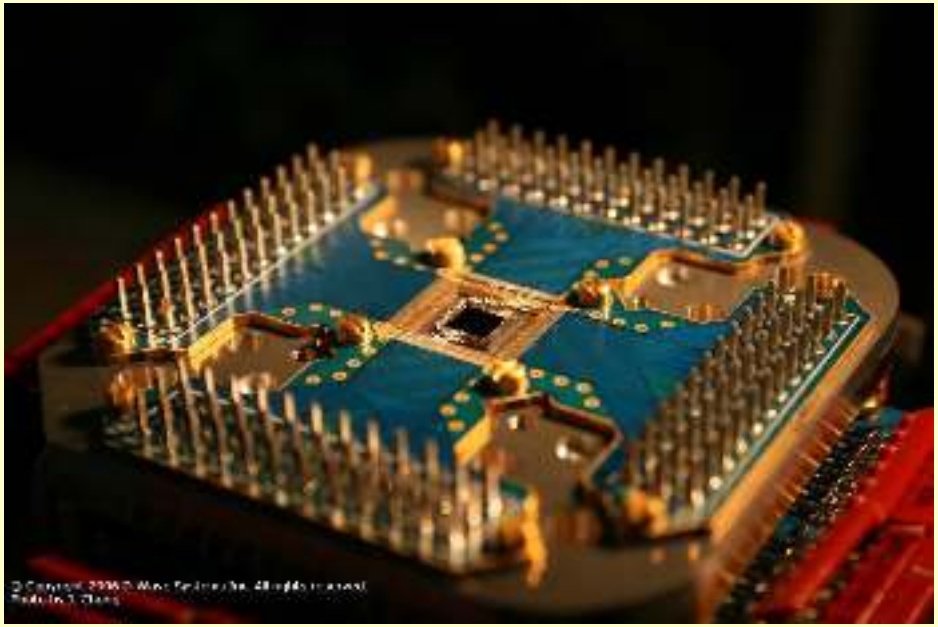
J. H. Plantenberg et al, Nature **447**, 836-839 (2007)

Enter D-Wave



- 4x4 superconducting qubits of flux or phase type.
- Claims:
  - Adjustable interactions.
  - Implements adiabatic quantum algorithms
  - Should solve optimization problems.

Enter D-Wave



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Enter D-Wave

SCIENTIFIC AMERICAN

News - February 13, 2007

First "Commercial" Quantum Computer Solves Sudoku Puzzles

Quantum computing company banks on a long-shot form of quantum computing

c|net NEWS.COM http://www.news.com/

Start-up demos quantum computer

By Michael Kanellos

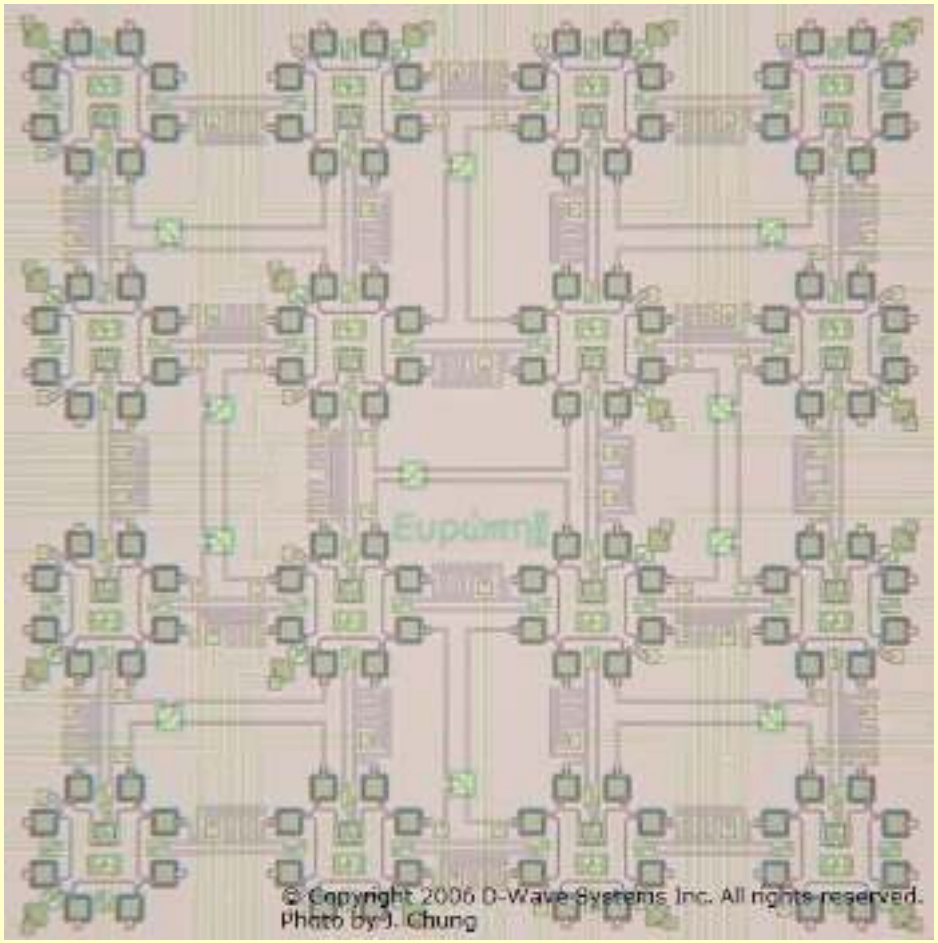
The New York Times
nytimes.com

April 8, 2007

SLIPSTREAM

A Giant Leap Forward in Computing?

Problems



- Adiabatic algorithm requires coherence
  - Otherwise annealing
- In D-Wave qubits are closer to classical bits with noise.
- But we cannot discard it as a possibility!