Superconducting qubits

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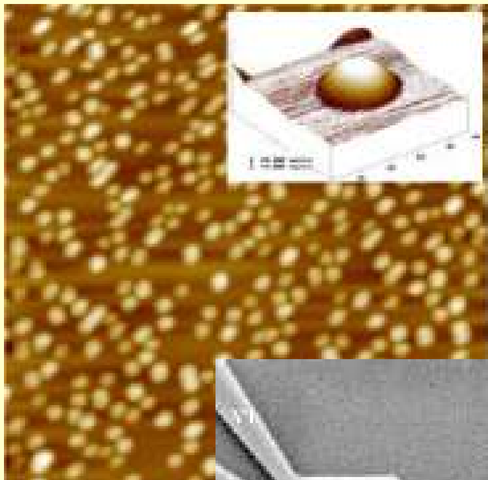
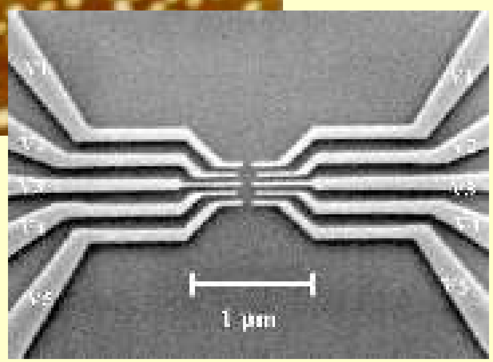
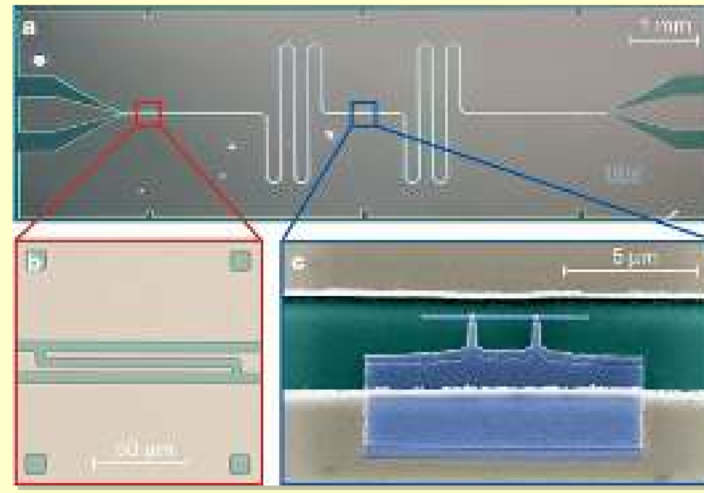
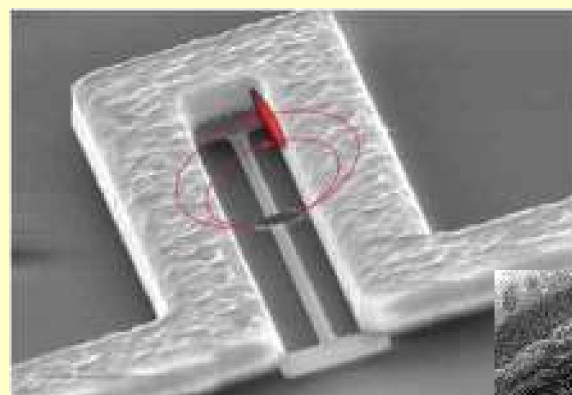
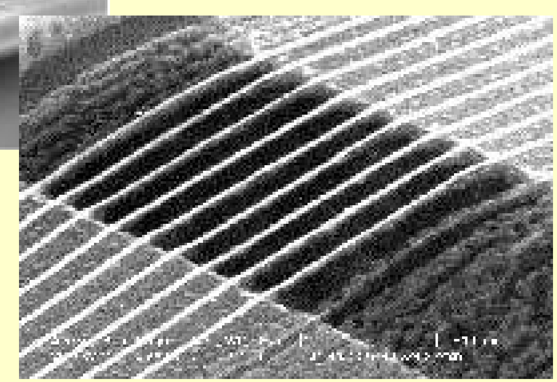
Mesoscopic quantum sytems





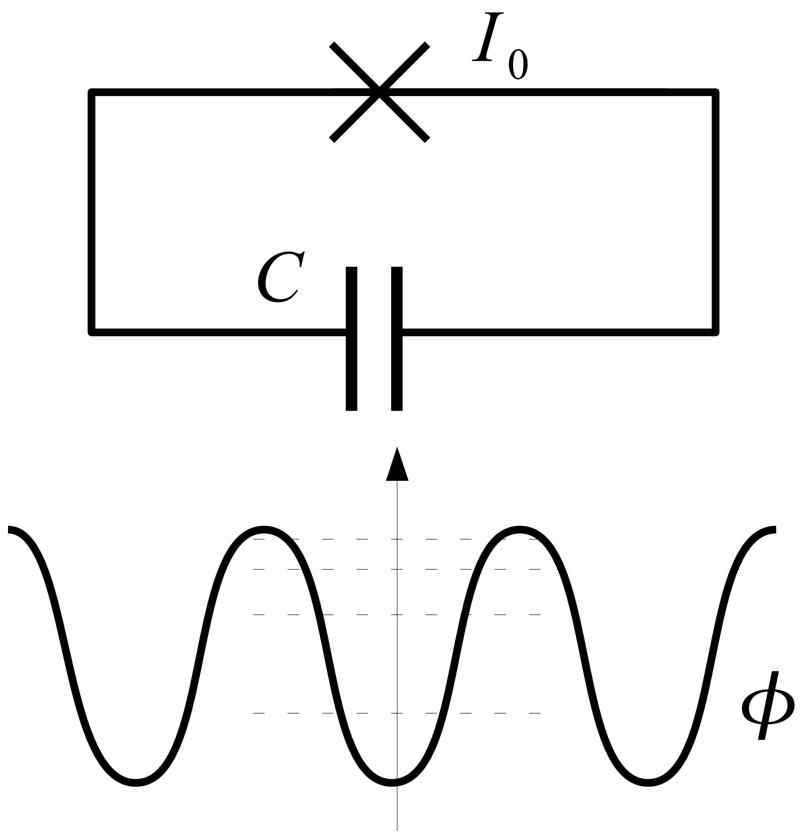






Quantum mechanical degrees of freedom reveal at the meso / macroscopic level.

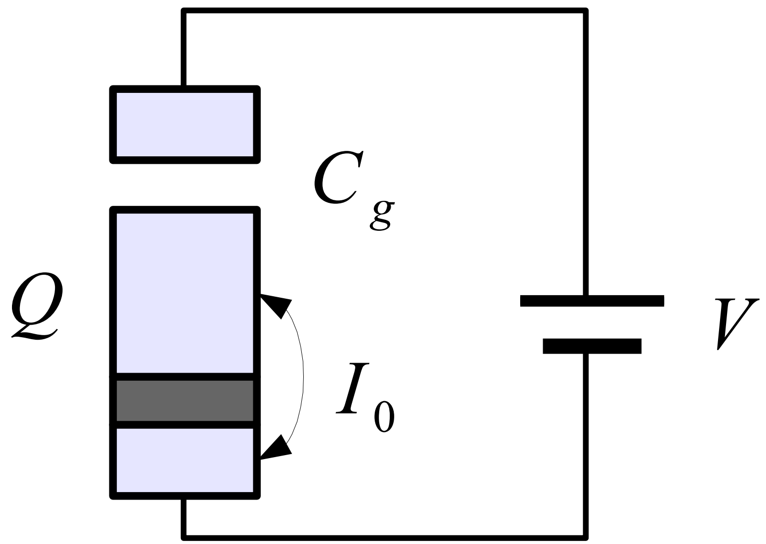
Superconducting qubits



$$H = \frac{Q^2}{2C} - \Phi_0 I_0 \cos\left(2\pi\phi/\Phi_0\right)$$

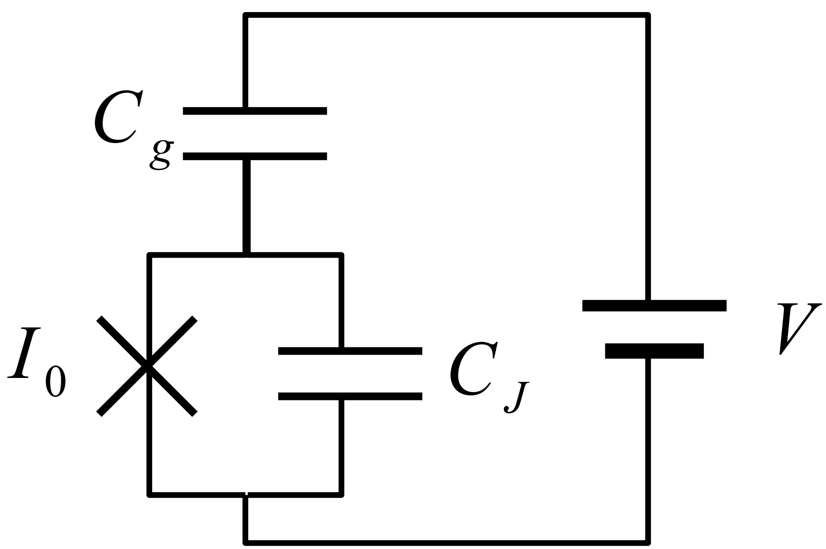
- We are going to use the anharmonicity to obtain spectroscopically resolved qubit states
  - Small capacitance, or **charge qubits**.
  - Large capacitance, with a current flowing through, or **phase qubits**.
  - Loops of junctions, or **flux qubits**.

Charge qubit



- Superconducting island with very small capacitance.
- Connected via a Josephson junction to a reservoir of charges.
- Influenced capacitively by an external potential.
  - Charges may tunnel in and out via junction.
  - The total quanta of charge can be tuned.

Charge qubit



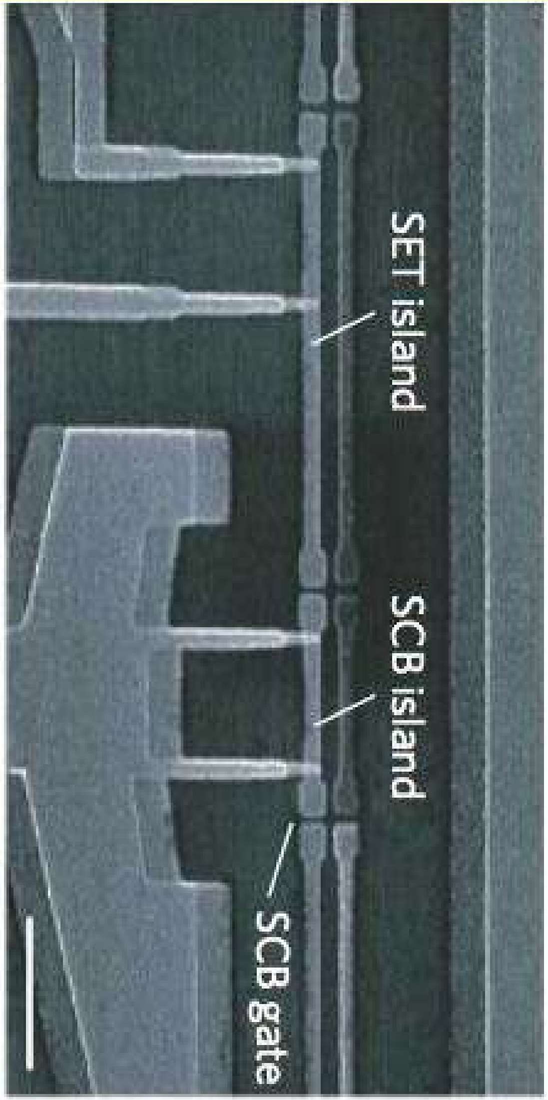
- Superconducting island with very small capacitance

$$H=\frac{(Q-Q_c)^2}{2C_\Sigma}-\Phi_0 I_0\cos\left(2\pi\phi/\Phi_0\right)$$

- Charge offset influenced by the external potential

$$Q_c=C_g V/2e$$

Charge qubit



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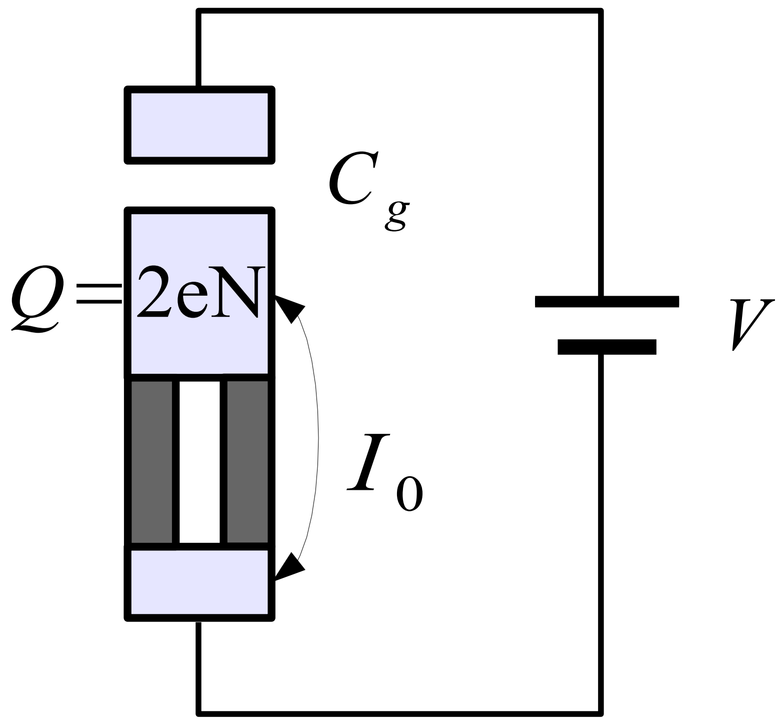
Charge qubit



- We introduce number states associated to the # of Cooper pairs $|N\rangle$

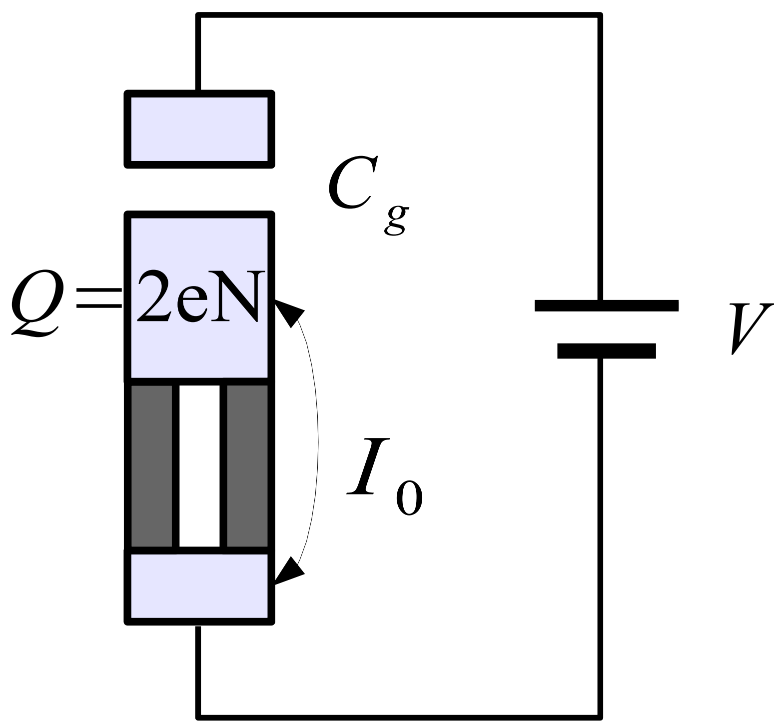
$$\hat{N} = \hat{Q}/2e$$

- Conjugate to this variable we find the supeconductor phase

$$\hat{\theta} = 2\pi\hat{\phi}/\Phi_0$$

$$[\hat{\phi}, \hat{Q}] = i\hbar \rightarrow [\hat{\theta}, \hat{N}] = i$$

Charge qubit



- Commutation relations

$$N\theta=\theta N-i$$

$$N e^{i\theta}=N\sum_n \frac{1}{n!}(i\theta)^n$$

$$=e^{i\theta}N-i\sum_n \frac{n}{n!}i(i\theta^{n-1})$$

$$=e^{i\theta}N+e^{i\theta}$$

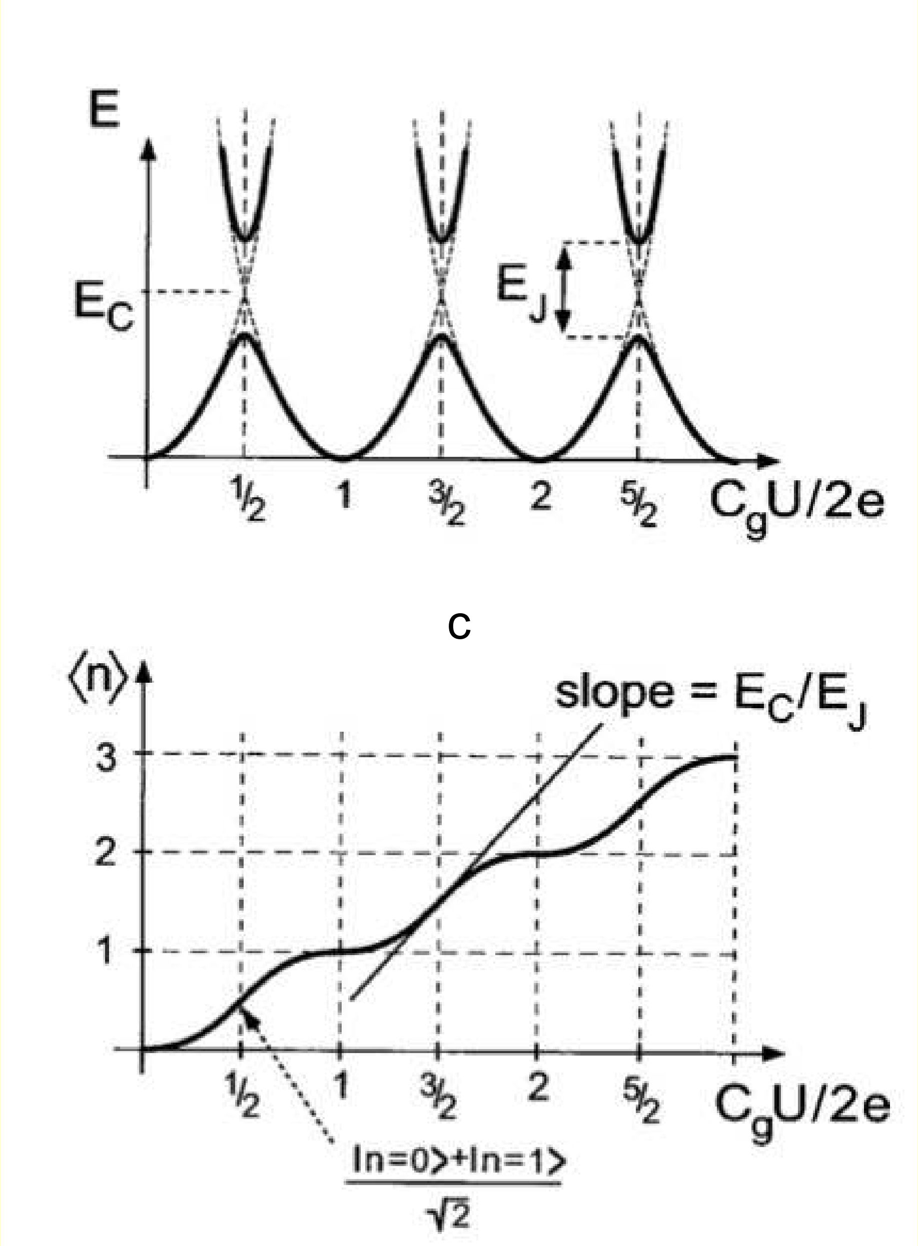
- That means

$$e^{i\theta}|N\rangle=|N+1\rangle$$

$$\cos(\theta)|N\rangle=\tfrac{1}{2}\left[|N-1\rangle+|N+1\rangle\right]$$

phase is the generator of displacements.

Charge qubit



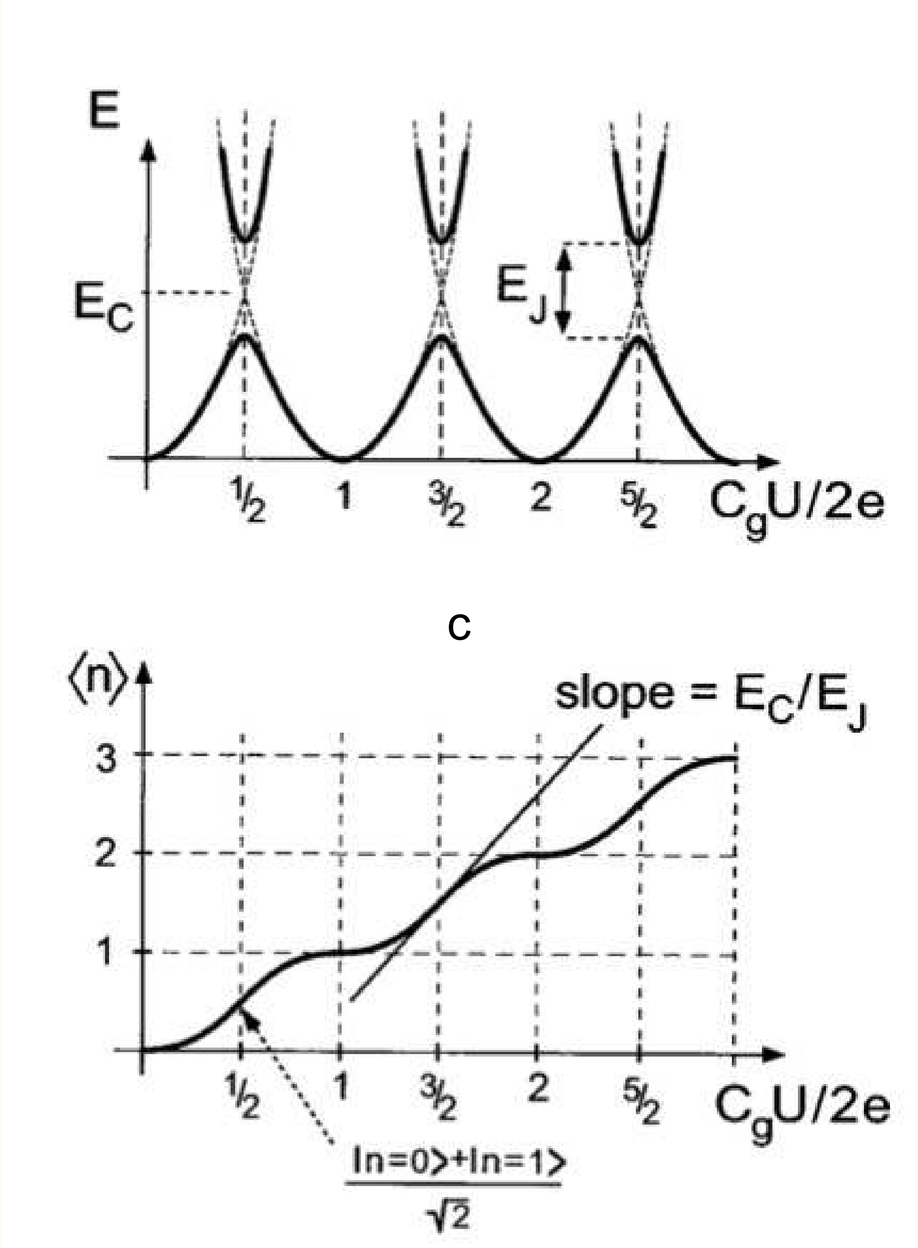
- Hamiltonian

$$H = \sum_n E_C (n - N_0)^2 |N\rangle\langle N| + - \sum_n E_J \left(|N+1\rangle\langle N| + H.c. \right)$$

$$E_C = \frac{(2e)^2}{C_\Sigma} \gg E_J = \Phi_0 I_0$$

- Dominant term is quadratic with a weak coupling among different occupation states.
- Since $N_0 = N_{eq} + C_g V / (2e)^2$ energy degeneracy points between pairs, N and N+1.

Charge qubit



- Around degeneracy

$$H \simeq \sigma_z V_{ext} + \sigma_x E_J$$
$$\equiv \tilde{\sigma}_x V_{ext} + \hbar \Omega \tilde{\sigma}_z$$

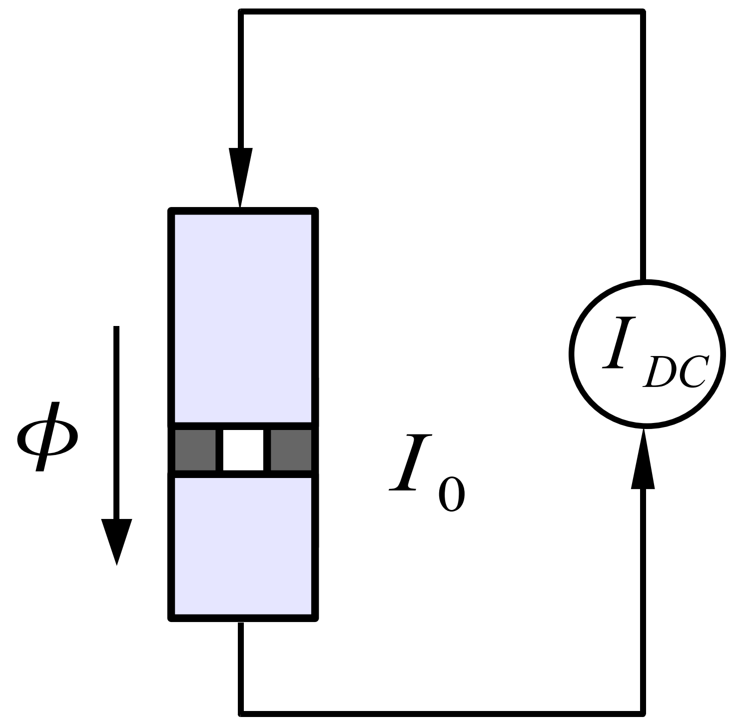
- Characteristically
  - Ω ~ 10 GHz
  - V ~ 50 – 100 MHz
- Qubit states are linear superpositions

$$|0\rangle \propto |N_{eq}\rangle - |N_{eq}+1\rangle$$
$$|1\rangle \propto |N_{eq}\rangle + |N_{eq}+1\rangle$$

Current biased JJ

Current biased JJ

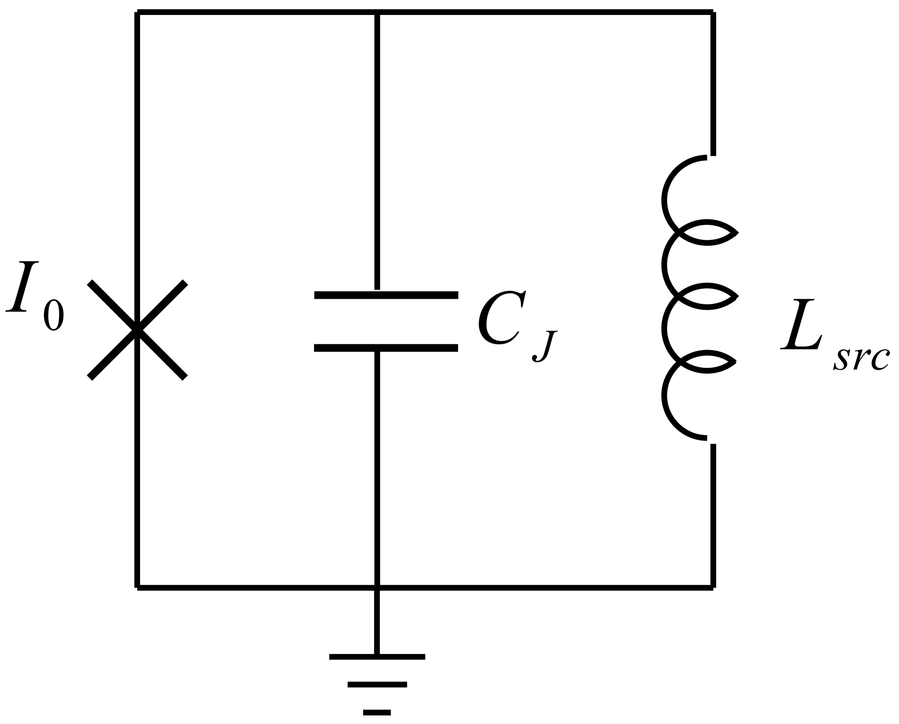
- We plug a JJ to a source of constant intensity.
- The junction now has a large capacitance
  - Nonlinear inductance is relevant.
- We expect the qubit states to be related to current states.



Current biased JJ

- Write the equivalent circuit.
- The big inductance on the right is chosen so that it mimics a large DC source

$$\tilde{\phi}/L = I_{DC}$$



Current biased JJ

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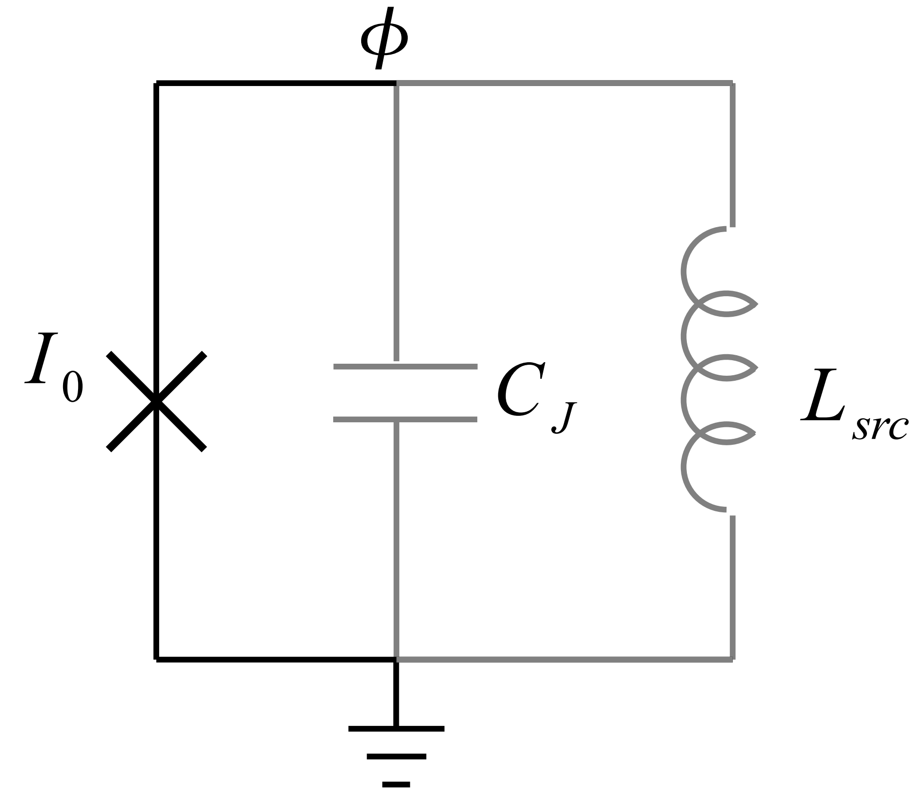
$$\tilde{\phi}/L = I_{DC}$$

- Identify ground and a set of open paths going through inductors to every vertex.
- Assign a flux to every reached vertex.



Current biased JJ

- Write the equivalent circuit.
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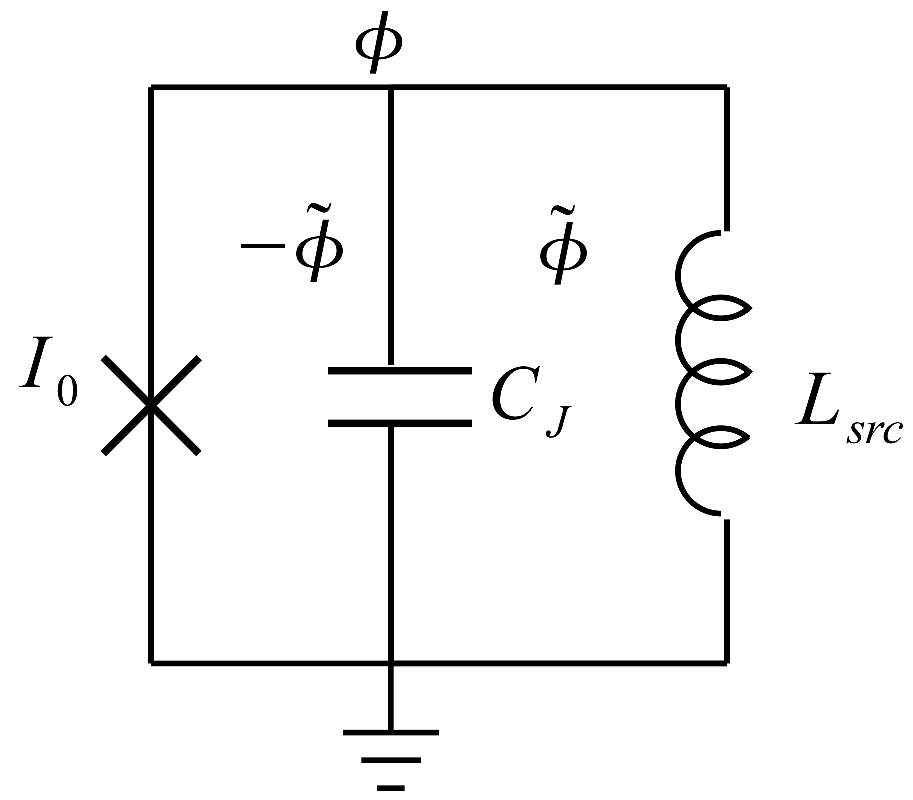
$$\tilde{\phi}/L = I_{DC}$$

- Identify ground and a set of open paths going through inductors to every vertex.
- Assign a flux to every reached vertex.
- Identify static fluxes in loops.



Current biased JJ

- Write down the Lagrangian

$$L = \sum_c \frac{1}{2} C_c \dot{\phi}_c^2 - \sum_i U(\phi_i)$$

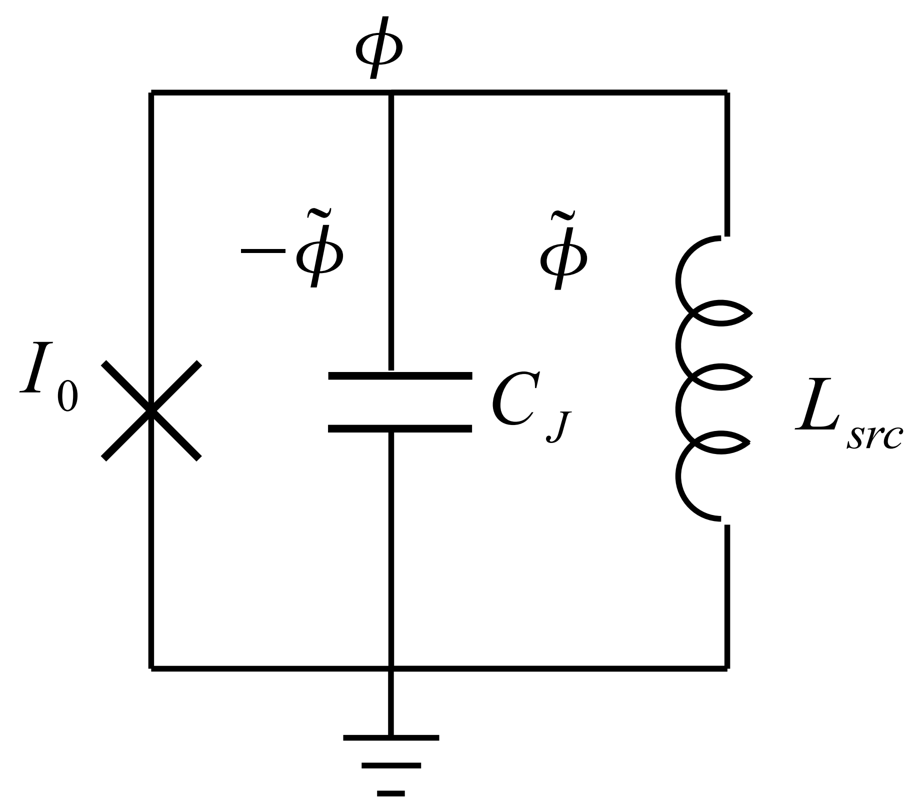
where ϕ_c, ϕ_i denote the fluxes along branches.

- In our example

$$L = \frac{1}{2} C_J \dot{\phi}^2 - (\phi - \tilde{\phi})^2 / L_{src}$$
$$- \Phi_0 I_0 \cos(\phi / \Phi_0)$$

where we approximate

$$\frac{d}{dt} \tilde{\phi} \sim 0$$



Current biased JJ

- We can further approximate the Lagrangian

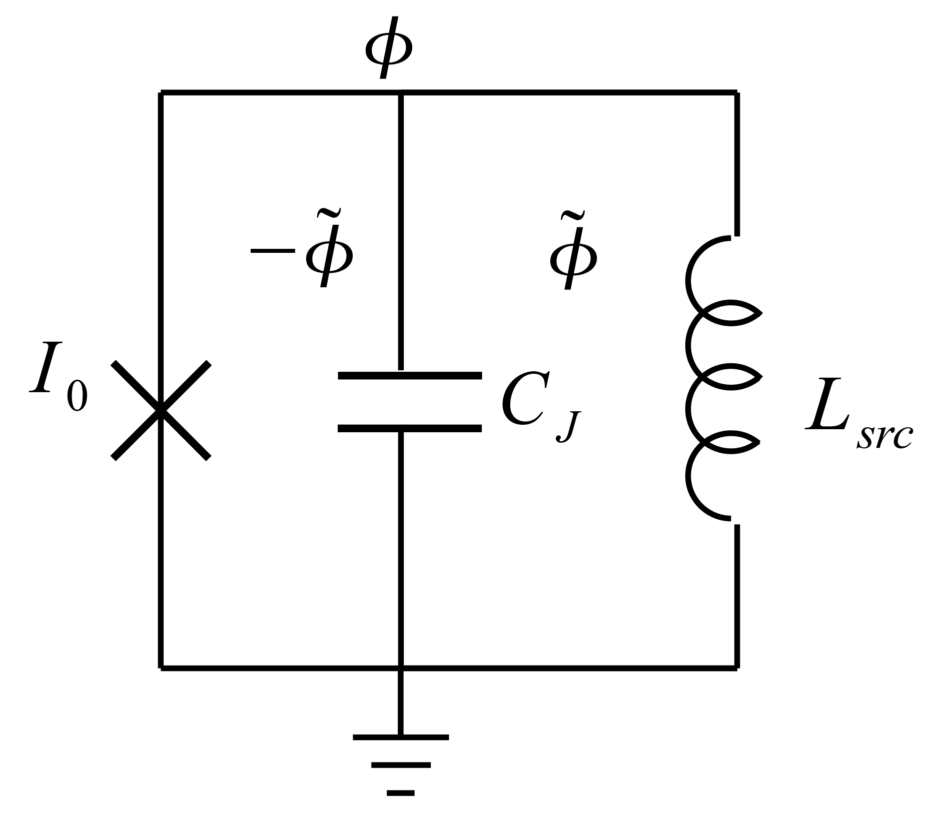
$$L = \frac{1}{2} C_J \dot{\phi}^2 - \phi \tilde{\phi} / L_{src}$$
$$- \Phi_0 I_0 \cos(2\pi \phi / \Phi_0)$$

- Now we introduce the conjugate variable "q"

$$q = \frac{\partial L}{\partial \dot{\phi}} = C_J \dot{\phi}$$

and compute the Hamiltonian

$$H = \sum_i q_i \dot{\phi}_i - L$$



Current biased JJ

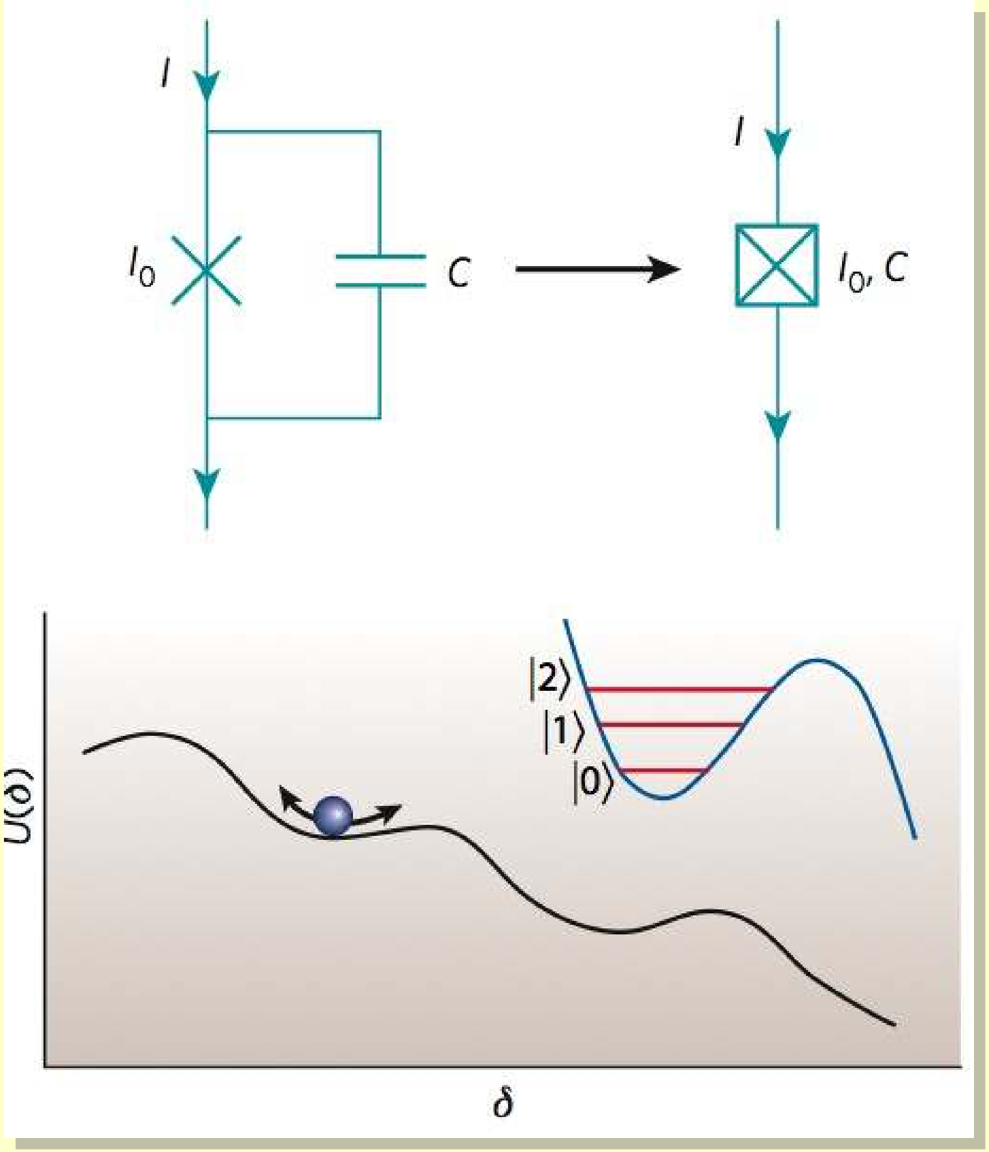
- The result is

$$H=\frac{1}{2C_J}q^2-\phi\tilde{I}_{DC}$$
$$-\Phi_0 I_0 \cos(2\pi\phi/\Phi_0)$$

- Remember that "q" is conjugate to the flux

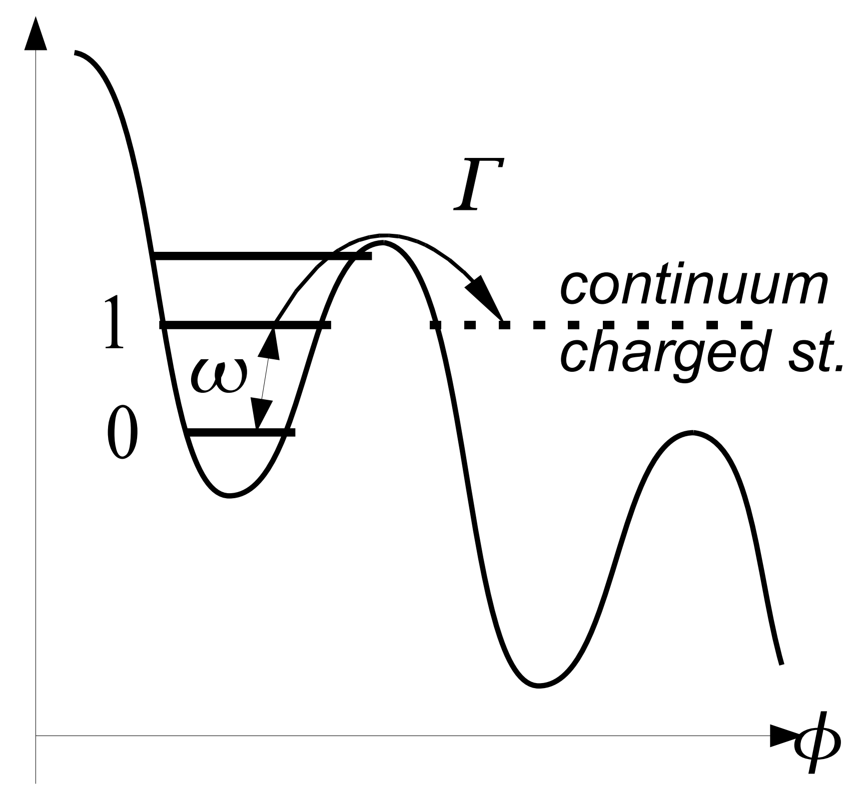
$$q\sim -i\hbar\frac{\partial}{\partial\phi}$$

- Everything is equivalent to a particle moving in a washboard potential



Current biased JJ

- The qubit states are two metastable phase states.
- These states have long lifetimes.
- Lifetimes reduce by 1000 from state to the other, Γ_2~100 Mhz
- Qubit states have larger energy splitting than excited states
  - Spectroscopically resolved



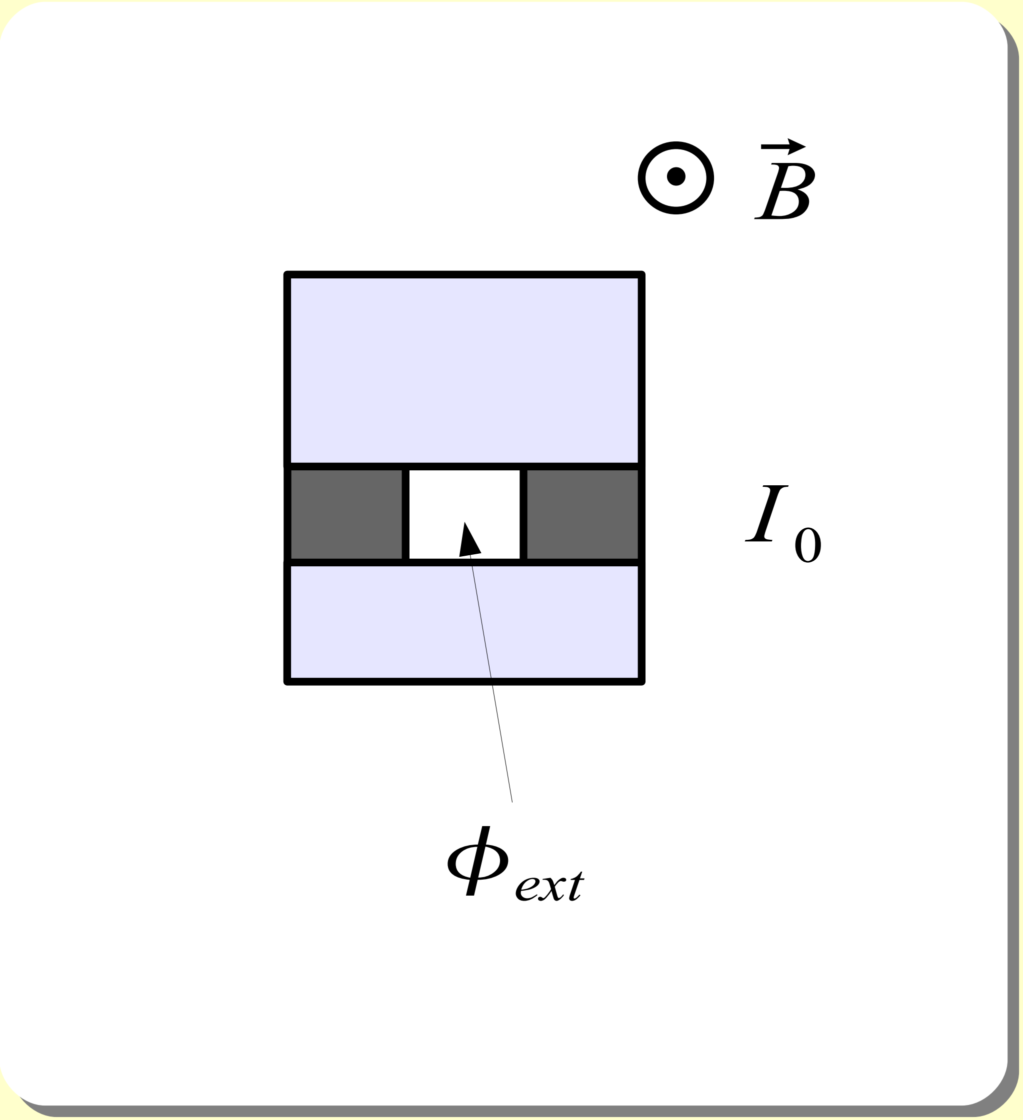
Single-qubit operations

Single-qubit operations

- External potentials induce coupling between qubit states

$$H = \tilde{\sigma}_x V_{ext} + \hbar \Omega \tilde{\sigma}_z$$

- Via magnetic fields, we can tune the JJ critical current, changing the energy levels.
- With σ_x and σ_z we have enough for approximating arbitrary local unitaries
  - Solovay- Kitaev



Important numbers

- Rather fast single-qubit ops

$$\Omega \sim 2 - 10\, GHz$$

with rather long lifetimes

$$1/\Gamma \sim 1\, \mu s$$

improving every day.

- Fast two-qubit operations

$$J \sim 100\, MHz$$

with room for improvement

$$J \sim 1 - 10\, GHz$$