Quantum resonators

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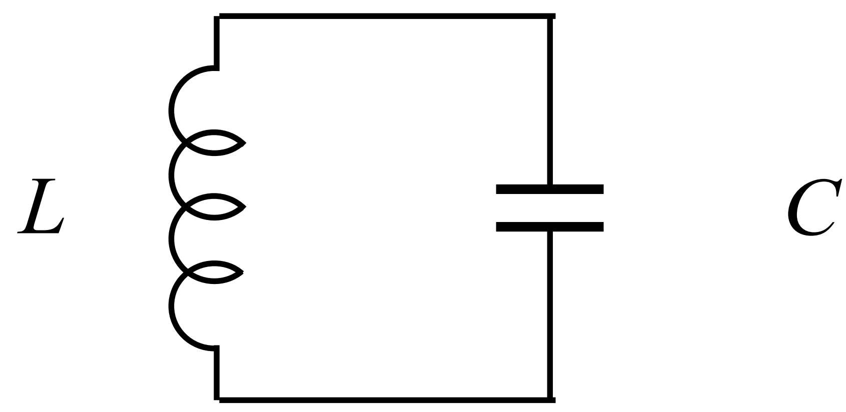
Quantum circuits

L-C circuit

- An inductor, L, adds “inertia” of the current to change.
- A capacitor, C, which reflects the potential energy accumulated by charges
- They together form a classical oscillator
- Two physical variables:
  - flux along inductor
  - accumulated charge
- Effective Hamiltonian H for the classical variables.



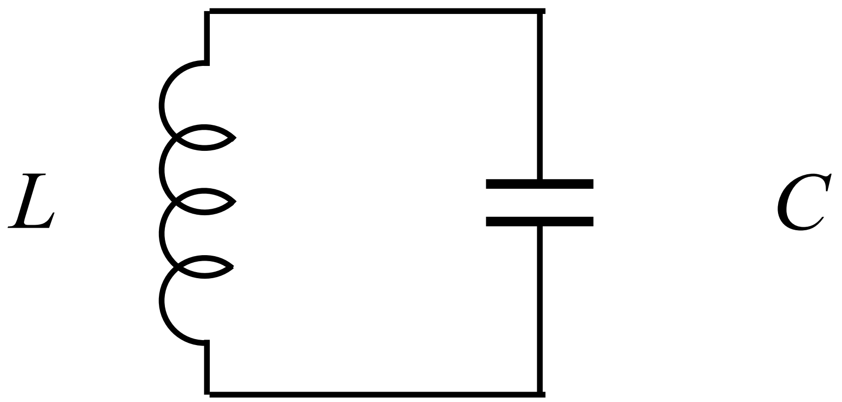
$$H = \frac{1}{2L}\Phi^2 + \frac{1}{2C}Q^2$$

$$\omega = 1/\sqrt{LC}$$

L-C circuit: quantization

- If we build the circuit using superconducting materials
  - We expect long lived oscillations.
  - We may reach very low temperatures.
  - Quantum effects should be apparent.
- The flux-charge variables become conjugate quantum observables

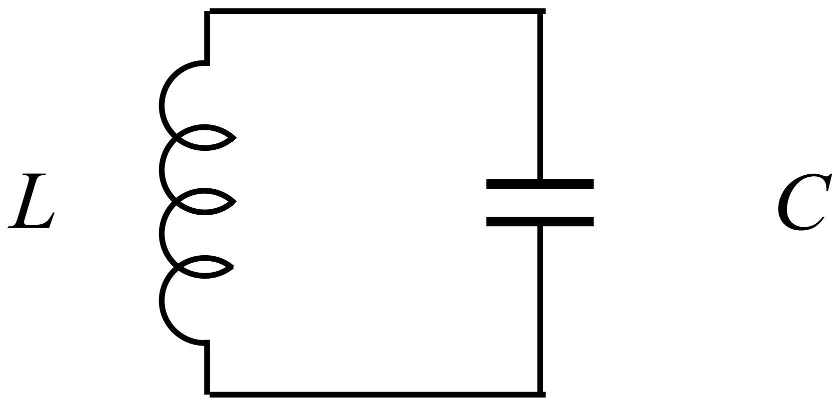
$$[\phi, Q] = i\hbar$$



$$H = \frac{1}{2L}\Phi^2 + \frac{1}{2C}Q^2$$
$$= \hbar\omega\left(a^+ a + \tfrac{1}{2}\right)$$
$$\omega = 1/\sqrt{LC}$$

L-C circuit: quantization

- Typical values
  - d = 10 µm
  - L = 0.1 nH
  - C = 1 pF
  - ω = 2π x 16 GHz
- Characteristic temperature
  - kB = 2.08 10^{10}
  - T = 0.8 K
- In dilution refrigerators
  - T = 20 mK



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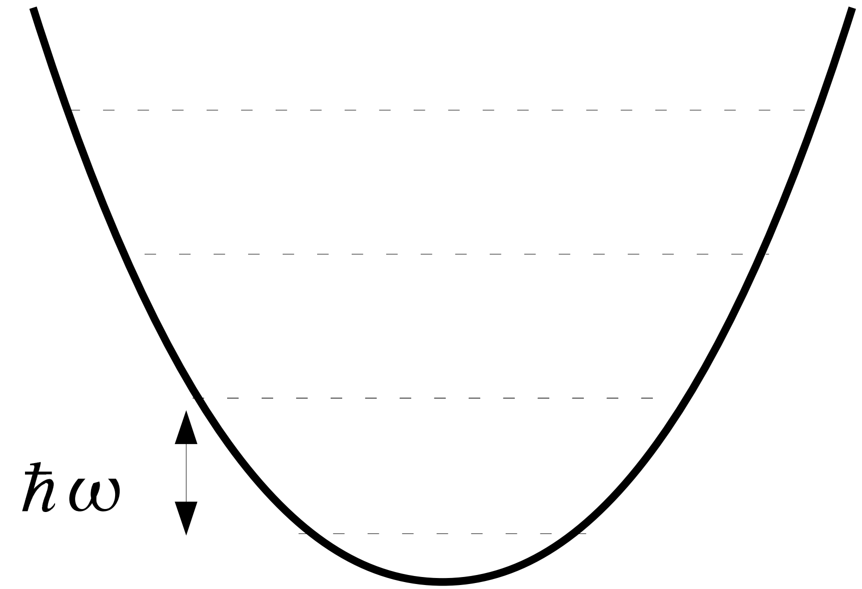

$$P(n) \sim \frac{\exp(-\hbar\omega n / k_B T)}{1 - \exp(-\hbar\omega / k_B T)}$$

L-C circuit: quantization

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- In dilution refrigerators
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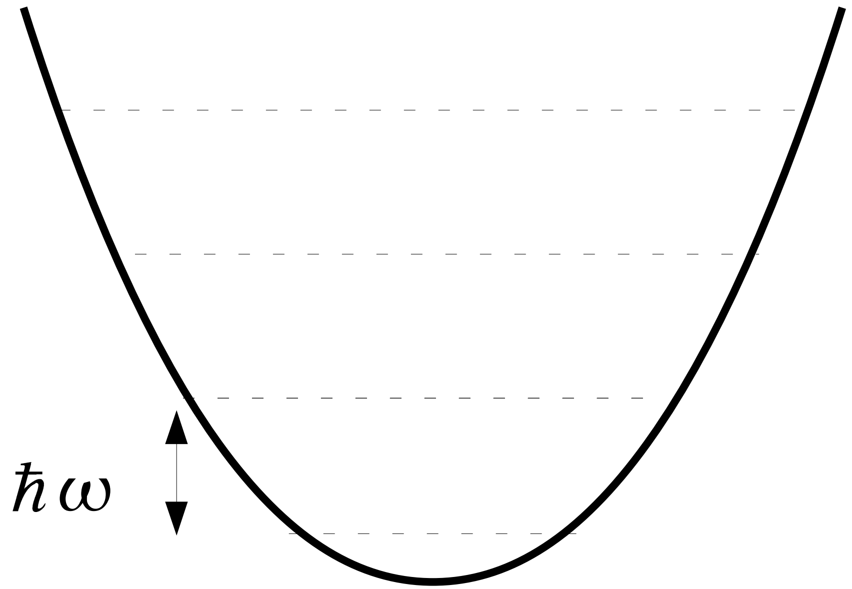
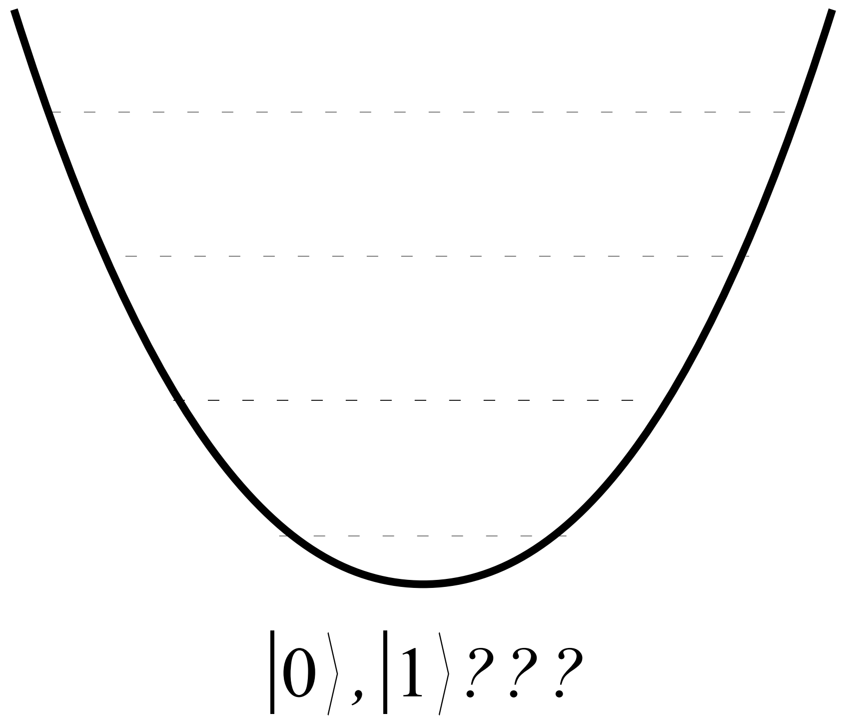
  - $P_{exc} \sim 0$



$$\langle n \rangle = \frac{1}{\exp(\beta \hbar \omega) - 1} \sim 10^{-18}$$

Enough for a qubit?

- Can we use similar quantum circuits for encoding qubits?



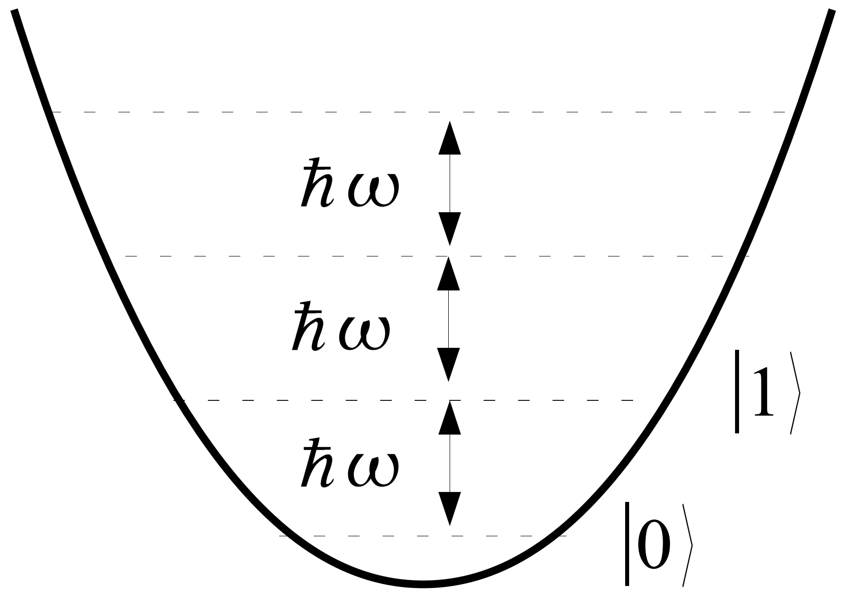
Enough for a qubit?

- Can we use similar quantum circuits for encoding qubits?
- We can encode a qubit

$$|0\rangle \equiv |vac\rangle$$

$$|1\rangle \equiv a^{+}|vac\rangle$$

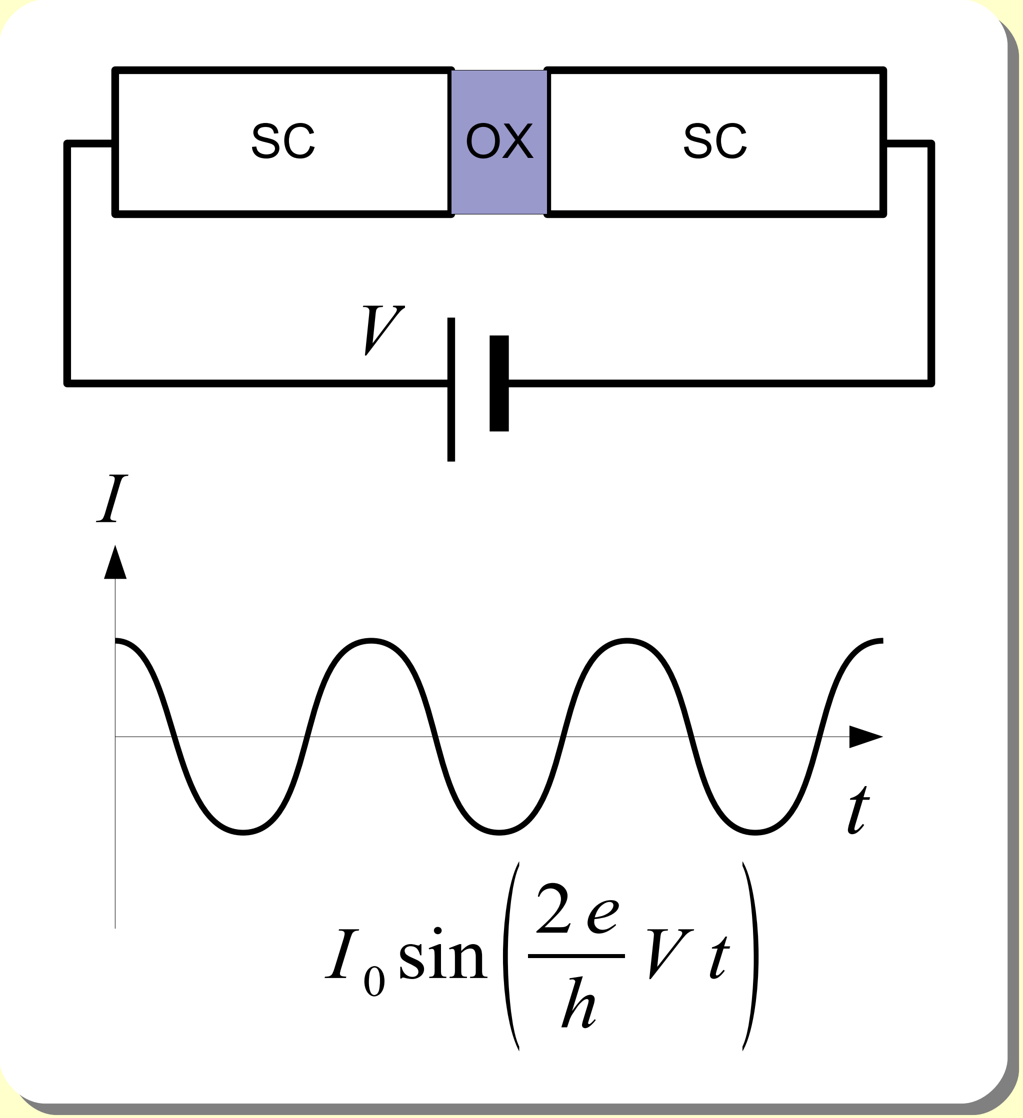
- But any external perturbation couples any two neighboring levels: no useful single-qubit and two-qubit unitaries.
- Remember the case of linear optics.
- Need some **nonlinearity**.



Josephson effect

Josephson effect

- Two superconductors joined by a thin insulating layer.
- An oscillating current appears.

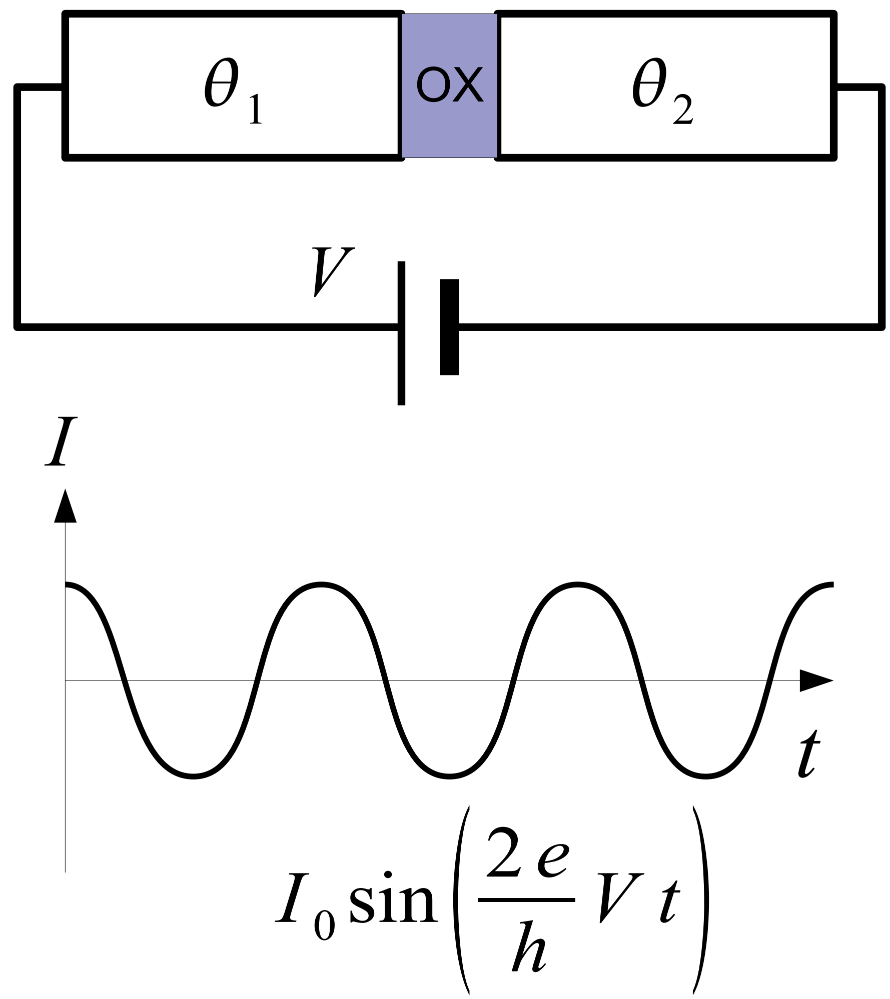


Josephson effect

- Two superconductors joined by a thin insulating layer.
- An oscillating current appears.
- Each superconductor has its own macroscopic phase, θ.
- Due to the different potential energy

$$\theta_2 - \theta_1 = \frac{2e}{h} V t = 2\pi \frac{\Delta\phi}{\Phi_0}$$

- A phase gradient gives rise to a quantum current.



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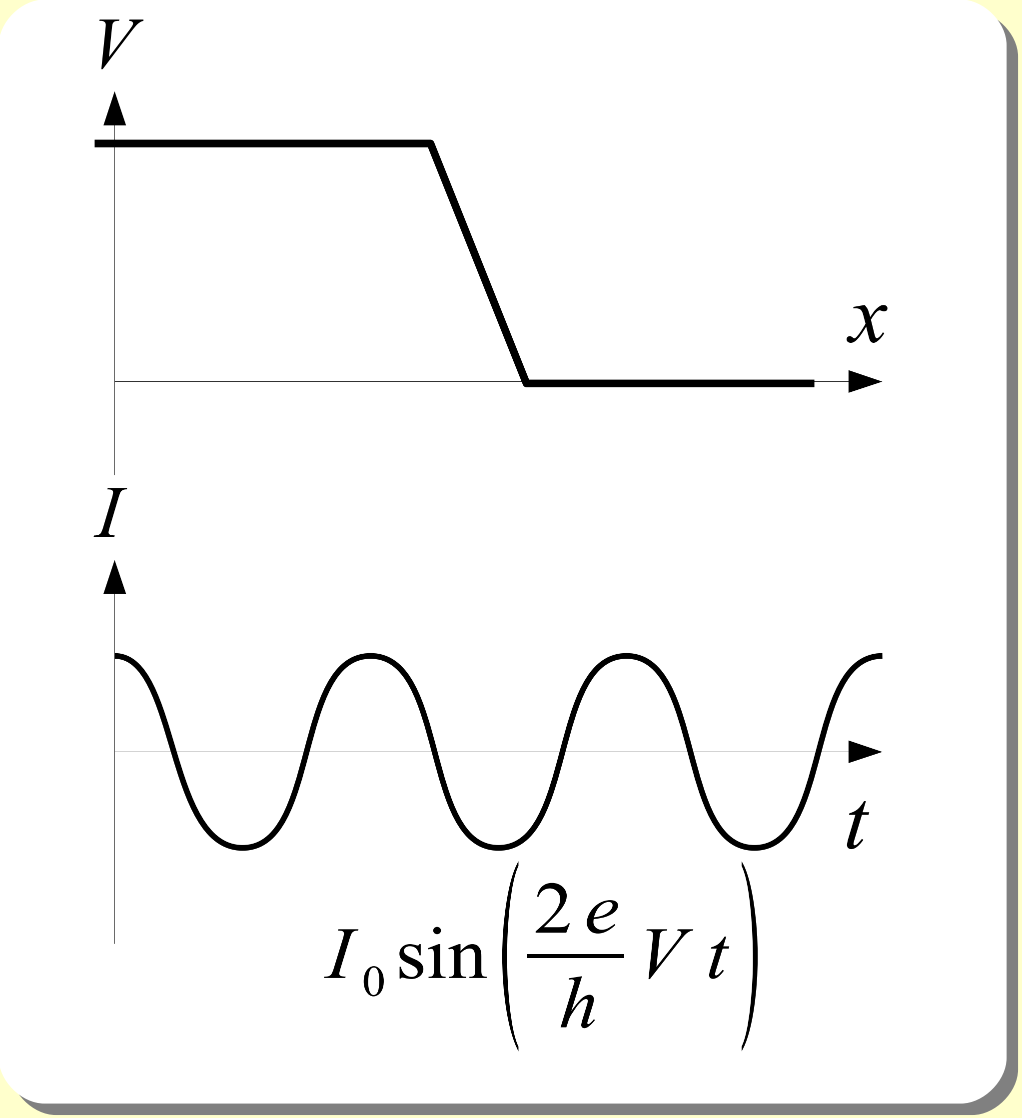

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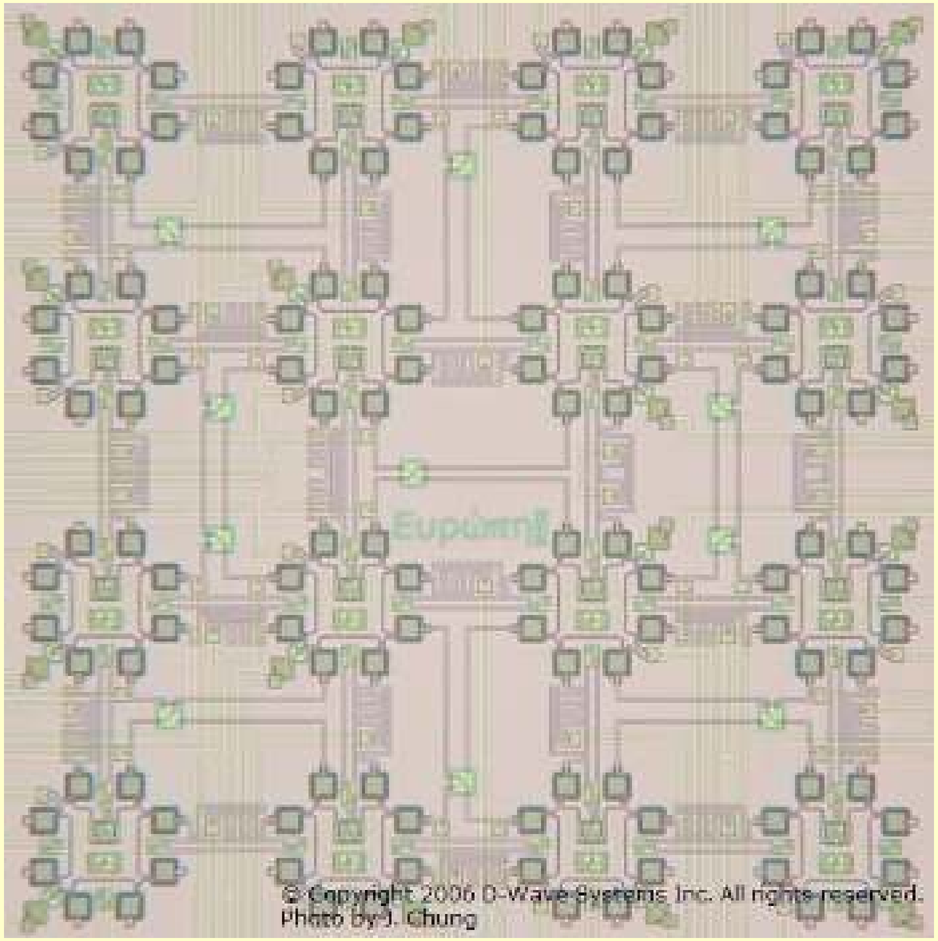
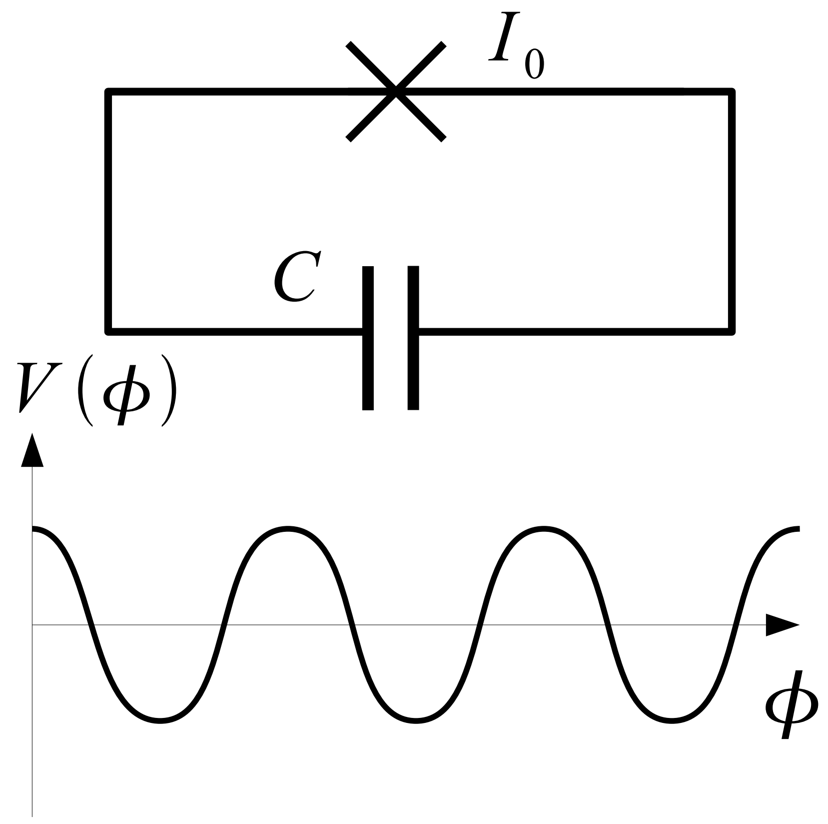


- We see evidence of quantum mechanics influencing dynamics of charge and current.
- We still have to find out
  - the actual equations
  - states it gives rise to
  - energy scales

Josephson oscillator

- Josephson junction provides a nonlinear inductance

$$I = I_0 \sin(2\pi\phi/\Phi_0)$$



Josephson oscillator

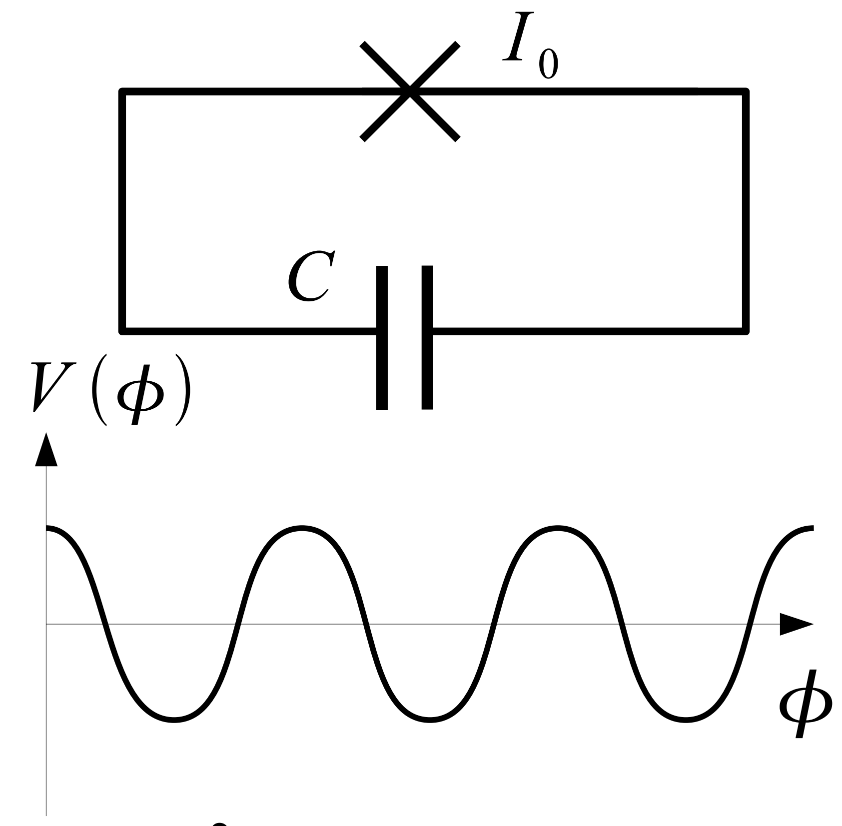
- Josephson junction provides a nonlinear inductance

$$I = I_0 \sin(2\pi\phi/\Phi_0)$$

- This can be associated to a Hamiltonian contribution

$$-\Phi_0 I_0 \cos(2\pi\phi/\Phi_0)$$

- The junction also has an associated capacitive energy
- The flux and charge remain conjugate variables.



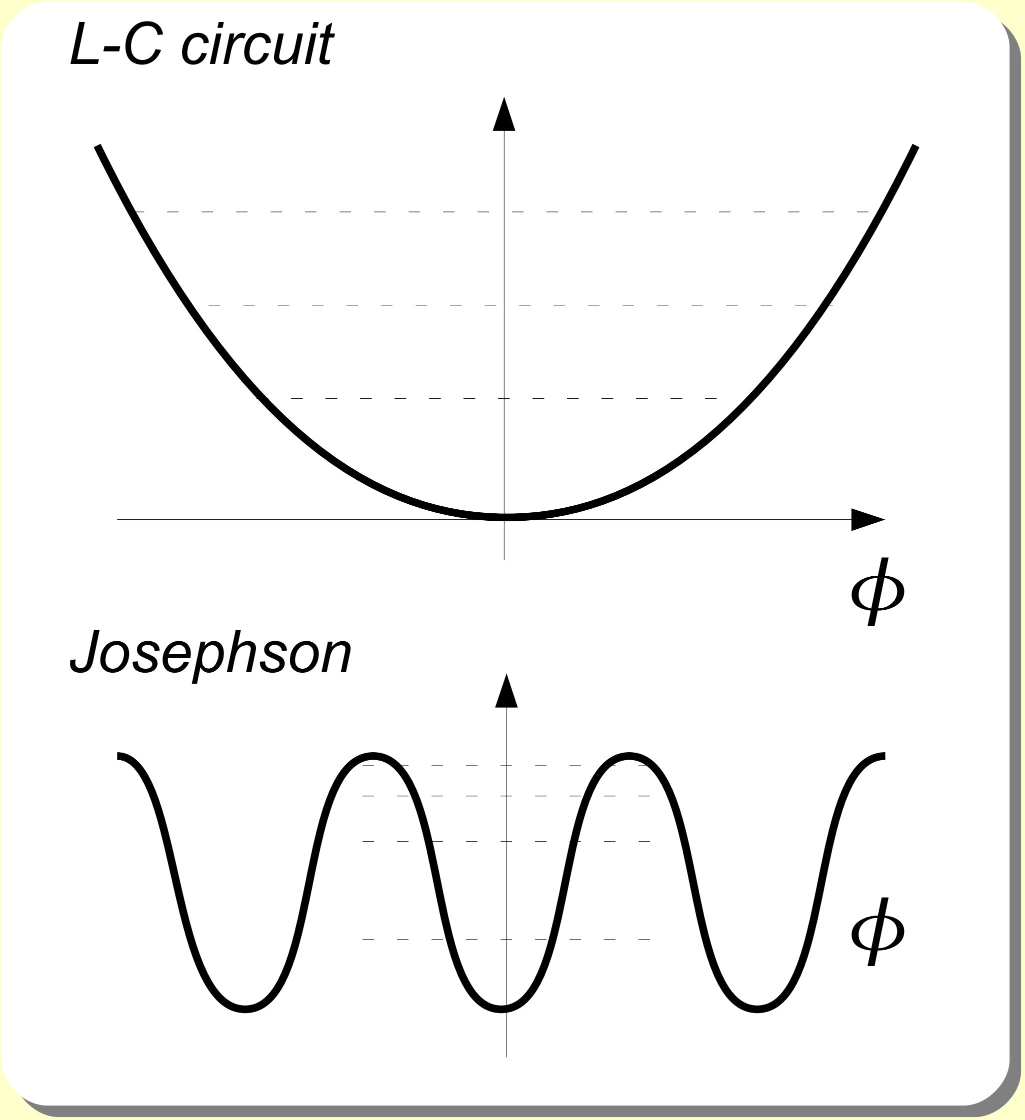
$$H = \frac{Q^2}{2C} - \Phi_0 I_0 \cos(2\pi\phi/\Phi_0)$$

Josephson oscillator

- We can model this as an **anharmonic** potential

$$V(\phi) = -\Phi_0 I_0 \cos(2\pi\phi/\Phi_0)$$

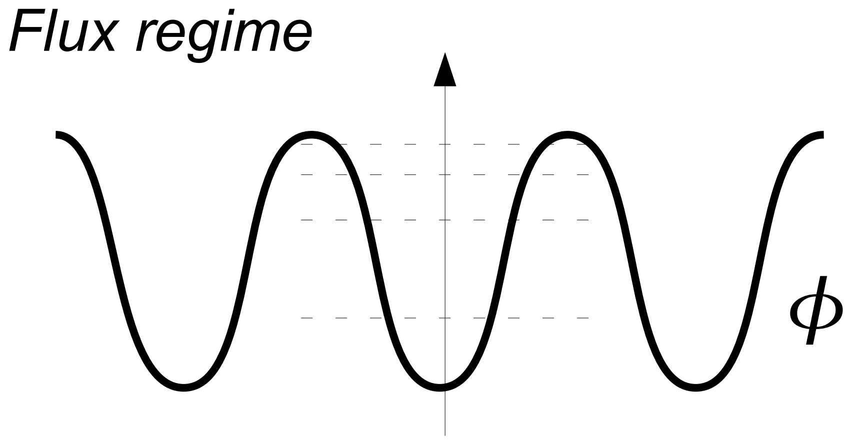
- The charge operator is like the momentum.
- Particle moves in this periodic potential
  - Similar to optical lattice
- We can probably resolve different levels.



Josephson oscillator

- We expect two dynamical regimes
  - Large capacitance, large Josephson energy

$$E_C = \frac{(2e)^2}{2C} \ll E_J = \Phi_0 I_0$$



Josephson oscillator

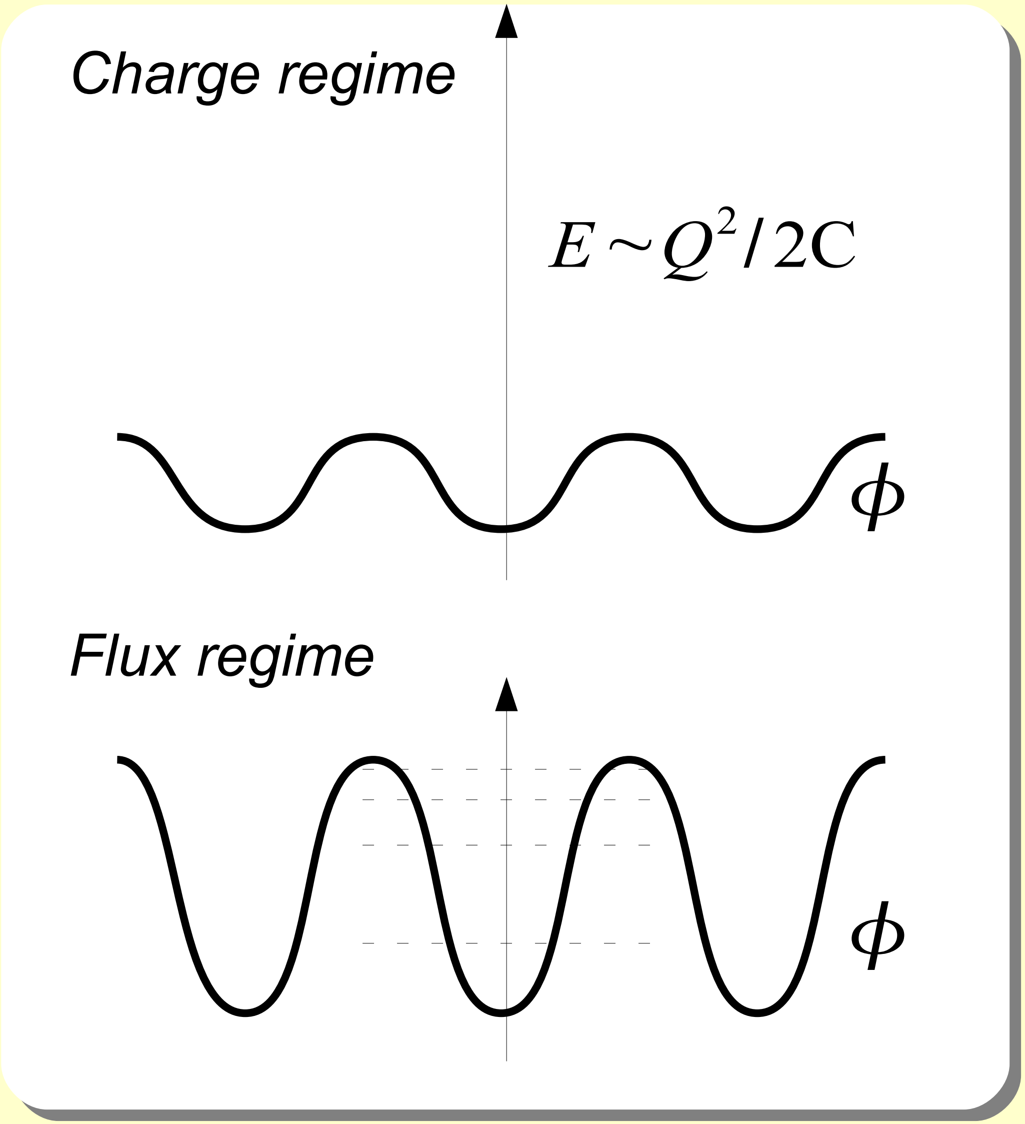
- We expect two dynamical regimes
  - Large capacitance, large Josephson energy

$$E_C = \frac{(2e)^2}{2C} \ll E_J = \Phi_0 I_0$$

  - Small capacitance, weak periodic potential

$$E_C > E_J$$

- These are like weak or strong lattices.



Controlling parameters

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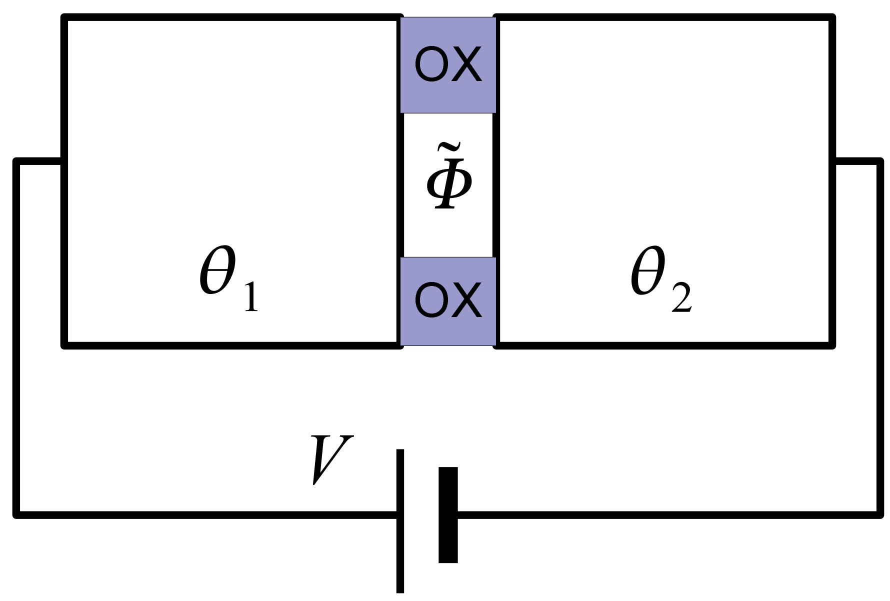
- We can make a hole through the junction, and let a magnetic field flow through.



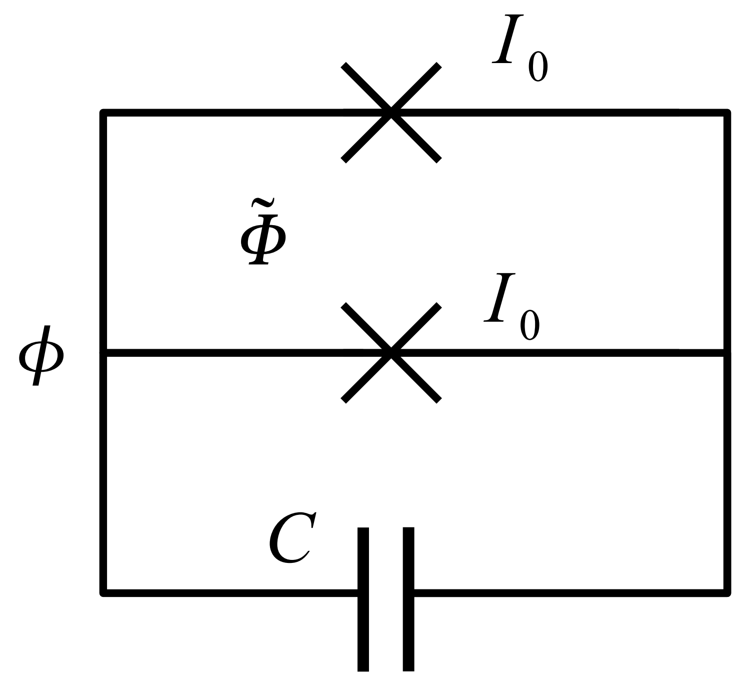
Controlling parameters

- We can make a hole through the junction, and let a magnetic field flow through.
- The equivalent circuit could be two parallel and similar junctions.

$$\Phi_0 I_0 \cos(2\pi\phi/\Phi_0) + \Phi_0 I_0 \cos[(2\pi\phi - \tilde{\Phi})/\Phi_0]$$

- Summing cosines

$$V(\phi) \sim \Phi_0 I_0 \cos(2\pi\tilde{\Phi}/\Phi_0) \times \times \cos(2\pi\phi/\Phi_0)$$



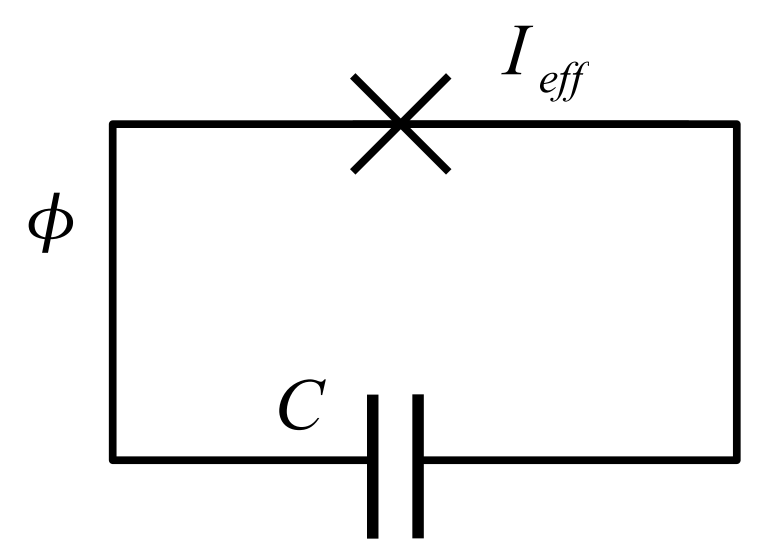
Controlling parameters

- Equivalently we have

$$V(\phi) \sim -\Phi_0 I_{eff} \cos(2\pi \phi / \Phi_0)$$

where the critical current can be modulated

$$I_{eff} = I_0 \cos(2\pi \tilde{\Phi} / \Phi_0)$$



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- A very sensitive magnetometer,

$$\Phi_0 \sim 2.07 \times 10^{-15}\, T\, m^2$$