Superconductors: Quantum circuits

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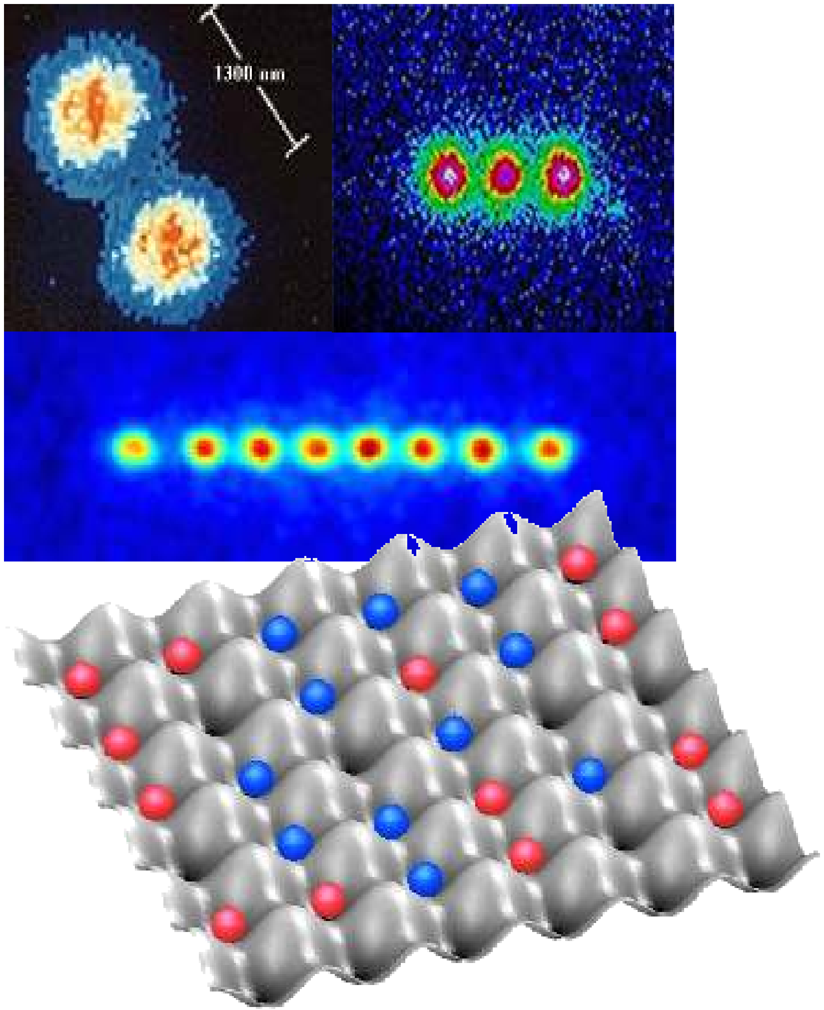
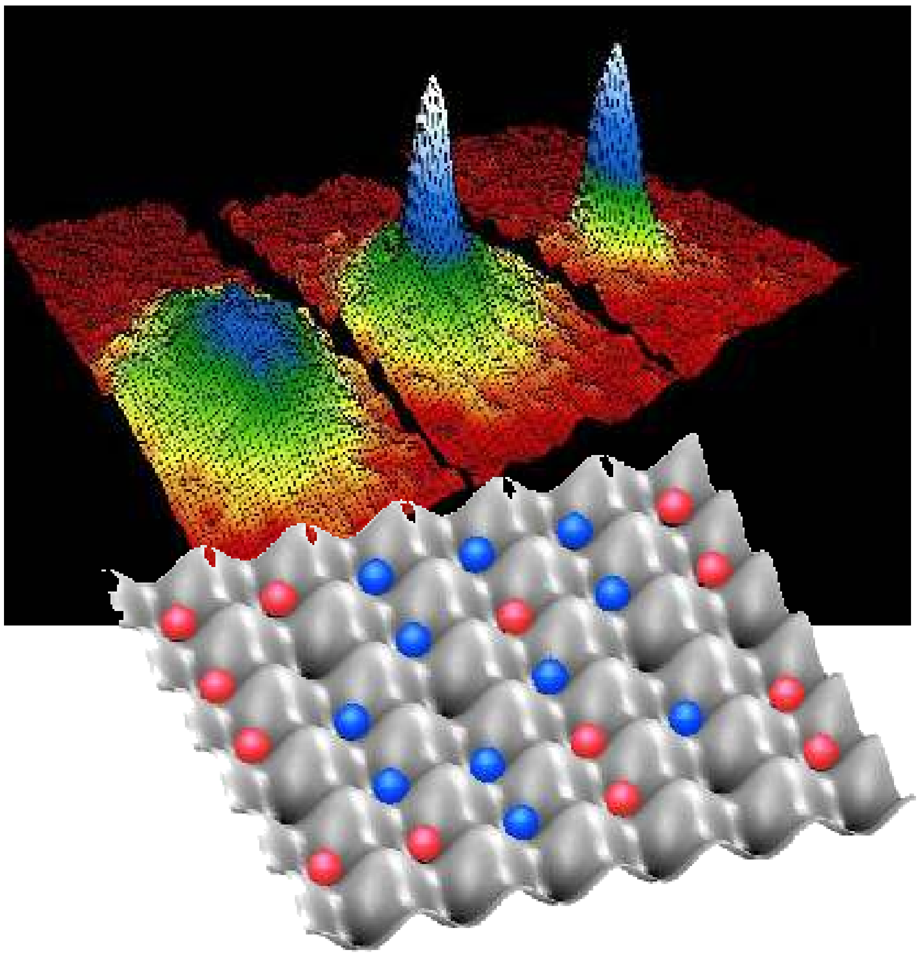
Mesoscopic QIPC

Small systems



- So far we have only seen small systems to store and process QI
  - Individual atoms
  - As trapped ions
  - As pseudo-crystals
- Other successful systems which are very similar
  - Molecules
  - For NMR
  - Or for AMO type exp.

But not so small

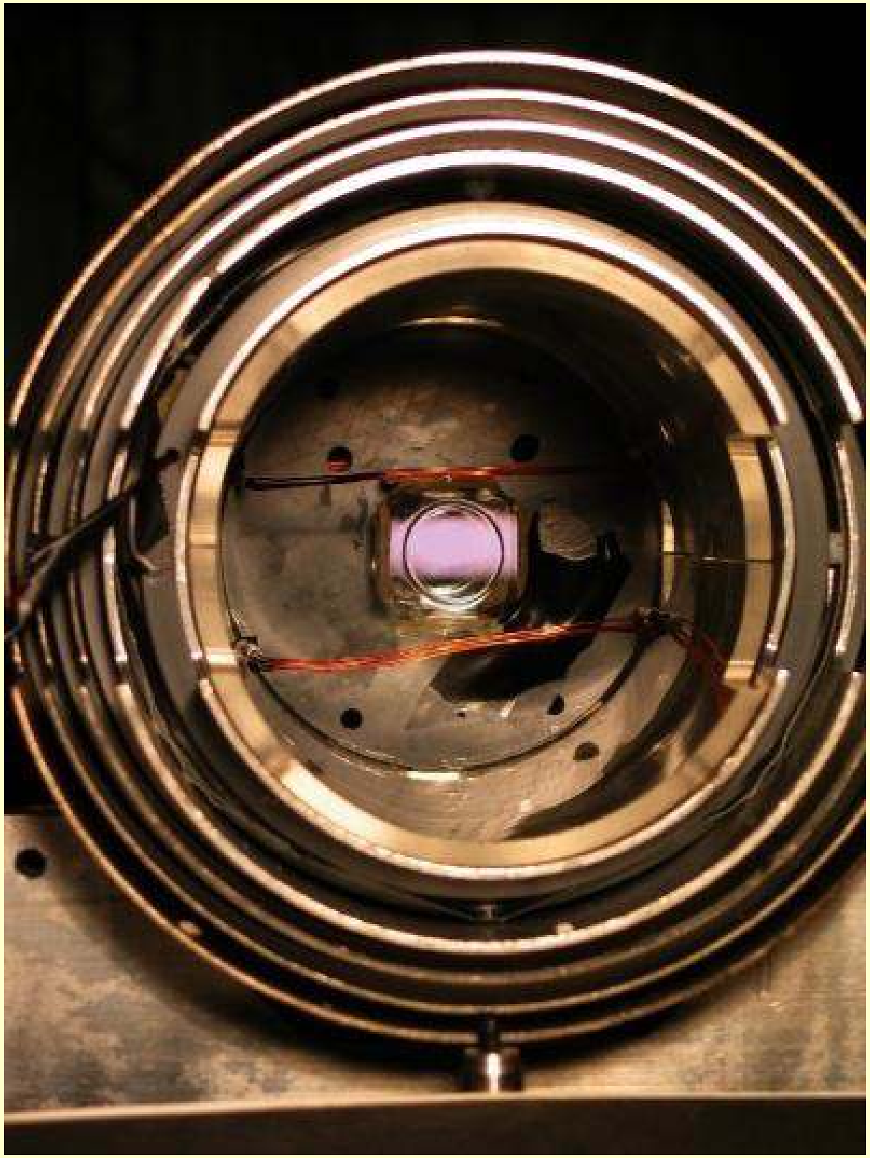


- Even though each individual register qubit is small, we have a mesoscopic number of them
  - $10^5 - 10^6$ atoms
- So many atoms that the law of averages works!
- We can define collective observables

$$L_z = \frac{1}{2} \sum_i \sigma_i^z$$

where quantum fluctuations can be measured.

Atomic ensembles



- Trap a gass of atoms in a glass cell.
- Polarize them

$$\langle L_z \rangle = \frac{1}{2} \sum_i \langle \sigma_i^z \rangle \sim N/2$$

- Study the fluctuations of transverse components

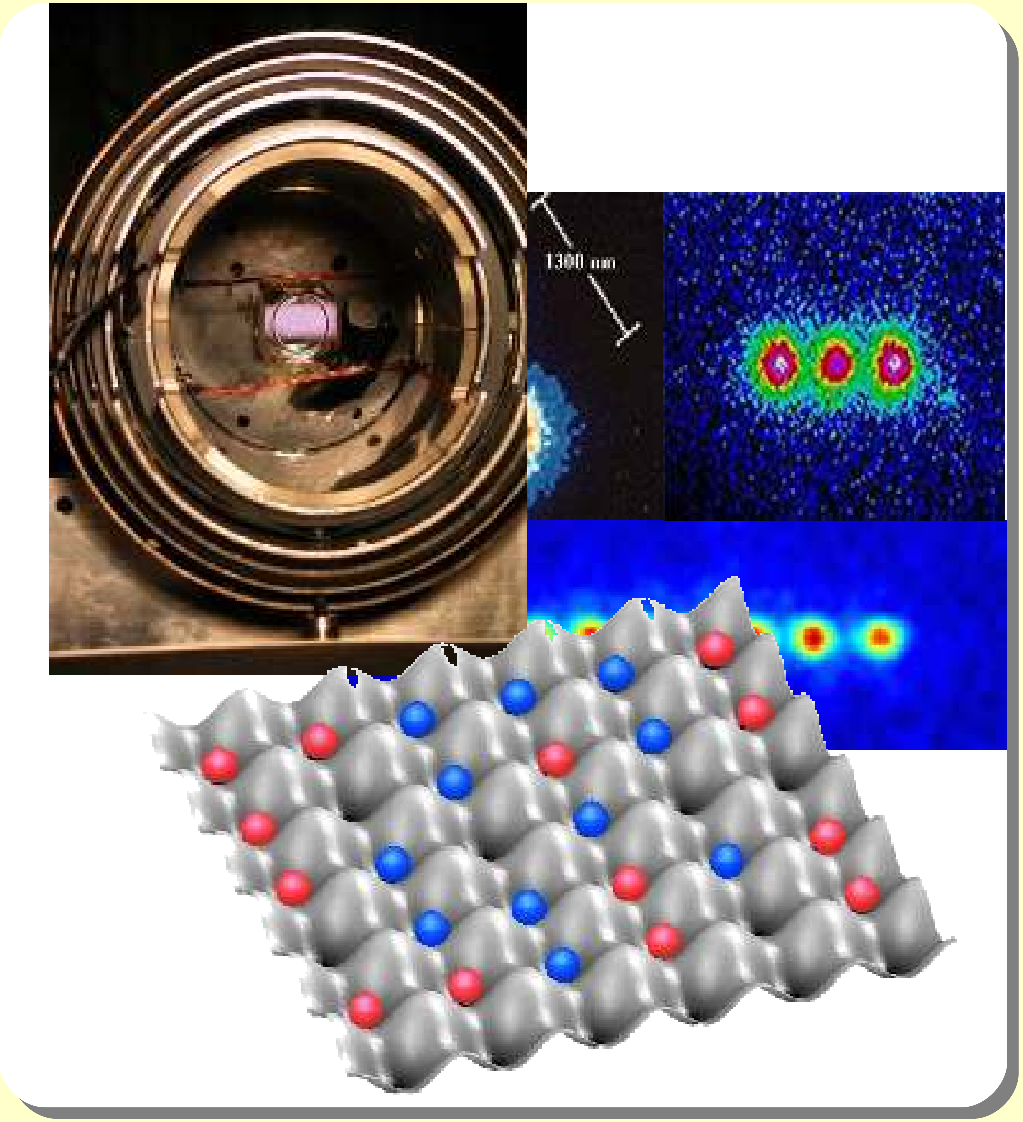
$$[L_x, L_y] = i\hbar L_z \sim i\hbar N/2$$

$$Q \sim L_x \sqrt{2/N} \quad \langle Q \rangle = 0$$

$$P \sim L_y / \sqrt{2/N} \quad \langle P \rangle = 0$$

- Use them for continuous variables QIPC.

Atomic ensembles



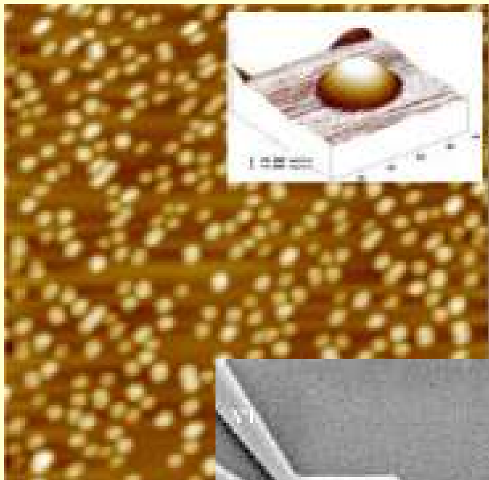
- All these systems (ions, atoms, molecules) are ideal
  - negligible or controlled interactions
  - isolated,
  - little or no dissipation.
- Can we use real-life materials and systems for QIPC in a coherent way?
  - What about interactions
  - What about T?
  - What about decoherence

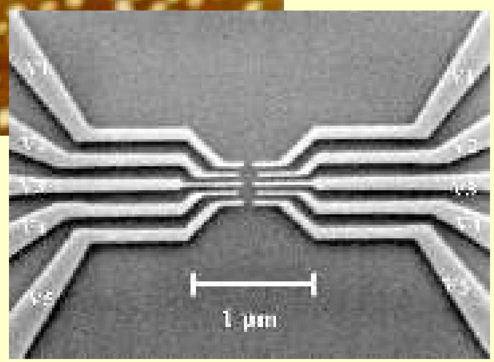
Mesoscopic quantum systems

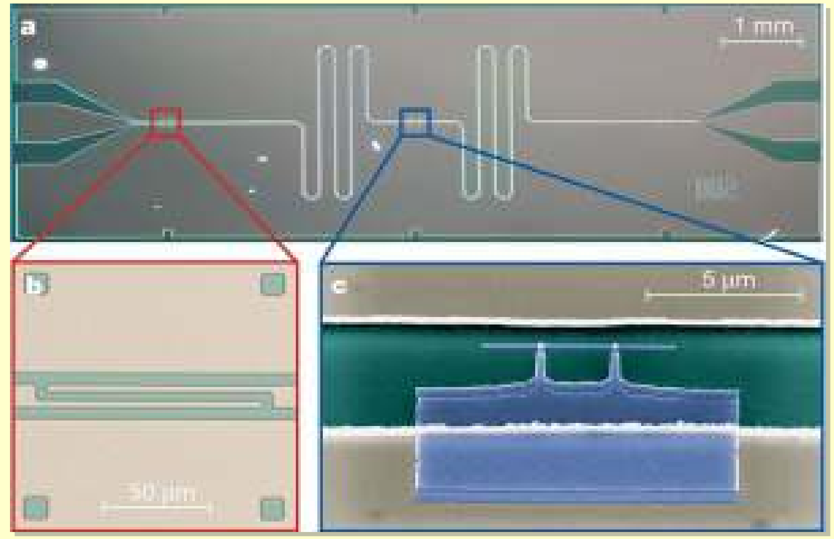
Mesoscopic quantum systems

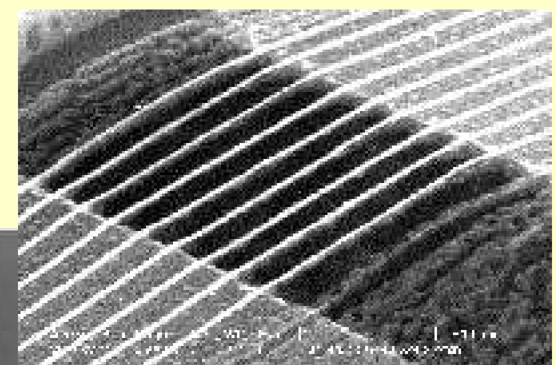
- Any system is candidate for QIPC if suitable degrees of freedom exist.
- What matters
  - Energy scales of the degree of freedom.
  - Temperature.
- In addition to this
  - Decoherence time scales
  - Dimensionality
  - Controllability

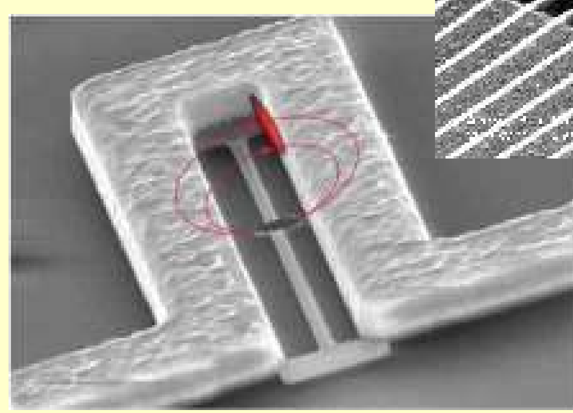
Mesoscopic quantum systems







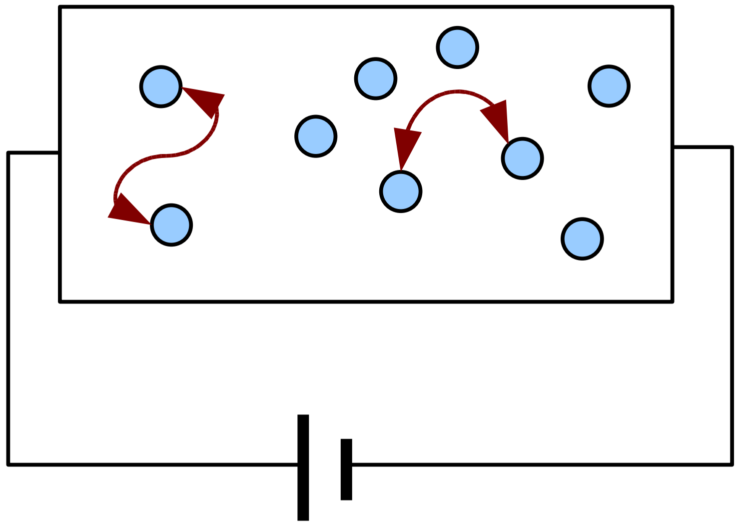




A lot of systems in the last years have reached the quantum degenerate limit.

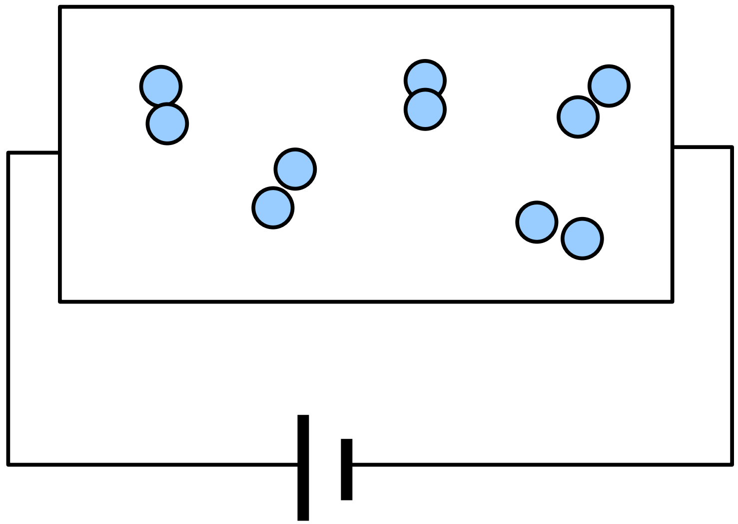
Superconductors

Superconductivity



- Electrons are the charge carriers in a solid.
- They may interact through the phonons.
- This attraction couples pairs of fermions.

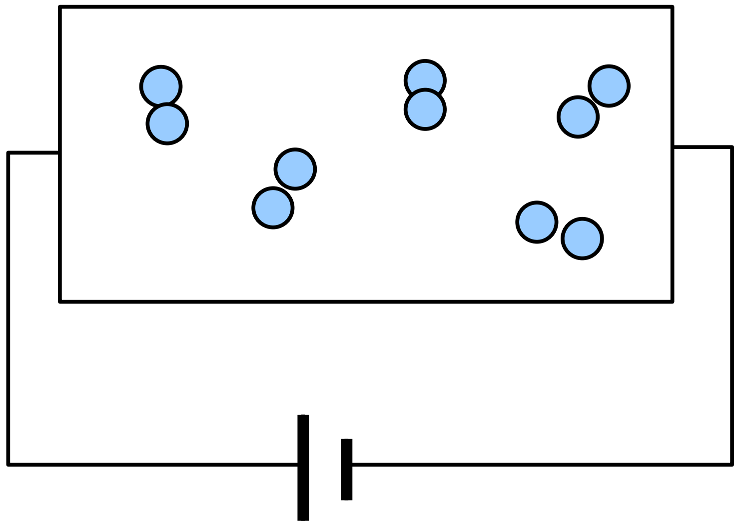
Superconductivity



Cooper pair
(according to Google)

- Electrons are the charge carriers in a solid.
- They may interact through the phonons.
- This attraction couples pairs of fermions.

Superconductivity



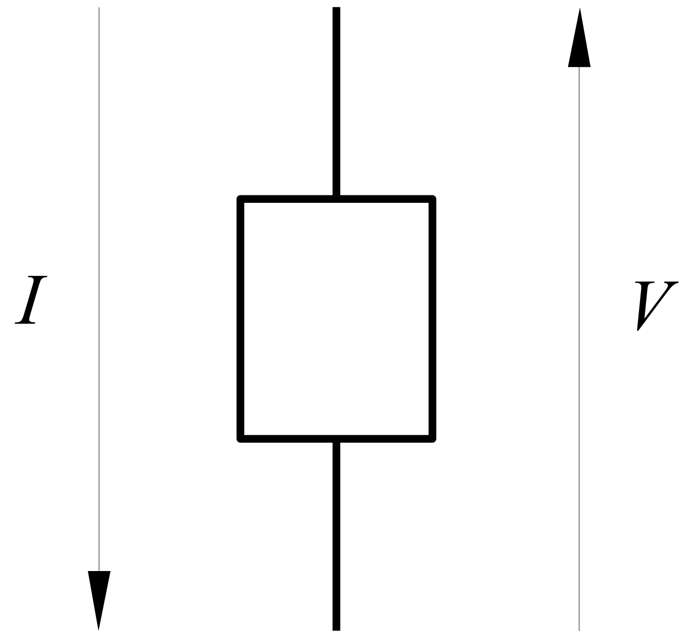
$$\prod_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}\right)\left|vac\right\rangle$$

Cooper pair
(sort of)

- Electrons are the charge carriers in a solid.
- They may interact through the phonons.
- This attraction couples pairs of fermions.
  - Bosonic particles
  - Charge carriers
  - Superfluid
- Result: **a conductor without resistance.**

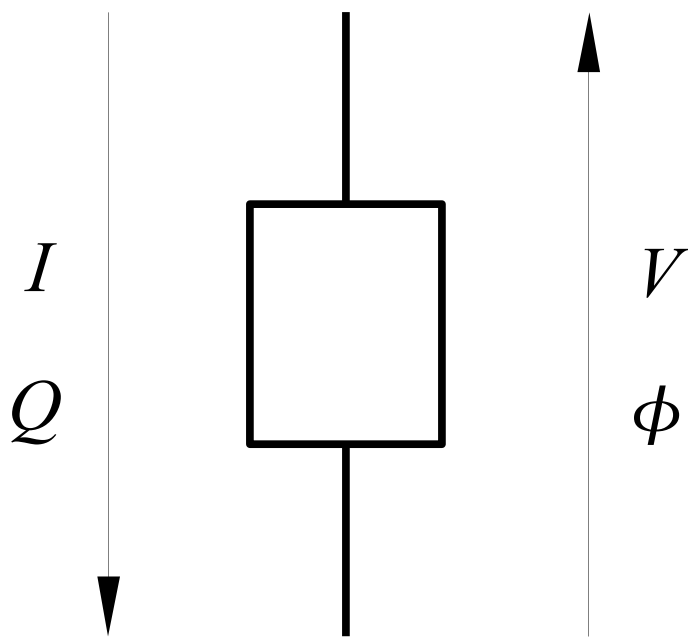
Quantum degrees of freedom

Electric variables



- We can use
  - Electric potential, V
  - Current intensity, I

Electric variables



- We can use
  - Electric potential, V
  - Current intensity, I
- Or their integrals
  - Flux, ϕ

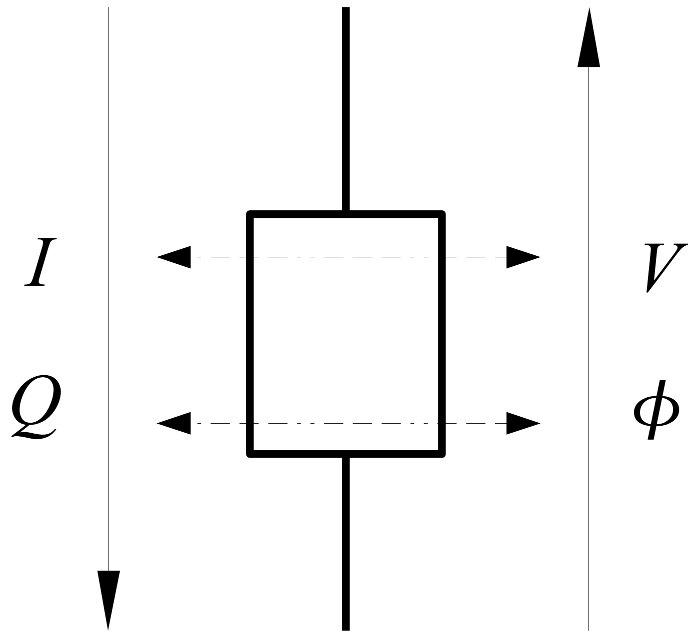
$$\phi = \int_{-\infty}^{t} V(t)\,dt$$

  - Charge, Q

$$Q = \int_{-\infty}^{t} I(t)\,dt$$

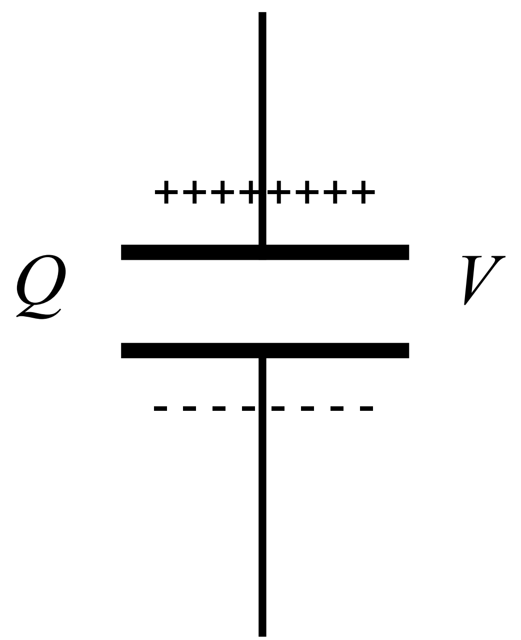
- All of them, macroscopic.

Linear response



- The different circuit elements are described by one relation between variables.

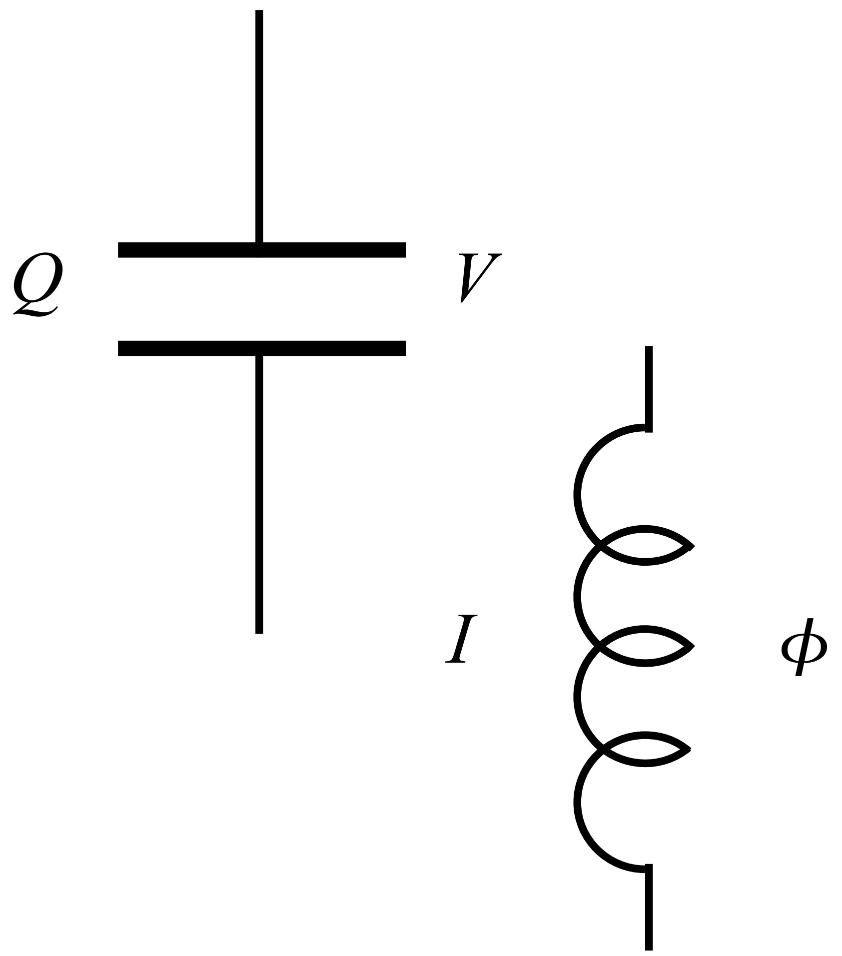
Linear response



- The different circuit elements are described by one relation between variables.
- Capacitive elements

$$V = g(Q)$$

Linear response



- The different circuit elements are described by one relation between variables.
- Capacitive elements

$$V = g(Q)$$

- Inductive elements

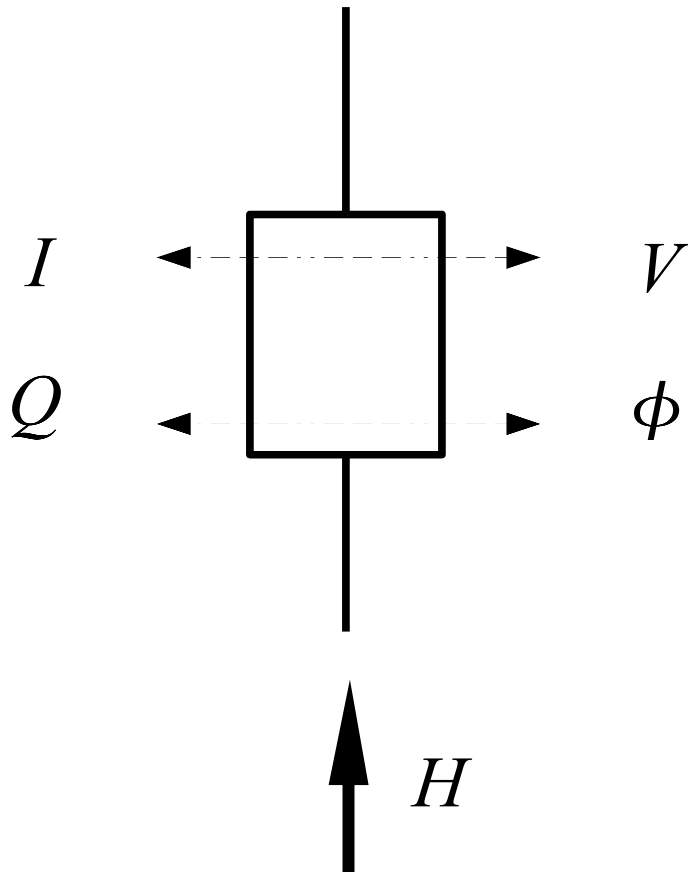
$$I = f(\phi)$$

- Linear cases
  - Capacitance(C) $V = Q/C$
  - Inductance(L) $I = \phi/L$

Energies

Where does that come from?

Power flow



- The work or power that flows into a circuital element is given by

$$\dot{H} = V\,I = V\,\dot{Q} = \dot{\phi}\,I$$

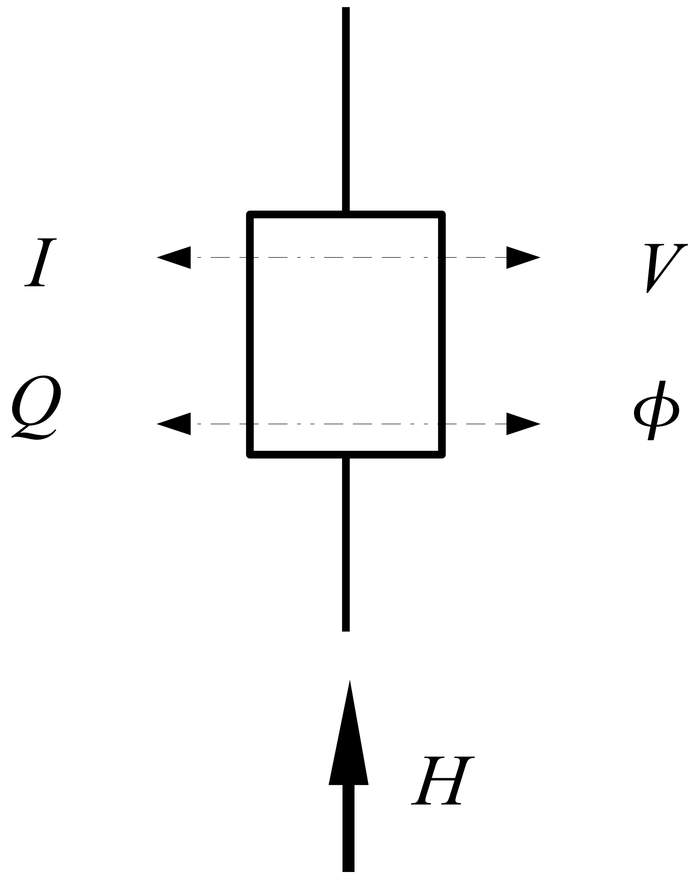
- Using the previous relations

$$H_{cap} = \int_0^{Q_{end}} g(Q)\,dQ$$

$$H_{ind} = \int_0^{\phi_{end}} f(\phi)\,d\phi$$

for capacitive and inductive elements.

Quadratic approximations



- If we use only the linear part of "f" and "g" we obtain

$$H_{cap} = \frac{Q^2}{2C}$$

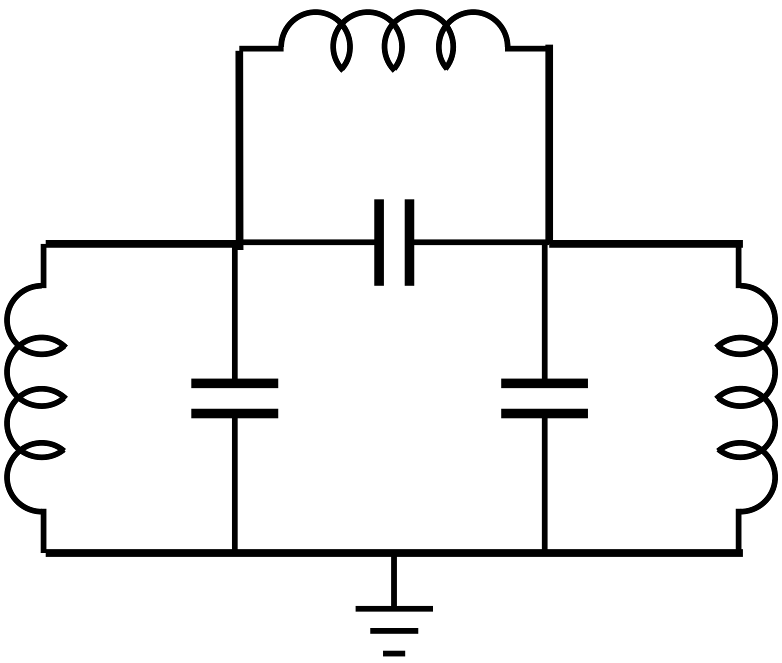
$$H_{ind} = \frac{\phi^2}{2L}$$

- These can be understood as the quadratic approximations of the electric and magnetic energies.

Circuit quantization

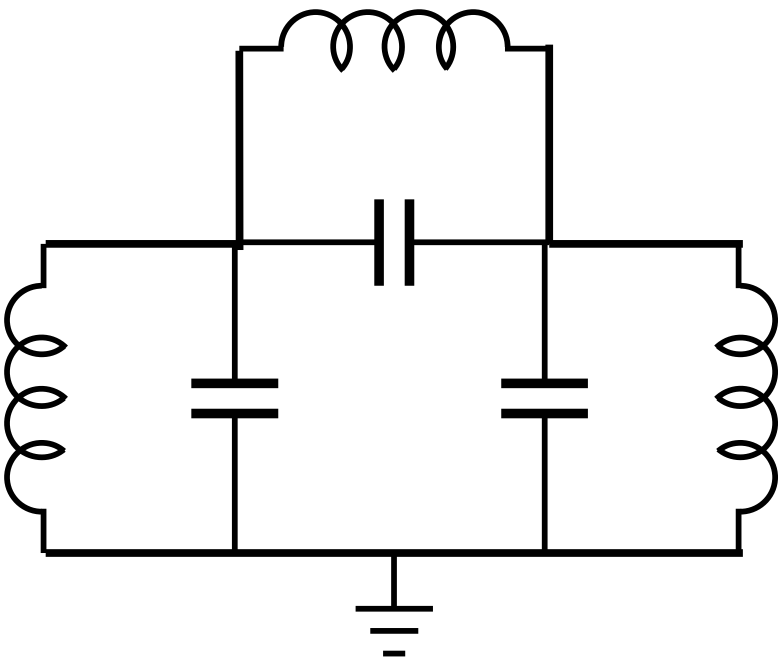
How to obtain the quantum degrees of freedom

Outline: Lagrangian



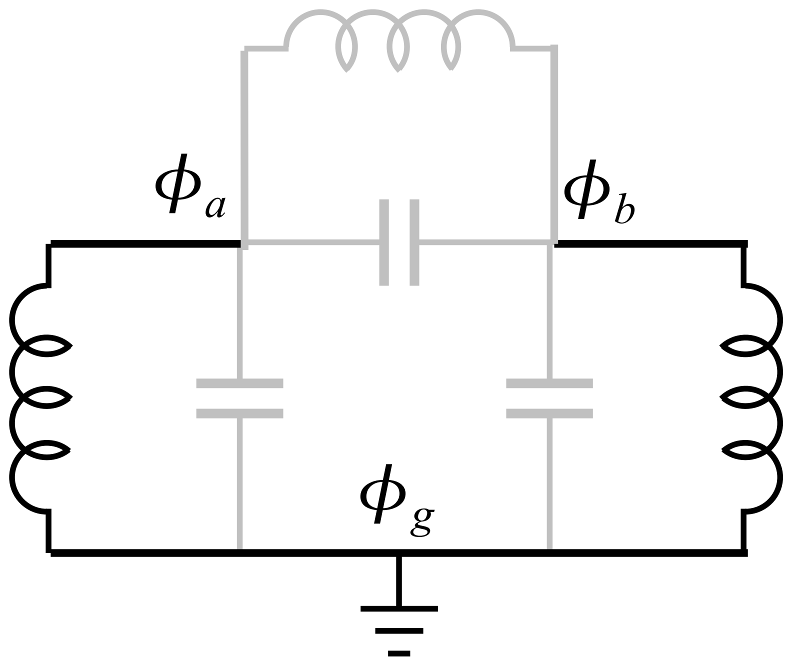
- Quantization prescription:
  - Choose set of quantum variables
  - Find evolution equations
  - Find the originating Lagrangian
  - Compute the Hamiltonian
  - Find canonical variables
  - Impose commutation relations
  - (Solve quadratic H)

Step 1: identify variables



- Identify or define a ground point.
- Trace unique paths to all other vertices, following the inductors.
- Each line is associated a flux, charge, potential or intensity variable.
- Each vertex is given a topology-dependent value, the difference of which forms the branch values.

Step 1: identify variables



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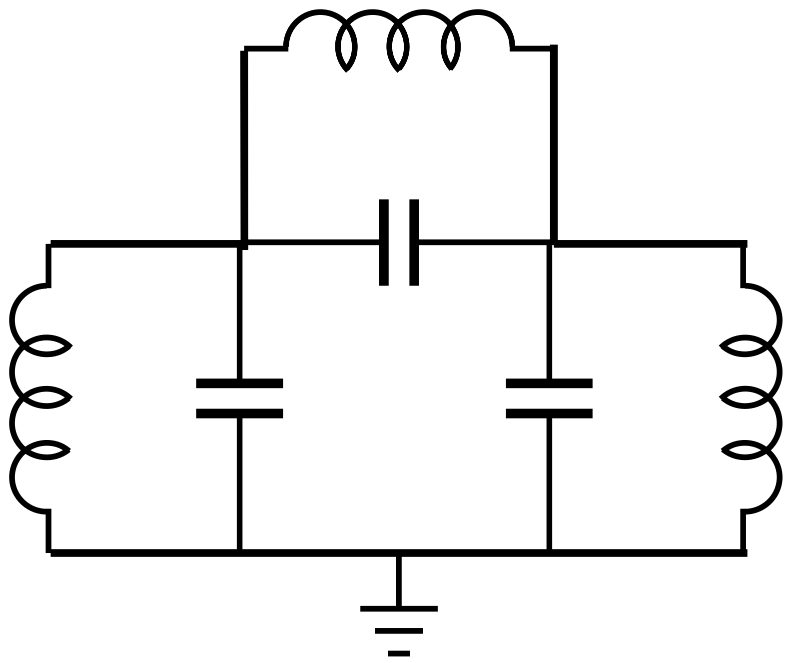
Step 2: Kirchoff laws



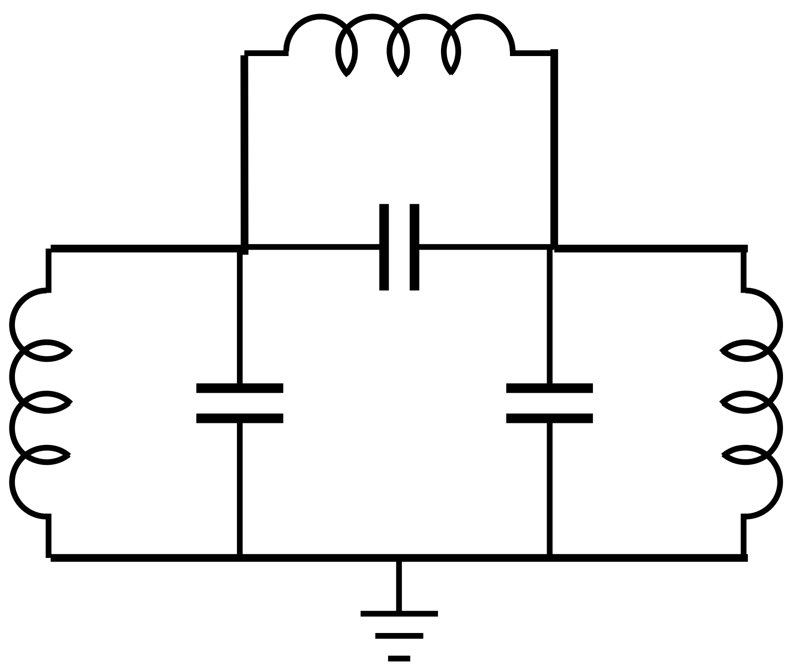
- Charge conservation:
  - The sum of intensities arriving to a node is zero

$$\sum_{b\ \text{arriving at}\ v} I_b = 0$$

- Voltage law
  - The sum of potential differences along a loop is zero

$$\sum_{b \in loop} V_b = 0$$

Step 2: Kirchoff laws



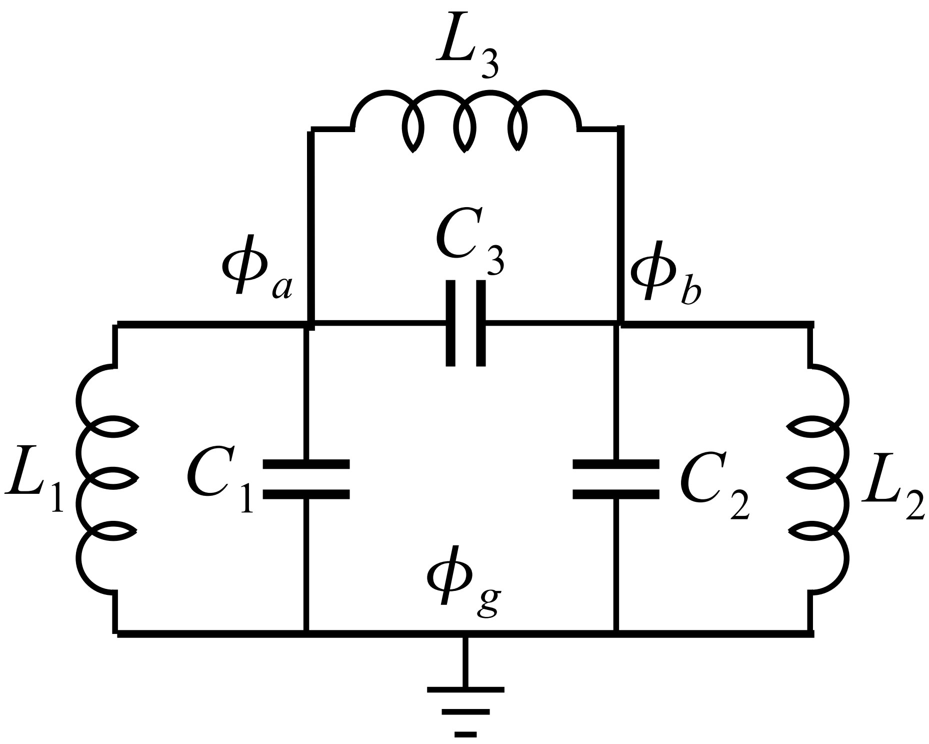
- Charge conservation:
  - The sum of charges arriving to a node is constant

$$\sum_{b\,arriving\,at\,v} Q_b = \tilde{Q}_v$$

- Voltage law
  - The sum of fluxes along a loop is constant

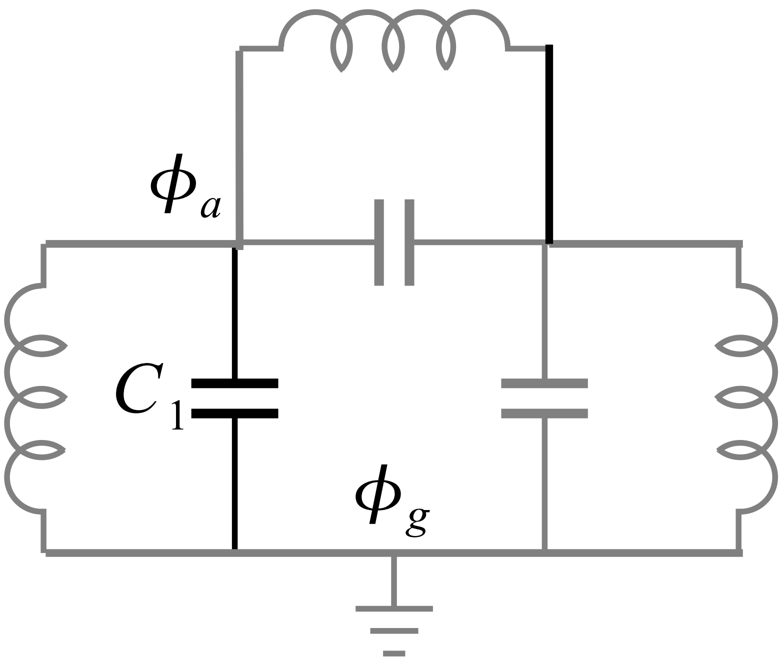
$$\sum_{b \in loop} \phi_b = \tilde{\phi}_l$$

Step 3: Evolution equations



- For each active node
  - Sum of currents arriving from inductors equal to
  - sum of currents going into capacitors.

Step 3: Evolution equations

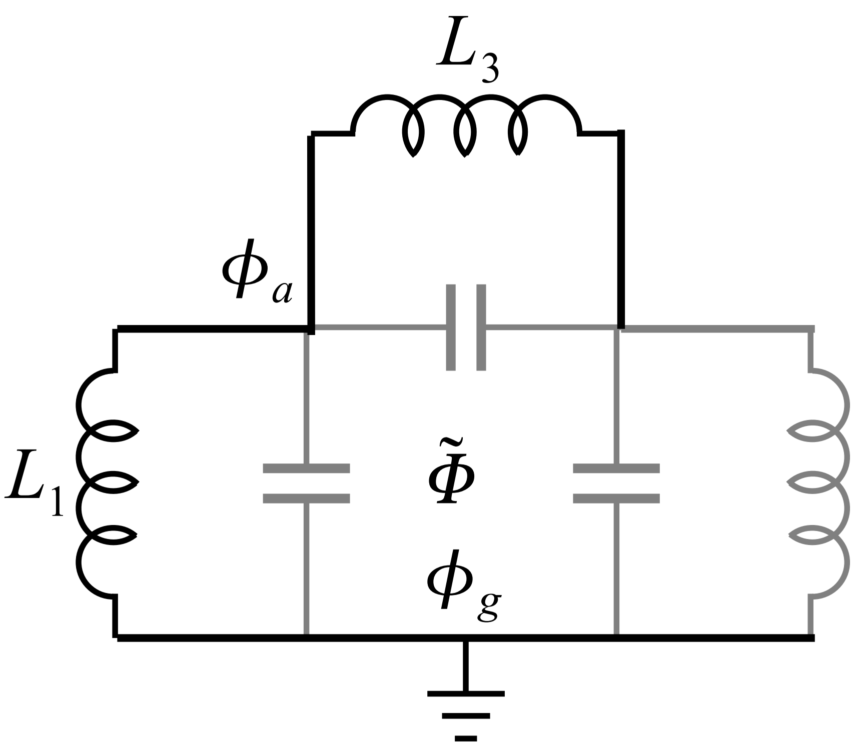


- For each active node
  - Sum of currents arriving from inductors equal to
  - sum of currents going into capacitors.
- Along capacitor 1

$$Q_1 = C_1 V_1$$

$$I_1 = C_1 \frac{d}{dt} V_1 = C_1 (\ddot{\phi}_a - \ddot{\phi}_g)$$
$$= C_1 \ddot{\phi}_a$$

Step 3: Evolution equations



- For the inductive element 1

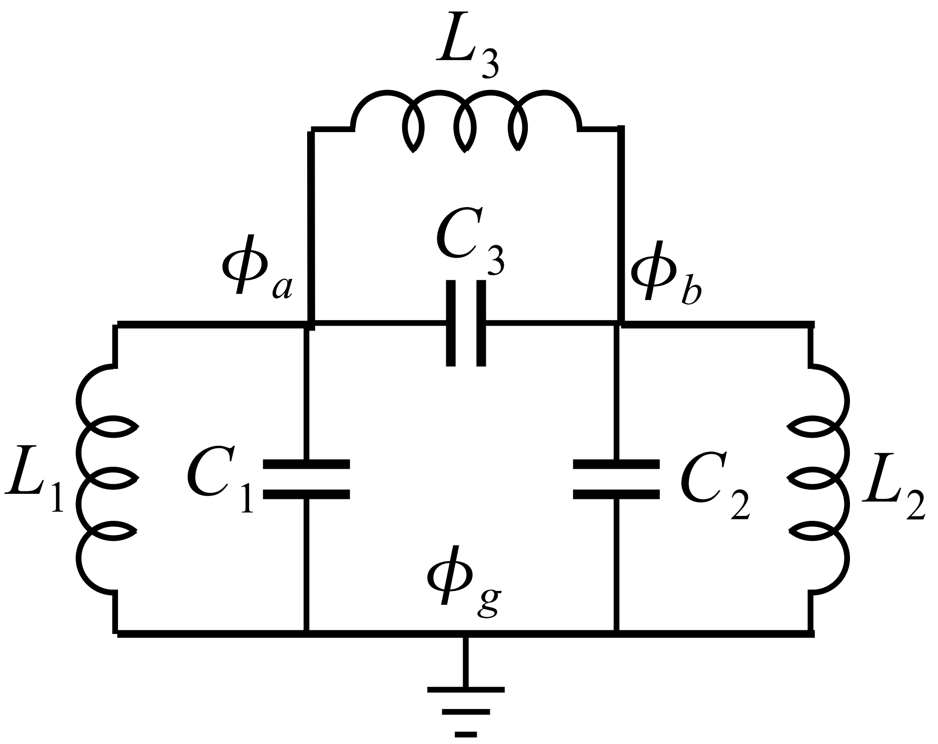
$$I_1 = (\phi_a - \phi_g) / L_1$$

- For the inductive element 3

$$I_3 = (\phi_a - \phi_b + \tilde{\Phi}) / L_3$$

where we have used the Kirchoff law to obtain the upper flux.

Step 3: Evolution equations

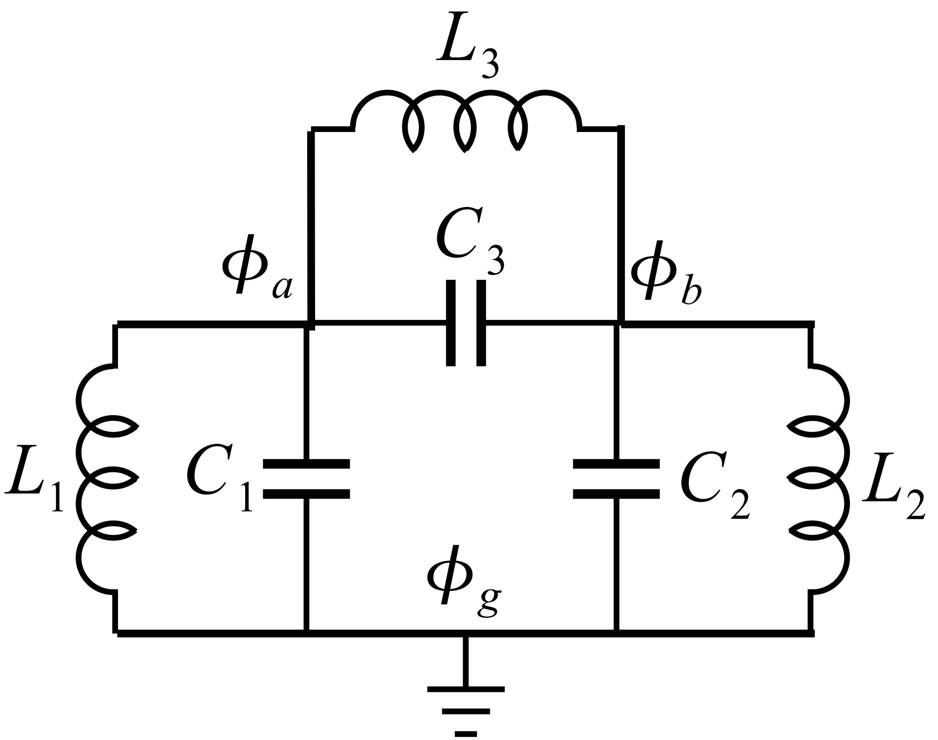


- Finally

$$C_1\ddot{\phi}_a + C_3(\ddot{\phi}_a - \ddot{\phi}_b) = \frac{\phi_a}{L_1} + \frac{\phi_a - \phi_b + \tilde{\Phi}}{L_3}$$

$$C_2\ddot{\phi}_b + C_3(\ddot{\phi}_b - \ddot{\phi}_a) = \frac{\phi_b}{L_2} + \frac{\phi_b - \phi_a - \tilde{\Phi}}{L_3}$$

Step 4: Lagrangian



- Finally

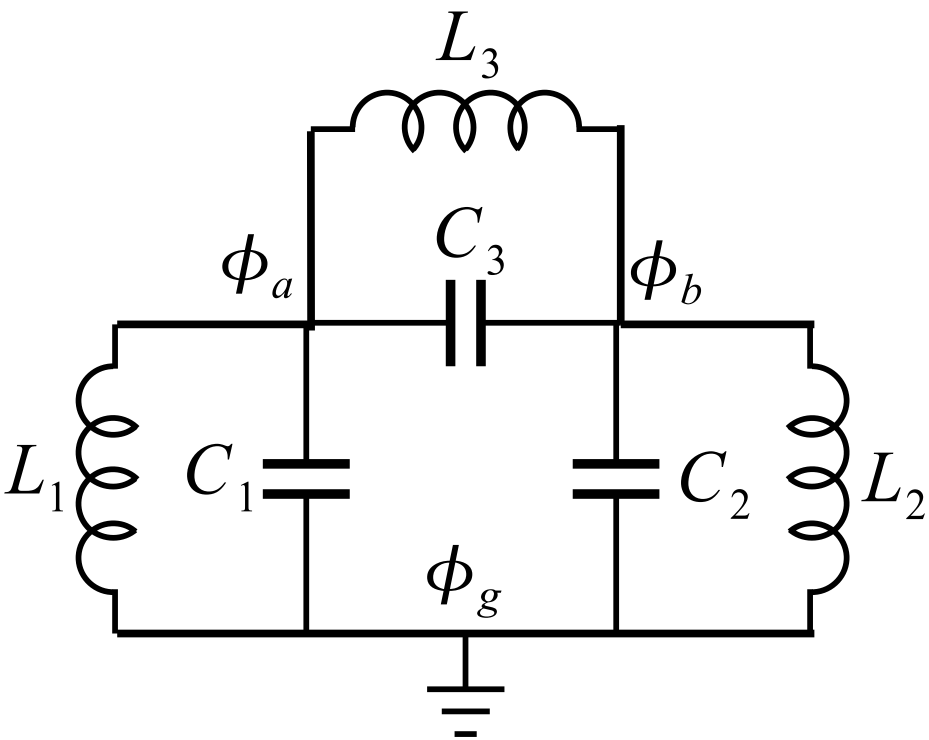
$$C_1\ddot{\phi}_a+C_3(\ddot{\phi}_a-\ddot{\phi}_b)=\frac{\phi_a}{L_1}+\frac{\phi_a-\phi_b+\tilde{\Phi}}{L_3}$$

$$C_2\ddot{\phi}_b+C_3(\ddot{\phi}_b-\ddot{\phi}_a)=\frac{\phi_b}{L_2}+\frac{\phi_b-\phi_a-\tilde{\Phi}}{L_3}$$

- This are Lagrange equations

$$L=C_1\frac{\dot{\phi}_a^{\,2}}{2}+C_2\frac{\dot{\phi}_b^{\,2}}{2C_2}+C_3\frac{(\dot{\phi}_a-\dot{\phi}_b)^2}{2}+$$
$$+\frac{\phi_a^2}{L_1}+\frac{\phi_b^2}{L_2}+\frac{(\phi_a-\phi_b+\tilde{\Phi})^2}{2L_3}$$

Step 4: Lagrangian



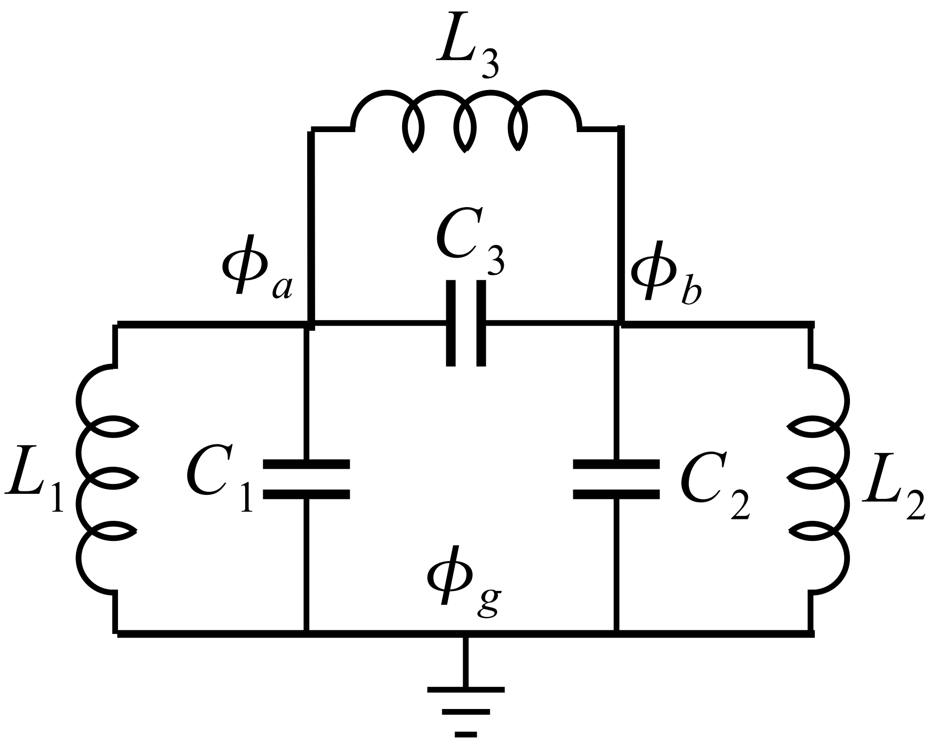
- From this Lagrangian

$$L = C_1 \frac{\dot{\phi}_a^{\,2}}{2} + C_2 \frac{\dot{\phi}_b^{\,2}}{2} + C_3 \frac{(\dot{\phi}_a - \dot{\phi}_b)^2}{2} + \\ - \left[\frac{\phi_a^2}{L_1} + \frac{\phi_b^2}{L_2} + \frac{(\phi_a - \phi_b + \tilde{\Phi})^2}{2 L_3} \right]$$

the equations are given as

$$\frac{d^2}{dt^2} \phi_x = \frac{\partial L}{\partial \dot{\phi}_x}$$

Step 4: Canonical variables



- We obtain the conjugate variables, charges, as

$$q_x = \frac{\partial L}{\partial \dot{\phi}_x}$$

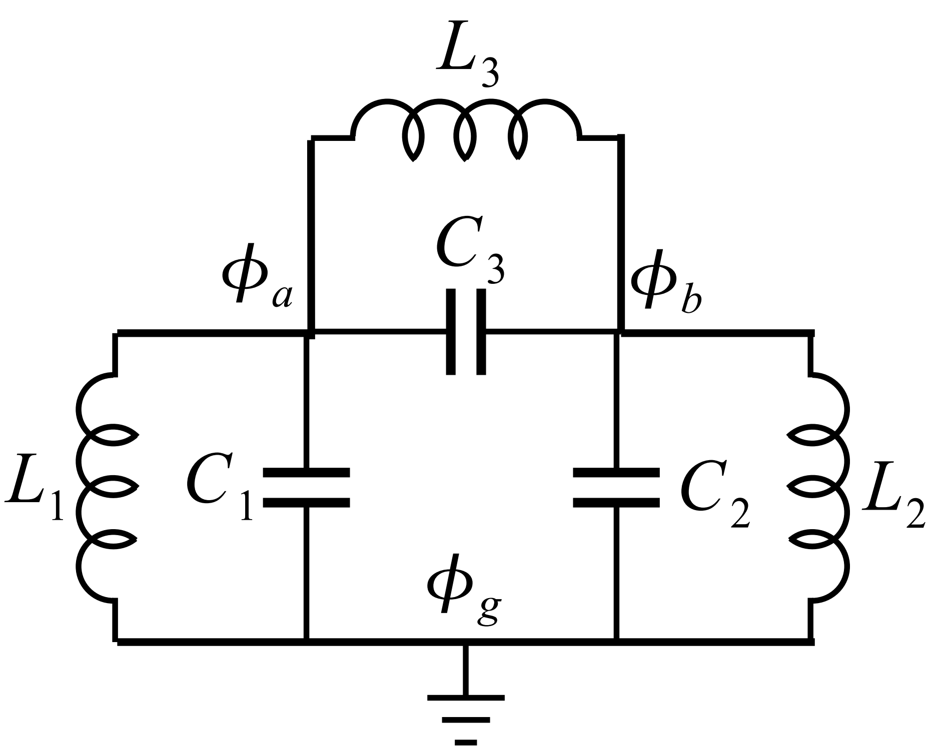
in our case

$$q_a = C_1 \dot{\phi}_a + C_3 (\dot{\phi}_a - \dot{\phi}_b)$$

$$q_b = C_2 \dot{\phi}_b - C_3 (\dot{\phi}_a - \dot{\phi}_b)$$

- Note that they mix both fluxes
- This will give rise to long range capacitive interactions

Step 5: Hamiltonian



- We obtain the conjugate variables, charges, as

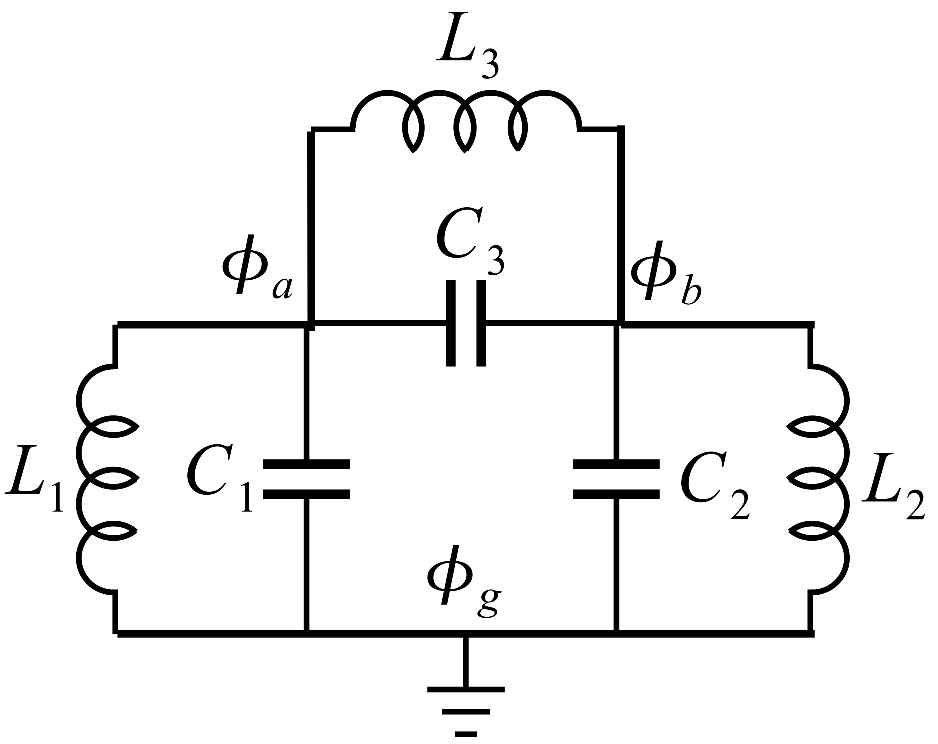
$$H = \sum_x q_x \dot{\phi}_x - L$$

in our case

$$H = \frac{q_a^2}{2C_a} + \frac{q_b^2}{2C_b} + \frac{(q_a - q_b)^2}{2C_{ab}} + \left[\frac{\phi_a^2}{L_1} + \frac{\phi_b^2}{L_2} + \frac{(\phi_a - \phi_b + \tilde{\Phi})^2}{2L_3} \right]$$

$$C_a = \frac{C_1 C_2 + C_1 C_3 + C_2 C_3}{C_2 + C_3}$$

Step 5: Quantization



- The charge and fux variables are imposed to have

$$[\phi_x, q_x] = i\hbar$$

- They are like position – momentum variables.
- If the Hamiltonian is quadratic, we can diagonalize to “normal modes”, like for ions.